Exercise (3.8.1 from textbook)

Given the following DFA:

(a) give all regular expressions $R_i^0$
(b) simplify $R_i^0$, simplifying as much as possible
(c) give a regular expression for the language of the automaton
(d) starting from the diagram above, give a regular expression for the language by eliminating state $q_2$

Solution (c):

\[
R_{ij}^{(k)} = R_{ij}^{(k-1)} + \frac{R_{ik}^{(k-1)} \cdot R_{kj}^{(k-1)}}{R_{ik}^{(k-1)}}
\]

\[
R_{ii}^{(0)} = 1 + \epsilon
\]
\[
R_{ii}^{(1)} = 0
\]
\[
R_{ii}^{(2)} = \emptyset
\]
\[
R_{ii}^{(3)} = 1
\]
\[
R_{ii}^{(4)} = \epsilon
\]
\[
R_{ii}^{(5)} = 0
\]
\[
R_{ii}^{(6)} = A
\]
\[
R_{ii}^{(7)} = 0 + \epsilon
\]
Solution (e)

Elimination of state $q_2$:

$$ E = ((101) 00 (010)^* 11)^* 00 (0110)^* $$

final regular expression

Reminder:

leads to expression

$$(R^* + SU^T)^* SU^*$$
Exercise (3.2.4 from textbook)

Convert the following regular expressions to $\varepsilon$-NFAs:

(a) $0^*$
(b) $(0+1)0$
(c) $00(0+1)^*$

Solution (a):

\[\text{Final automaton:}\]
### Solution (b)

![Diagram](image)

Except 0 and 1 respectively.

Starting and accepting states:

- ![Diagram](image)
  Accepts 0+1

Having final state of the first automaton and initial state of the second (see Exercise 3.2.7 from textbook) for concatenation:

- ![Diagram](image)

### Solution (c)

![Diagram](image)

Accepts 00

- ![Diagram](image)
  Accepts 0+1

From the latter, we add ε-transitions from starting state to accepting state and vice versa (exercise 3.2.7 from textbook):

- ![Diagram](image)
  Accepts (0+1)*

- ![Diagram](image)
  Accepts ε(0+1)*

We can make the two rightmost states (with respect to the figure) collapse, thus eliminating the residual ε-transitions:

- ![Diagram](image)
\[ \delta_\sigma(q_2,0) = \text{Eclose}(\delta(q_2,0)) = \emptyset \]
\[ \delta_\sigma(q_3,1) = \text{Eclose}(\delta(q_3,1)) = \text{Eclose}(g_3) = \{q_4, q_3, q_2\} \]
\[ \delta_\sigma(q_0,0) = \text{Eclose}(\delta(q_0,0) \cup \delta(q_0,0)) = \text{Eclose}(\emptyset \cup \emptyset) = \emptyset \]
\[ \delta_\sigma(q_4,1) = \emptyset \text{ (as in previous step)} \]
\[ \delta_\sigma(q_5,0) = \emptyset \text{ (trivial)} \]
\[ \delta_\sigma(q_3,1) = \emptyset \text{ (trivial)} \]

Notice that it is extremely redundant.

By applying merging of states as in cases (b) and (c), we would have immediately obtained the following:

\[ \rightarrow 0 \rightarrow 0 \quad \text{accepts } 0 \]
\[ \rightarrow 1 \rightarrow 1 \quad \text{accepts } 1^* \]
\[ \rightarrow 0 \rightarrow 0 \quad \text{accepts } 01^* \quad \text{(no } \epsilon\text{-transitions)} \]