Exercise

Construct a DFA that accepts the language \( \{a^{2m}b\}, m \geq 0 \)

Exercise

Construct a DFA that accepts strings containing an even number of \( a \) or (not exclusive) an even number of \( b \), on the alphabet \( \{a, b\} \).

Exercise

Construct a NFA accepting strings on alphabet \( \{a, b\} \) in which the symbol before the last one is \( b \).
Exercise (2.3.4 from textbook)

Give non-deterministic finite automata that accept the following languages:

(a) strings over \{0, ..., 9\} such that the last digit has appeared before
(b) strings over \{0, ..., 9\} such that the last digit has not appeared before
(c) strings over \{0, 1\} such that there are two zeros separated by a number of digits that is a multiple of four (including 0)

solution

(a) \[ q_0 \xrightarrow{0, ..., 9} q_0, \quad q_0 \xrightarrow{0, 9} q_0, \quad q_0 \xrightarrow{9} q_f \]

(b) \[ q_0 \xrightarrow{\neq 0} q_0, \quad q_0 \xrightarrow{0} q_0, \quad q_0 \xrightarrow{\neq 9} q_0, \quad q_0 \xrightarrow{9} q_0, \quad q_0 \xrightarrow{\neq 9} q_0, \quad q_0 \xrightarrow{9} q_f \]

We use states \( q_i \) with \( 0 \leq i \leq 9 \) to guess that final digit is \( i \)
Exercise

Construct a NFA on alphabet \{a, b\} accepting strings that end with \(ba, bb\) or \(baa\). Construct a DFA that is equivalent to it.

Solution

We write the transition function \(\delta\) of the required DFA directly, in a sort of breadth-first visit of the automaton.

\[
\begin{align*}
\delta([q_0], a) &= [q_0] \\
\delta([q_0], b) &= [q_0, q_1] \\
\delta([q_0, q_1], a) &= [q_0, q_2, q_3] \\
\delta([q_0, q_1], b) &= [q_0, q_1, q_2] \\
\delta([q_0, q_2, q_3], a) &= [q_0, q_2] \\
\delta([q_0, q_2, q_3], b) &= [q_0, q_1] \\
\delta([q_0, q_2], a) &= [q_0, q_2] \\
\delta([q_0, q_2], b) &= [q_0, q_1, q_3] \\
\delta([q_0], a) &= [q_0] \\
\delta([q_0], b) &= [q_0, q_1]
\end{align*}
\]

Final DFA:
Exercise

Prove that for every regular language $E$ we have $E^* = (E^*)^*$.

Solution

We need to prove both inclusions $E^* \subseteq (E^*)^*$ and $(E^*)^* \subseteq E^*$.

$E^* \subseteq (E^*)^*$ trivial

$(E^*)^* \subseteq E^*$

Consider $w \in (E^*)^*$. We want to prove that $w \in E^*$.

We know that

$$w = w_1 \cdots w_m$$

with $w_i \in E^*$, $1 \leq i \leq m$, $m \in \mathbb{N}$

On the other hand, for all $i \in \{1, \ldots, m\}$:

$$w_i = w_{i_1} \cdots w_{i_{m_i}}$$

with $w_{i_j} \in E$, $1 \leq j \leq m_i$, $m_i \in \mathbb{N}$.

Therefore

$$w = (w_{i_1} \cdots w_{i_{m_1}}) \cdots (w_{i_{m_1}} \cdots w_{i_{m_2}}) \cdots (w_{i_{m_1}} \cdots w_{i_{m_m}})$$

Since $w$ is a concatenation of strings of $E$, the thesis follows.
Exercise (2.3.5 from textbook)

**Base step:** $|w| = 1$

Let $w = a$, $a \in \Sigma$.

We have $\hat{\delta}_N(q_w) = \hat{\delta}_N(q, a) = [\delta_b(q, a)] = \{p\}$ by construction.

**Inductive step:** $|w| > 1$

Let $w = xa$, $x \in \Sigma^*$, $a \in \Sigma$.

We have by definition:

$p = \hat{\delta}_b(q, w) = \hat{\delta}_b(q, xa) = \delta_b(\hat{\delta}_b(q, x), a) = \delta_b(r, a)$

where we have denoted $r = \hat{\delta}_b(q, x)$.

By induction hypothesis we know

$\hat{\delta}_N(q, x) = [\delta_b(q, x)] = \{r\}$

Again, by definition

$\hat{\delta}_N(q, w) = \hat{\delta}_N(q, xa) = \bigcup_{h \in \delta_b(q, x)} \hat{\delta}_N(h, a) = \bigcup_{h \in \{r\}} \hat{\delta}_N(h, a) = \hat{\delta}_N(r, a) = \{p\}$ (by induction hyp.)

By construction we have

$\delta_N(r, a) = \{\delta_b(r, a)\} = \{p\}$ q.e.d.