Planning Problems for Graph Structured Data in Description Logics

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Motivation The complex structure and increasing size of information that has to be managed in today’s applications calls for flexible mechanisms for storing such information, making it easily and efficiently accessible, and facilitating its change and evolution over time. The paradigm of graph structured data (GSD) has gained popularity recently as an alternative to traditional relational databases that provides more flexibility and thus can overcome the limitations of an a priori imposed rigid structure on the data. Indeed, differently from relational data, GSD do not require a schema to be fixed a priori. This flexibility makes them well suited for many emerging application areas such as managing Web data, information integration, persistent storage in object-oriented software development, or management of scientific data. Concrete examples of models for GSD are RDFS, object-oriented data models, and XML.

Here we build on recent work that advocates the use of Description Logics (DLs) for managing change in GSD that happens as the result of (agents or users) executing actions. We consider GSD understood in a broad sense, as information represented by means of a node and edge labeled graph, in which the labels convey semantic information. We identify GSD with the finite structures over which DLs are interpreted, and use DL knowledge bases as descriptions of constraints and properties of the data. We express actions using a specially tailored action language in which actions are finite sequences of (possibly conditional) insertions and deletions performed on the extensions of labels. For this setting, the static verification problem, which consists on deciding whether the execution of a given action will preserve some given integrity constraints on any possible GSD, has been studied in. Here we discuss further problems that can be considered as variants of planning, such as deciding whether there is a sequence of actions that leads a given structure into a state where some property (either desired or not) holds, or whether a given sequence of actions leads every structure into a state where some property necessarily holds. We develop algorithms for variations of these problems, and characterize their computational complexity.

We refer the reader to [1] for the extended version of this paper, which also includes an extensive discussion of related work.

Updating GSD For manipulating GSD we use a specially tailored language in which a basic action can take, for example, the form \((A \oplus C)\) for a concept name \(A\) and an arbitrary concept \(C\). Intuitively, when this action is applied to an interpretation

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We assume constants \( p \) work only for

\[ (P1) \text{ the classic plan existence problem, formulated in the setting of GSD.} \]

\[ (P2) \text{ also} \]

\[ I \text{ (i.e., a GSD instance)} \]

\[ \langle \text{I} \rangle \]

The sequence

\[ \alpha \]

aims at deciding plan existence, but rather than the full actual state of the data, we have

\[ \langle \text{I} \rangle \text{ the domain of} I \text{ with a finite set of domain elements.} \]

Example 1. The following interpretation \( I \) represents (part of) the project database of a research institute. There are two active projects and three employees working in them.

\[
\begin{align*}
\text{Emp}^I &= \{e_1, e_3, e_7\} \\
\text{ActPrj}^I &= \{p_1, p_2\} \\
\text{Prj}^I &= \{p_1, p_2\} \\
\text{FinPrj}^I &= \{\}
\end{align*}
\]

\[ \text{worksFor}^I = \{(e_1, p_1), (e_3, p_1), (e_7, p_2)\} \]

We assume constants \( p_i \) with \( p_i^I = p_i \) for projects, and analogously constants \( e_i \) for employees. The KB \( K_1 \) expresses constraints on this project database: all projects are active or finished, the domain of worksFor are the employees, and its range the projects.

\[ (\text{Prj} \sqsubseteq \text{ActPrj} \sqcup \text{FinPrj}) \land (\exists \text{worksFor}. \top \sqsubseteq \text{Emp}l) \land (\exists \text{worksFor}^- . \top \sqsubseteq \text{Prj}) \]

The following goal KB requires that \( p_1 \) is not an active project, and that \( e_1 \) is an employee. Consider the following actions \( \alpha_1 \) and \( \alpha_2 \): here \( \varepsilon \) stands for the empty action. Action \( \alpha_1 \) moves \( p_1 \) from the active to the finished projects, and removes the employees that work only for \( p_1 \). Action \( \alpha_2 \) transfers an employee \( x \) from project \( p_1 \) to project \( p_2 \), if the necessary preliminary checks are successful.

\[ K_y = \neg(p_1 \cdot \text{ActPrj}) \land e_1 \cdot \text{Emp}l \]

\[ \alpha_1 = \text{ActPrj} \sqcap \{p_1\} \cdot \text{FinPrj} \sqcup \{p_1\} \cdot \text{Emp}l \sqcup \exists \text{worksFor}. \{p_1\} \cdot \text{worksFor} \sqcup \text{worksFor}^I(p_1) \]

\[ \alpha_2 = (p_2 \cdot \text{Prj} \sqcap \{x, p_1\} \cdot \text{worksFor} ? \cdot \exists \text{worksFor} \sqcap \{(x, p_1)\} \cdot \exists \text{worksFor} \sqcup \{(x, p_2)\} \parallel \varepsilon) \]

The sequence \( \langle \alpha_2', \alpha_1 \rangle \) is a plan for \( K_y \) from \( I \), where \( \alpha_2' \) is the result of applying to \( \alpha_2 \) the substitution \( \sigma : \{x \mapsto e_1\} \), that is, parameter \( x \) takes the value \( e_1 \). The interpretation \( I' \) that reflects the resulting status of the data looks as follows (note that \( I' \models K_1 \land K_y \)):

\[
\begin{align*}
\text{Emp}l^I &= \{e_1, e_7\} \\
\text{ActPrj}^I &= \{p_2\} \\
\text{Prj}^I &= \{p_1, p_2\} \\
\text{FinPrj}^I &= \{p_1\} \\
\text{worksFor}^I &= \{(e_1, p_2), (e_7, p_2)\}
\end{align*}
\]

Planning Problems for GSD We define the following planning problems:

(P1) Given a set \( Act \) of actions, a finite interpretation \( I \), and a goal KB \( K \), does there exist a plan for \( K \) from \( I \)?

(P2) Given a set \( Act \) of actions and a pair \( K_{pre} \), \( K \) of formulae, does there exist a substitution \( \sigma \) and a plan for \( \sigma(K) \) from some finite \( I \) with \( I \models \sigma(K_{pre}) \)?

(P1) is the classic plan existence problem, formulated in the setting of GSD. (P2) also aims at deciding plan existence, but rather than the full actual state of the data, we have as an input a precondition KB, and we are interested in deciding the existence of a plan from some of its models. To see the relevance of (P2), consider the complementary
This is formalized via the following problems: The first such problem is to ‘certify’ that a candidate plan is always a plan for the goal. Variants of the so-called conformant fragments. Identifying meaningful restricted fragments of lower complexity, in particular tractable in DLs. Interesting lines for further research are developing practicable algorithms and executing complex actions on GSD in the presence of integrity constraints expressed complete for co-DL-Lite. Problem (S) is undecidable already for DL-Lite KBs and a quite restricted form of actions. For (P1), the intuition behind this is that we do not know how many fresh objects we need to add to the domain of I. If we put a bound on the number of these fresh objects, we regain decidability. (P2) remains undecidable even if the domain is fixed, but it becomes decidable if we place a bound on the length of plans. (P1b) Given a set Act of actions, a finite interpretation I, a goal KB K, and a positive integer k, does there exist a plan for K from I such that at most k elements are added to the domain of I?

(P2b) Given a set of actions Act, a pair K_pre, K of formulae, and a positive integer k, does there exist a substitution σ and a plan of length at most k for σ(K) from some finite interpretation I with I |= σ(K_pre)?

In what follows, we assume that the integers k given as bounds are encoded in unary. The problem (P1b) is PSPACE-hard already for settings more restricted than DL-Lite, and it can be solved in polynomial space even for the very expressive ALCHOTQbr (an extension of ALCHOTQ with further role constructors and Boolean KBs). Note that the problem is not harder than deciding plan existence in standard planning formalisms such as propositional STRIPS. The problem (P2b) is NEXPTIME-complete for ALCHOTQbr, and the complexity drops to NP-complete for DL-Lite and suitably restricted actions.

Next we consider problems that are related to ensuring that plans always achieve a goal K, given a possibly incomplete description K_pre of the initial data. They are variants of the so-called conformant planning, which deals with incomplete information. The first such problem is to ‘certify’ that a candidate plan is always a plan for the goal. (C) Given a sequence P of actions and formulae K_pre, K, is σ(P) a plan for σ(K) from every finite interpretation I with I |= σ(K_pre), for every substitution σ?

Finally, we are interested in deciding the existence of a plan that always achieves the goal, for every possible state satisfying the precondition. Solving this problem corresponds to the automated synthesis of a program for reaching a certain condition. This is formalized via the following problems:

(S) Given a set Act of actions and formulae K_pre, K, does there exist a sequence P of actions from Act such that σ(P) is a plan for σ(K) from every finite I with I |= σ(K_pre), for every substitution σ?

(Sb) Given a set Act of actions, formulae K_pre, K, and a positive integer k, does there exist a sequence P of actions from Act such that σ(P) is a plan for σ(K) of length at most k, from every finite I with I |= σ(K_pre), for every substitution σ?

Problem (S) is undecidable already for DL-Lite. For ALCHOTQbr, (C) and (Sb) are complete for coNEXPTIME. For DL-Lite, (C) is complete for coNP and (Sb) for NP.

Conclusions We believe this work provides powerful tools for analyzing the effects of executing complex actions on GSD in the presence of integrity constraints expressed in DLs. Interesting lines for further research are developing practicable algorithms and identifying meaningful restricted fragments of lower complexity, in particular tractable fragments.
References


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