Repairable systems

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Repairable system

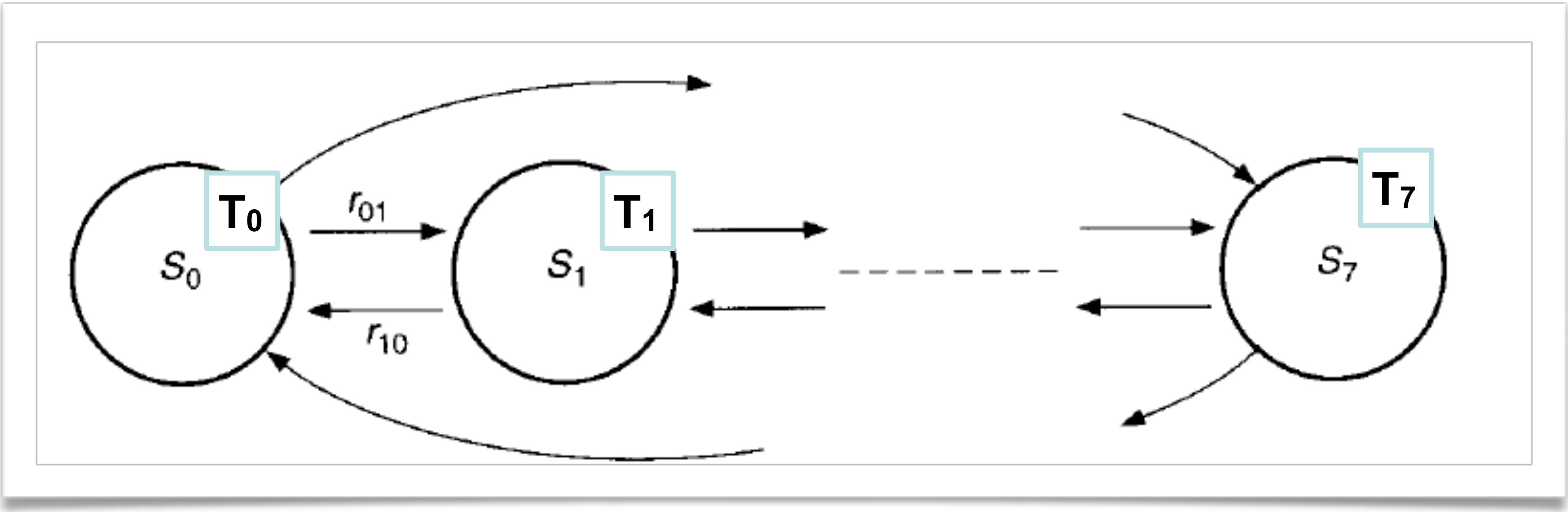
- A reparable system is obtained by *glueing individual non-repairable systems* each around a single failure

- To describe this gluing process we need to review the concept of *stochastic process*

Stochastic process

- Stochastic processes are classified by their
  - **State Space** – the range of possible values
  - The **index set** of the state space
  - The **dependence structure** among random variables X_t that make up the whole stochastic process

Stochastic process



Failure occurrences

- For each index i, there is a new random variable **Ti** representing the *Time of the i-th Failure*
- Each **Ti** has its own probability density function **fi**, cumulative distribution function **Fi** and hazard rate **hi**
- these are all functions defined on *local time (the time around the i-th failure)*

Failure occurrences

it can only be introduced in repairable systems!

- We introduce a new random variable which is the **Time Between Failures**

$$X_1=T_1 \text{ and } X_i=T_i-T_{i-1}$$

- <u>Note:</u> giving Time of Failure variables you can easily derive the Time Between Failures ones and vice versa

Failure occurrences

- *Each Time Between Failures X_i (or T_i) is a random variable for each state of the process*
- These variables may be
  - Dependent or independent
  - Identically distributed or not identically distributed

Independent random variables

- Two random variables X and Y say, are said to be **independent** if and only if the value of X has no influence on the value of Y and vice versa

Independent random variables

- For **discrete independent random variables** X and Y, the mass function of the process described by X and Y is

$$P(X = x_i \,;\, Y = y_j) = P(X = x_i)*P(Y=y_j)$$

  - for each pair (x_i,y_j).

Independent random variables

- Same for density functions

$$f(x,y) = g(x)*h(y)$$

- where g(x) and h(y) are the marginal density functions of the random variables X and Y respectively, for all pairs (x,y).

Identically distributed random variables

- Two random variables X and Y are said **identically distributed** if they have the same cumulative distribution function F (or density function)

Repairable systems

- The assumption of independent and identically distributed **times between failures** is usually invalid for software repairable systems

- Why?

Repairable systems

- In classical **hardware theory**, we simply **replace** failed components with **identical working** new ones
  - We might have that **all density functions** to be **identical** and their cumulative distribution function as well

- Generally when you substitute a part of a car you do not expect that the car has better performance (density function) because you cannot intervene on its design!
- In few cases we may also replace a failed component with one of better quality

Repairable systems

- Once a software fault is completely removed it will **not cause the same failure again**, but ...
- **Dependency**: Removing faults may cause new failures: the variable **Times Between Failures X_i may be dependent**

- By fixing a failure we may also **improve the design** to minimize the likelihood of recurrence of the faults that have caused the failure
- Fault **prevention:** in operation settings, software reliability can also be improved by **testing** whereas for hardware one has to use better material

Repairable systems

- Altogether, we expect that the probability density function of X_i would be different than the one of X_{i-1}
  - For example, by improving design $E[X_{i-1}]$ tends to be less than the one of $E[X_i]$

Minimal/Perfect repair

- **Minimal repair (as bad as old):** the repair done on a system leaves the system in exactly the same condition as it was just before the failure
- **Perfect repair (as good as new):** the system is brought to a new state after the repair

- If every repair is a **perfect repair** then times between failures are *independent and identically distributed*

Reliability Growth

- Reliability Growth means:
  - PdF f_{i+1} of T_{i+1} is different from PdF f_i of T_i
  - $E[T_{i+1}] \geqq E[T_i]$
- Expected Failure times tend to increase!
- *Reliability Growth is not normally considered in hardware reliability*
  - It is a goal of software maintenance!