

Approximate Matching of Hierarchical Data Using *pq*-Grams

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Abstract

When integrating data from autonomous sources, exact matches of data items that represent the same real world object often fail due to a lack of common keys. Yet in many cases structural information is available and can be used to match such data. As a running example we use residential address information. Addresses are hierarchical structures and are present in many databases. Often they are the best, if not only, relationship between autonomous data sources. Typically the matching has to be approximate since the representations in the sources differ.

We propose *pq*-grams to approximately match hierarchical information from autonomous sources. We define the *pq*-gram distance between ordered labeled trees as an effective and efficient approximation of the well-known tree edit distance. We analyze the properties of the *pq*-gram distance and compare it with the edit distance and alternative approximations. Experiments with synthetic and real world data confirm the analytic results and the scalability of our approach.

1 Introduction

When integrating data from autonomous sources, exact matches of data items representing the same real world object often fail due to missing global keys and different data representations. Approximate matching

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techniques must be applied instead. We focus on hierarchical data, where, in addition to data values, the data structure must also be considered.

As a running example we use an application from our local municipality. The GIS Office wants to relate data about apartments stored in different databases and display this information on a map. This requires a join on the address attributes. An equality join gives extremely poor results, mainly due to the different street names in various databases. Street names vary because different conventions are used to represent them. They may even be stored in different languages, which prevents the use of standard string comparison techniques. To overcome this problem we exploit the hierarchical organization of addresses. Instead of comparing street names we look for similarities in the hierarchical structure imposed by the addresses of a street.

Hierarchical data can be represented as ordered labeled trees. Data is then matched based on similarities of the corresponding trees. A well-known measure for comparing trees is the tree edit distance. It is computationally very expensive and leads to a prohibitively high run time. We propose the *pq*-gram distance as an effective and efficient approximation of the tree edit distance. The *pq*-grams of a tree are all its subtrees of a particular shape. Intuitively, two trees are close to each other if they have many *pq*-grams in common. For a pair of trees the *pq*-gram distance can be computed in $O(n \log n)$ time and $O(n)$ space, where n is the number of tree nodes.

In general, the *pq*-gram distance is a good approximation of the tree edit distance. In contrast to the tree edit distance, it places more emphasis on modifications to the structure of the tree. For example, deletions of nodes with a rich structure (many descendants) are more expensive than deletions of nodes with a poor structure (e.g., leaf nodes). We show that this property yields intuitive results.

At a technical level, our contribution is a new approximation for the tree edit distance with *pq*-grams.

this to formulate the original join as an *approximate tree join*

$$\text{SRO} \times [\text{dist}(\mathbf{T}(\text{SRO.id}), \mathbf{T}(\text{SLR.id})) \leq \tau] \text{SLR}.$$

Here $\mathbf{T}(id)$ are the address trees of the streets, $\text{dist}(\mathbf{T}_1, \mathbf{T}_2)$ is the distance between trees \mathbf{T}_1 and \mathbf{T}_2 , and τ is a distance threshold. The equality match between street names has been replaced by an approximate matching of the corresponding address trees.

Our goal is to find an effective approximation for the tree edit distance that can be efficiently computed and is scalable to large trees.

3 Related Work

A well known distance function for trees is the tree edit distance, which is defined as the minimum cost sequence of edit operations (node insertion, node deletion, and label change) that transforms one tree into another [19]. Zhang and Shasha [24] present an algorithm to compute the tree edit distance in $O(n^2 \min^2(l, d))$ time and $O(n^2)$ space for trees with n nodes, l leaves, and depth d . Other algorithms were presented in more recent works [7, 14]. All of them have more than $O(n^2)$ runtime complexity and do not scale to large trees.

By imposing restrictions on the edit operations that can be applied to transform a tree, suboptimal solutions with better runtime complexities can be found: Alignment distance [13], isolated subtree distance [20], and top-down distance [18, 22] have runtime at least $O(n^2)$, bottom-up distance can be computed in $O(n)$ time. Bottom-up distance tries to find the largest possible common subtrees of two trees, starting with the leaf nodes. It is very sensitive to differences between the leaf nodes. If the leaves are different, the inner nodes are never compared. This makes the bottom-up distance applicable in only very specific domains.

Guha et al. [11] present a framework for approximate XML joins based on tree edit distance, where XML documents are represented as ordered labeled trees. They give upper and lower bounds for the tree edit distance that can be computed in $O(n^2)$ time and use reference sets to take advantage of the fact that the tree edit distance is a metric, thus reducing the actual number of distances to compute in a join. The success of this method depends heavily on a good choice of the reference set. We do not try to limit the number of distance calculations with the expensive tree edit distance, rather we substitute it with an efficient approximation.

Chawathe et al. [6] use a variant of the tree edit distance for change detection. Lee et al. [15] tune the algorithm presented by Chawathe et al. to XML documents. Both algorithms first compute a match between the nodes of the trees, and based on this the distance is computed in $O(ne)$ time, where e is the edit distance between the trees. Whereas in a change

detection scenario typically trees with small differences are compared, for joins the distances between all pairs of trees have to be computed. For trees that are very different the edit distance e is $O(n)$, which yields $O(n^2)$ runtime for both algorithms.

A core operation in XML query processing is to find all occurrences of a twig pattern [3, 12]. The goal of our work is not to find occurrences of a pattern to answer queries. We split the tree into subtrees in order to calculate the distance between trees. Polyzotis et al. [17] build synopsis of an XML tree optimized for approximate query answering. They introduce the Element Simulation Distance to capture the difference between the original tree and the synopsis with respect to twig queries. This distance is tailored to measure the quality of a synopsis and is not suitable as an approximation for the tree edit distance.

Garofalakis and Kumar [9] investigate an algorithm for embedding the tree edit distance (with subtree move as an additional edit operation) into a numeric vector space equipped with the standard L_1 distance norm. The algorithm computes an approximation of the tree edit distance with subtree move (to within a $O(\log^2 n \times \log^* n)$ factor) in $O(n \times \log^* n)$ time and $O(n)$ space¹. We implement this approximation and empirically compare it to the pq -gram distance. The tree embedding distance gives less weight to structural changes than the tree edit distance. The sensitivity of the pq -gram distance to structural changes is controlled by the parameters p and q . The pq -gram distance typically weights them more than the edit distance.

Navarro [16] gives a good overview of the edit distance for *strings* and its variants. Ukkonen [21] introduces the q -gram distance as a lower bound for the string edit distance. The q -gram distance between two strings is based on the number of common substrings of length q . Gravano et al. [10] present algorithms for approximate string joins based on edit distance and use q -grams as a filtering algorithm. Approximate string matching techniques are successful if the distance between corresponding strings is smaller than that of other strings in the join set. This is typically the case for spelling mistakes, where only a few characters change. The distance between corresponding street names, however, is often larger than the length of the shorter string. If streets are renamed, string matching fails completely.

4 THE pq -GRAM DISTANCE

Hierarchical data can be represented as rooted, ordered, labeled trees, where the single data values are represented as labels of the tree nodes. In this section we first give a definition of trees and then define the pq -gram distance of trees.

¹ $\log^* n$ denotes the number of log applications required to reduce n to a quantity that is ≤ 1 , cf. [9].

4.1 Preliminaries

Let $G = (V, E)$ be a graph with nodes $V(G) = V$ and edges $E(G) = E$. A *tree* \mathbf{T} is a directed, acyclic, connected, non-empty graph. An *edge* is an ordered pair (p, c) , where $p, c \in V(\mathbf{T})$ are nodes, and p is the *parent* of c . Nodes with the same parent are *siblings*. An order \leq is defined on the nodes, and this order is total among siblings. The siblings $s_1 \leq s_2$ ($s_1 \neq s_2$) are *contiguous* if s_1 and s_2 have no sibling x ($s_1 \neq x \neq s_2$) with $s_1 \leq x \leq s_2$. Node c is the i -th *child* of p with $i = |\{x \in V(\mathbf{T}) \mid (p, x) \in E(\mathbf{T}), x \leq c\}|$. The number of p 's children is its *fanout* f_p . The node with no parent is the *root* node $r = \text{root}(\mathbf{T})$, and a node without children is a *leaf*.

Each node a in the path from the root node to a node v is called an *ancestor* of v . If there is a path of length $k > 0$ from a to v , then a is the ancestor of v at distance k . The parent of a node is its ancestor at distance 1. d is a *descendant* of v if v is an ancestor of d . The *level* of a node $\text{level}(v)$ is the length of the path from the root to v , the *depth* of a tree $\text{depth}(\mathbf{T})$ is the length of the longest path from the root to any one of the leaves.

A *label* is a symbol $\sigma \in \Sigma$, where Σ is a finite alphabet. Each node $v \in V(\mathbf{T})$ has assigned a label $l(v)$. A node o with the special label $l(o) = *$ is a *null node*.

In our graphical representation of trees we represent nodes as an (identifier, label)-pair, the edges are lines between the nodes, and siblings are ordered from left to right. Whenever possible we omit the identifiers of the nodes to avoid clutter (e.g., in Figure 3).

Example 4.1 Figure 4 shows a tree $\mathbf{T}_1 = (V, E)$ with $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E = \{(v_1, v_2), (v_1, v_5), (v_1, v_6), (v_2, v_3), (v_2, v_4)\}$, and the order $v_2 \leq v_5 \leq v_6, v_3 \leq v_4$. v_1 has 3 children, where v_2 is the first, v_5 the second, and v_6 the third child. The root node $\text{root}(\mathbf{T}) = v_1$. v_1 is the ancestor of all other nodes. v_3, v_4, v_5 and v_6 are leaf nodes. The node labels of our example tree are $l(v_1) = a$, $l(v_2) = a$, $l(v_3) = e$, $l(v_4) = b$, $l(v_5) = b$, and $l(v_6) = c$.

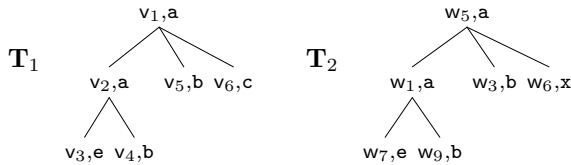


Figure 4: Graphical representation of trees.

A *subtree* $\mathbf{S} \subseteq \mathbf{T}$ is a tree with $V(\mathbf{S}) \subseteq V(\mathbf{T})$ and $E(\mathbf{S}) \subseteq E(\mathbf{T})$, retaining the node order. A *preorder traversal* of a tree visits the root node first, and then recursively traverses all the subtrees rooted in its children in preorder, preserving the children's order. We call a node v the i -th node of \mathbf{T} in preorder if v is visited as the i -th node in a preorder traversal.

Two trees \mathbf{T} and \mathbf{T}' are *isomorphic* if there is a bijective mapping m between the nodes $V(\mathbf{T})$ and $V(\mathbf{T}')$ such that the following holds true: (v, w) is an edge of \mathbf{T} and w is the i -th child of v if and only if $(m(v), m(w))$ is an edge of \mathbf{T}' and $m(w)$ is the i -th child of $m(v)$.

Example 4.2 Consider Figure 4. The tree $\mathbf{S}_1 = (\{v_2, v_3, v_4\}, \{(v_2, v_3), (v_2, v_4)\})$, $v_3 \leq v_4$ is a subtree of \mathbf{T}_1 . The preorder traversal of \mathbf{T}_1 visits the nodes in the following order: $v_1, v_2, v_3, v_4, v_5, v_6$. Tree \mathbf{T}_2 is isomorphic to \mathbf{T}_1 with $m = \{(v_1, w_5), (v_2, w_1), (v_3, w_7), (v_4, w_9), (v_5, w_3), (v_6, w_6)\}$.

4.2 The pq-Gram Distance

In the following paragraphs we define the notion of *pq*-grams and a distance measure based on *pq*-grams. Intuitively, the *pq*-grams of a tree are all subtrees of a specific shape. To ensure that each node of the tree appears in at least one of the *pq*-grams, we extend the tree with *null nodes*. The *pq*-grams are then defined as subtrees of the extended tree.

Definition 4.1 (pq-Extended Tree) Let \mathbf{T} be a tree, and $p > 0$ and $q > 0$ be two integers. The *pq*-extended tree, \mathbf{T}^{pq} , is constructed from \mathbf{T} by adding $p-1$ ancestors to the root node, inserting $q-1$ children before the first and after the last child of each non-leaf node, and adding q children to each leaf of \mathbf{T} . All newly inserted nodes are null nodes that do not occur in \mathbf{T} .

Example 4.3 Figure 5 shows the graphical representation of $\mathbf{T}_1^{2,3}$, the 2,3-extended tree of our example tree \mathbf{T}_1 .

Definition 4.2 (pq-Gram Pattern) For $p > 0$ and $q > 0$, the *pq*-gram pattern is a tree that consists of an anchor node with $p-1$ ancestors and q children.

Example 4.4 An example of a 2,3-gram pattern is the tree $(\{p_1, p_2, p_3, p_4, p_5\}, \{(p_1, p_2), (p_2, p_3), (p_2, p_4), (p_2, p_5)\})$, $p_3 \leq p_4 \leq p_5$. p_2 is the anchor node, and it has 1 ancestor (p_1) and 3 children (p_3, p_4 , and p_5).

Definition 4.3 (pq-Gram) For $p > 0$ and $q > 0$, a *pq*-gram \mathbf{G} of a tree \mathbf{T} is defined as a subtree of the extended tree \mathbf{T}^{pq} with the following properties: \mathbf{G} is isomorphic to the *pq*-gram pattern, and contiguous siblings in \mathbf{G} are contiguous siblings in \mathbf{T}^{pq} .

Definition 4.4 (Label-tuple) Let \mathbf{G} be a *pq*-gram with the nodes $V(\mathbf{G}) = \{v_1, \dots, v_p, v_{p+1}, \dots, v_{p+q}\}$, where v_i is the i -th node in preorder. The tuple $l(\mathbf{G}) = (l(v_1), \dots, l(v_p), l(v_{p+1}), \dots, l(v_{p+q}))$ is called the *label-tuple* of \mathbf{G} .

Subsequently, if the distinction is clear from the context, we use the term *pq*-gram for both, the *pq*-gram itself and its representation as a label-tuple.

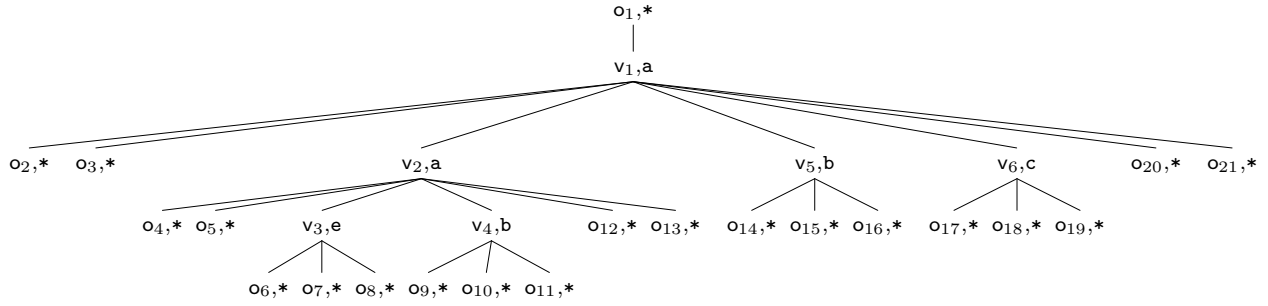


Figure 5: Graphical representation of the extended tree $\mathbf{T}_1^{2,3}$.

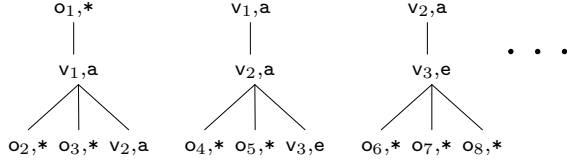


Figure 6: Some of the 2,3-grams of \mathbf{T}_1 .

Example 4.5 Figure 6 shows some of the 2,3-grams of the example tree \mathbf{T}_1 . They are constructed by moving the 2,3-gram pattern over the extended tree $\mathbf{T}_1^{2,3}$ (see Figure 5). We start at the top of the tree. For the first pq -gram the anchor node of the pattern is mapped to v_1 , and the children of the anchor are mapped to two null nodes and v_2 . The corresponding label-tuple is $(*, a, *, *, a)$.

Definition 4.5 (pq -Gram Profile) For $p > 0$ and $q > 0$, the pq -gram profile, $P^{p,q}(\mathbf{T})$, of a tree \mathbf{T} is defined as the bag of label-tuples $l(\mathbf{G}_i)$ of all pq -grams \mathbf{G}_i of \mathbf{T} .

The tables in Figure 7 show the 2,3-gram profile of \mathbf{T}_1 and \mathbf{T}_2 , respectively. Note that pq -grams might appear more than once in a pq -gram profile, e.g., $(a, b, *, *, *)$ appears twice in the profile of \mathbf{T}_1 .

$P^{2,3}(\mathbf{T}_1)$	$P^{2,3}(\mathbf{T}_2)$
labels	labels
$(*, a, *, *, a)$	$(*, a, *, *, a)$
$(a, a, *, *, e)$	$(a, a, *, *, e)$
$(a, e, *, *, *)$	$(a, e, *, *, *)$
$(a, a, *, e, b)$	$(a, a, *, e, b)$
$(a, b, *, *, *)$	$(a, b, *, *, *)$
$(a, a, e, b, *)$	$(a, a, e, b, *)$
$(a, a, b, *, *)$	$(a, a, b, *, *)$
$(*, a, *, a, b)$	$(*, a, *, a, b)$
$(a, b, *, *, *)$	$(a, b, *, *, *)$
$(*, a, a, b, c)$	$(*, a, a, b, x)$
$(a, c, *, *, *)$	$(a, x, *, *, *)$
$(*, a, b, c, *)$	$(*, a, b, x, *)$
$(*, a, c, *, *)$	$(*, a, x, *, *)$

Figure 7: 2,3-Gram profiles of \mathbf{T}_1 and \mathbf{T}_2 .

We subsequently define the pq -gram distance as a measure for the similarity of two trees. The pq -gram distance is based on the number of pq -grams that the profiles of the compared trees have in common.

Definition 4.6 (pq -Gram Distance) For $p > 0$ and $q > 0$, the pq -gram distance, $\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2)$, between two trees \mathbf{T}_1 and \mathbf{T}_2 is defined as follows:

$$\Delta^{p,q}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2 \frac{|P^{p,q}(\mathbf{T}_1) \cap P^{p,q}(\mathbf{T}_2)|}{|P^{p,q}(\mathbf{T}_1) \cup P^{p,q}(\mathbf{T}_2)|} \quad (1)$$

Example 4.6 Consider the 2,3-gram distance between \mathbf{T}_1 and \mathbf{T}_2 . The corresponding 2,3-gram profiles are shown in Figure 7. The bag-intersection of the two profiles is $\{(*, a, *, *, a), (a, a, *, *, e), (a, e, *, *, *) , (a, a, *, e, b), (a, b, *, *, *) , (a, a, e, b, *) , (a, a, b, *, *) , (*, a, *, a, b), (a, b, *, *, *)\}$, which yields $|P^{2,3}(\mathbf{T}_1) \cap P^{2,3}(\mathbf{T}_2)| = 9$. For the cardinality of the bag-union we get $|P^{2,3}(\mathbf{T}_1) \cup P^{2,3}(\mathbf{T}_2)| = |P^{2,3}(\mathbf{T}_1)| + |P^{2,3}(\mathbf{T}_2)| = 26$. Thus, the pq -gram distance is

$$\Delta^{2,3}(\mathbf{T}_1, \mathbf{T}_2) = 1 - 2 \frac{9}{26} = 0.31.$$

The pq -gram distance is 1 if two trees share no pq -grams. Trees at distance 0 have the same pq -gram profile. Note that distance 0 does not imply equality of trees. An example of two different trees with the same pq -gram profile is shown in Figure 8. The pq -grams responsible for detecting the swapped children of the root nodes of T' and T'' are those anchored in the root nodes. However, as all children of the root nodes have the same label, the pq -grams remain unchanged.

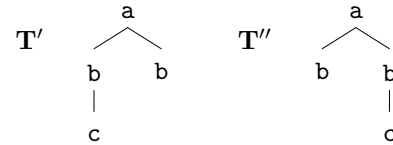


Figure 8: Different trees with the same pq -gram profile.

The pq -gram distance can be computed in $O(n \log n)$ time by computing the bag intersection of

the pq -gram profiles of size $O(n)$. Theorem 4.1 shows, how the size of the profile is related to the number of leaf and non-leaf nodes.

Theorem 4.1 *Let $p > 0, q > 0$, and \mathbf{T} be a tree with l leaf nodes and i non-leaf nodes. The size of the pq -gram profile is*

$$|P^{p,q}(\mathbf{T})| = 2l + qi - 1.$$

Proof 4.1 *By structural induction:*

$|V(\mathbf{T})| = 1$: *The tree consists of the root node only, and according to Definition 4.3 the pq -gram profile contains exactly one pq -gram. The number of leaves is 1, while the number of non-leaf nodes is 0, thus $|P^{p,q}(\mathbf{T})| = 2l + qi - 1 = 1$.*

$|V(\mathbf{T})| > 1$: *In this case $i \geq 1$ (at least the root node) and $l \geq 1$. First we delete all non-leaf nodes (except the root r) and get \mathbf{T}' . $|P^{p,q}(\mathbf{T})| - |P^{p,q}(\mathbf{T}')| = (i - 1) * q$. (Deleting a non-leaf node decreases the cardinality of the pq -gram profile by q). The number of leaves does not change with this operation, and the tree now consists of only the leaves and the root node. Now we delete all leaf nodes and get \mathbf{T}'' , $|P^{p,q}(\mathbf{T}')| - |P^{p,q}(\mathbf{T}'')| = 2(l - 1) + q$. (Deleting a leaf node decreases the cardinality of the pq -gram profile by q if the leaf has no siblings, otherwise by 2). \mathbf{T}'' consists only of the root node and $|P^{p,q}(\mathbf{T}'')| = 1$. This means, $|P^{p,q}(\mathbf{T})| = 1 + [2(l - 1) + q] + [(i - 1) * q] = 2l + qi - 1$.*

5 Algorithms

5.1 An Algorithm for the pq -Gram-Profile

The basic idea of the pq -GRAM-PROFILE algorithm in Figure 9 is to move the pq -gram pattern vertically and horizontally over the tree (see Figure 10a). After each move the nodes covered by the pattern form a pq -gram.

We use two shift registers, anc of size p and sib of size q , to represent the labels of the ancestor and the leaf nodes that are covered by the pq -gram pattern, respectively. A shift register reg supports a single operation $\text{shift}(reg, el)$, which returns reg with the oldest element dequeued and el enqueued. For example, $\text{shift}((a, b, c), x)$ returns (b, c, x) . The concatenation of the two registers, $anc \circ sib$, is a tuple in the pq -gram profile, i.e., for $anc = (l_1, \dots, l_p)$ and $sib = (l_{p+1}, \dots, l_{p+q})$ the label-tuple of the pq -gram is $(l_1, \dots, l_p, l_{p+1}, \dots, l_{p+q})$.

pq -GRAM-PROFILE takes as input a tree \mathbf{T} and the two values p and q and returns a relation that contains the pq -gram profile of \mathbf{T} . After the initialization, PROFILE calculates the pq -grams starting from the root node of \mathbf{T} . First PROFILE shifts the label of anchor node r into the register anc , which corresponds to moving the pq -gram pattern one step down. Now anc contains the labels of r and its $p - 1$ ancestors. The loop at line 13 moves the register sib from left to right over the children of r in order to produce all

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1  pq-GRAM-PROFILE( $\mathbf{T}, p, q$ )
2     $P$  : empty relation with schema (labels)
3     $anc$ : shift register of size  $p$  (filled with  $*$ )
4     $P = \text{PROFILE}(\mathbf{T}, p, q, P, \text{root}(\mathbf{T}), anc)$ 
5    return  $P$ 

6  PROFILE( $\mathbf{T}, p, q, P, r, anc$ )
7     $anc := \text{shift}(anc, l(r))$ 
8     $sib$ : shift register of size  $q$  (filled with  $*$ )
9
10   if  $r$  is a leaf then
11      $P := P \cup (anc \circ sib)$ 
12   else
13     for each child  $c$  (from left to right) of  $r$  do
14        $sib := \text{shift}(sib, l(c))$ 
15        $P := P \cup (anc \circ sib)$ 
16        $P := \text{PROFILE}(\mathbf{T}, p, q, P, c, anc)$ 
17     for  $k := 1$  to  $q - 1$ 
18        $sib := \text{shift}(sib, *)$ 
19        $P := P \cup (anc \circ sib)$ 
20
21   return  $P$ 

```

Figure 9: Calculating the pq -gram profile of a tree.

the pq -grams with anchor point r and calls PROFILE recursively for each child of r . Overall, PROFILE adds $f_r + q - 1$ label-tuples to P for each non-leaf node r , and 1 label-tuple for each leaf node. The pq -extended tree is calculated on the fly by an adequate initialization of the shift registers (lines 3, 8, 17-19).

Example 5.1 *Assume $p = 2, q = 3$, and the tree \mathbf{T}_1 from Figure 4. The main data structures of the PROFILE algorithm are visualized in Figure 10. After the initialization, $\text{PROFILE}(\mathbf{T}_1, 2, 3, \{\}, v_1, (*, *))$ is called.*

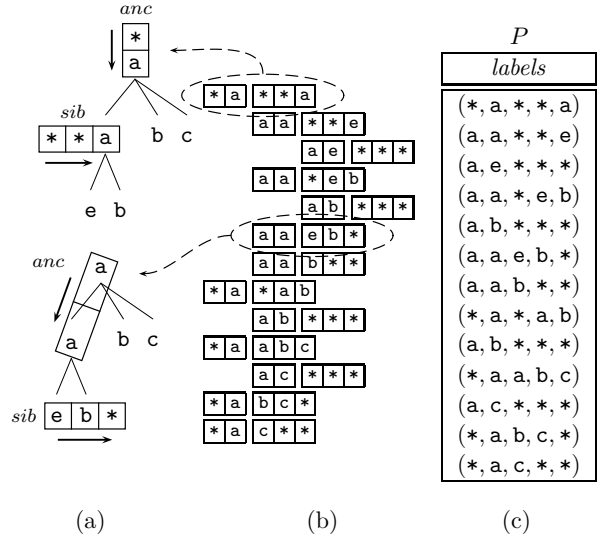


Figure 10: (a) Moving the pq -gram pattern in the tree, (b) Shift registers anc and sib , (c) Relation P produced by PROFILE.

Line 7 shifts the label of v_1 into the register *anc*, yielding $anc = (*, a)$, and line 8 initializes $sib = (*, *, *)$. Since v_1 is not a leaf we enter the loop at line 13 and process all children of v_1 . The label of the first child, v_2 , is shifted into *sib*, yielding $sib = (*, *, a)$, and the first label-tuple $(*, a, *, *, a)$ is added to the result set P . Figure 10b shows the values of *anc* and *sib* each time a label-tuple is added to P . The indentation illustrates the recursion. The table in Figure 10c shows the result relation P with the label-tuples in the order in which they are produced by the algorithm.

pq -GRAM-PROFILE has runtime complexity $O(n)$ for a tree \mathbf{T} , where $n = |V(\mathbf{T})|$: Each recursive call of PROFILE processes one node, and each node is processed exactly once.

5.2 Relational Implementation

The algorithm described above requires no particular encoding of trees. This section gives a scalable implementation for trees stored in a relational database. We use an interval representation of trees, where each node of a tree is represented by a pair of numbers (interval). The interval encoding is a technique for storing hierarchical data in relations [4, 5] and has been used to store and query XML data [1, 8, 23].

We associate a unique index number to each tree in the set. Each node of a tree is then represented as a quadruple of tree index, node label, and left and right endpoint of the node's interval.

Definition 5.1 (Interval Encoding) An interval encoding of a tree \mathbf{T} is a relation R that for each node $v \in \mathbf{T}$ contains a tuple $(id(\mathbf{T}), l(v), lft, rgt)$; $id(\mathbf{T})$ is a unique identifier of the tree \mathbf{T} , $l(v)$ is the label of v , lft and rgt are the endpoints of the interval representing the node. lft and rgt are constrained as follows:

- $lft < rgt$ for all $(id, l, lft, rgt) \in R$,
- $lft_a < lft_d$ and $rgt_a > rgt_d$ if node a is an ancestor of d , and $(id(\mathbf{T}), l(a), lft_a, rgt_a) \in R$, and $(id(\mathbf{T}), l(d), lft_d, rgt_d) \in R$,
- $rgt_v < lft_w$ if node v is a left sibling of node w , and $(id(\mathbf{T}), l(v), lft_v, rgt_v) \in R$, and $(id(\mathbf{T}), l(w), lft_w, rgt_w) \in R$,
- $rgt = lft + 1$ if node v is a leaf node, and $(id(\mathbf{T}), l(v), lft, rgt) \in R$.

We get an interval encoding for a tree by traversing the tree in preorder, using an incremental counter that assigns the left interval value lft to each node when it is visited first, and the right value rgt when it is visited last. Figure 11 shows an address tree of our application, where each node is annotated with the endpoints of the interval.

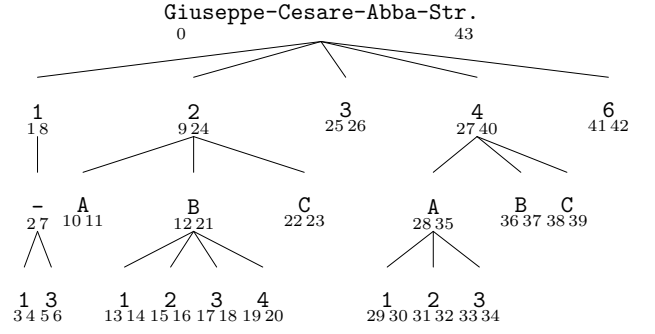


Figure 11: Address tree in interval encoding.

The interval encoding of a tree allows a scalable implementation of the algorithm pq -GRAM-PROFILE for a set of trees F stored in a relation F with schema $(treeID, label, lft, rgt)$. We define the following cursor:

```
cur = SELECT * FROM F ORDER BY treeID, lft
```

Then with a single scan all trees can be processed, and each tree is processed node-by-node in preorder. Our experiments in Section 7.1 confirm the scalability of this approach to large trees.

Figure 12 shows the algorithm adapted for interval encoding with the changes highlighted. Instead of a tree pq -GRAM-PROFILE gets a cursor as an argument. PROFILE processes all nodes of the tree in preorder, and when it terminates the cursor points to the root node of the next tree in the set.

```

1  pq-GRAM-PROFILE(cur, p, q)
2    P : empty relation with schema (labels)
3    anc: shift register of size p (filled with *)
4    P = PROFILE(cur, p, q, P, FETCH(cur), anc)
5    return P

6  PROFILE(cur, p, q, P, r, anc)
7    anc := shift(anc, l(r))
8    sib: shift register of size q (filled with *)
9
10a cur := NEXT(cur)
10 if ISLEAF(r) then
11   P := P ∪ (anc ∘ sib)
12 else
12a c := FETCH(cur)
13 while ISDESCENDANT(c, r) do
14   sib := shift(sib, l(c))
15   P := P ∪ (anc ∘ sib)
16   P := PROFILE(cur, p, q, P, c, anc)
16a c := FETCH(cur)
17 for k := 1 to q - 1
18   sib := shift(sib, *)
19   P := P ∪ (anc ∘ sib)
20
21 return P

```

Figure 12: Implementation of PROFILE using a cursor.

PROFILE calls the following two functions:

- **ISLEAF(v)**: Returns true iff v is a leaf node, i.e., $\text{lft}(v) + 1 = \text{rgt}(v)$.
- **ISDESCENDANT(d, a)**: Returns true iff d is a descendant of a , i.e., $\text{lft}(a) < \text{lft}(d)$ **and** $\text{rgt}(a) > \text{rgt}(d)$ **and** $\text{treeId}(a) = \text{treeId}(d)$ **and** $d \neq \text{null}$.

With the interval encoding it is easier to check whether a node is a descendant than whether it is a child. In our algorithm this amounts to the same thing: When the loop in line 13 is entered the first time, c is the next node after r in preorder (or null). Thus, if c is a descendant of r , it must be a child. The recursive call in line 16 will process c and all its descendants, and set the cursor on the next node after the processed nodes. Again, if this is a descendant of r , then it is a child. Thus the while-loop in Figure 12 is equivalent to the for-loop in Figure 9.

6 Sensitivity to Structural Changes

In this section we discuss the main properties of the pq -gram distance and compare it with the tree edit distance. We investigate two cases where the pq -gram distance behaves differently from the tree edit distance: structural and local changes. We consider the following standard edit operations [24]:

Update(T, v, σ): Updating a node $v \in V(\mathbf{T})$ means changing its label to $\sigma \in \Sigma$.

Delete(T, v): Deleting a node $v \in V(\mathbf{T}) \setminus \{\text{root}(\mathbf{T})\}$ means substituting v with its children (preserving the order), i.e., remove v and connect v 's children directly with v 's parent node.

Insert(T, v, p, i, k): Inserting a new node $v \notin V(\mathbf{T})$ as a child of a node $p \in V(\mathbf{T})$ at position i means substituting k consecutive children $v_i, v_{i+1}, \dots, v_{i+k-1}$ of p with v , and inserting them as children of v (preserving the order). If $k = 0$, a leaf node is inserted, and the number of p 's children increases by one.

The tree edit distance assigns a fixed cost to each operation. This disregards the fact that operations which change the structure (insert and delete) might have side effects on other nodes. For example, if a node is deleted, all children of this node are moved with their descendants to the parent node. This behavior leads to non-intuitive results, as shown in Figure 13: Tree \mathbf{T}' is the result of deleting the leaves with labels g and k from \mathbf{T} , \mathbf{T}'' is obtained from \mathbf{T} by deleting the nodes labeled c and e . Intuitively, \mathbf{T}' and \mathbf{T} are much more similar (in structure) than \mathbf{T}'' and \mathbf{T} , but the tree edit distance is 2 in both cases for a unit cost model.

The pq -gram distance depends directly on the number of affected pq -grams, which depends on the number

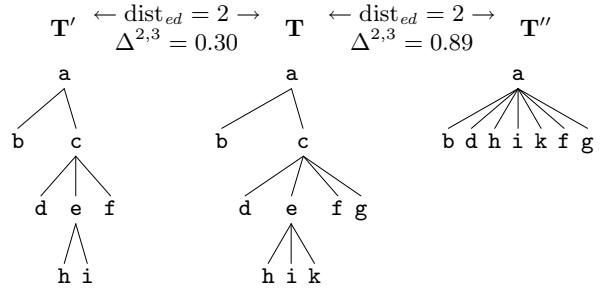


Figure 13: Tree edit distance and pq -gram distance for structural changes.

of descendants of v within distance p . Thus, changes to non-leaf nodes cost more than changes to leaves. The following theorem gives the number of pq -grams that contain a node v , which corresponds to the number of affected pq -grams if v is modified.

Theorem 6.1 *For a tree \mathbf{T} with all leaf nodes at level $d = \text{depth}(\mathbf{T})$ and a fixed fanout $f > 1$ for the non-leaf nodes, the number of pq -grams ($p > 0, q > 0$) that contain a node v of level $l = \text{level}(v)$ is:*

$$\text{cnt}_{pq}(\mathbf{T}, v) = q \text{sgn}(l) + \begin{cases} \frac{f^p - 1}{f - 1} (f + q - 1) & \text{if } p \leq d - l \\ \frac{f^{d-l} - 1}{f - 1} (f + q - 1) + f^{d-l} & \text{if } p > d - l. \end{cases}$$

Proof 6.1 *Consider how the pq -gram pattern with q leaves and p non-leaves is shifted over the tree. The leaves of the pattern are shifted over all nodes of the tree but the root node, which gives q pq -grams for each non-root node ($\text{sgn}(l)$ is 0 for the root, 1 for non-root nodes). If v is a non-leaf node, it appears in $f + q - 1$ pq -grams as the anchor node, otherwise in a single pq -gram. While v is in the pq -gram we recursively move the pattern down the tree. We exit the recursion earlier if the anchor node of the pq -gram pattern is a leaf. For the case $p \leq d - l$, we get $(f + q - 1) \sum_{i=0}^{p-1} f^i$, and for the case $p > d - l$, $(f + q - 1) \sum_{i=0}^{d-l-1} f^i$ additional pq -grams that contain v . For the latter case we add the term f^{d-l} that accounts for the pq -grams that have one of the f^{d-l} leaf descendants of v as an anchor node. We evaluate the partial sum of the geometric series to get the formula in Theorem 6.1.*

Theorem 6.1 assumes a tree with all leaves at the same depth and a fixed fanout. If f is the *maximum* fanout of v and its descendants within distance p , then $\text{cnt}_{pq}(\mathbf{T}, v)$ is an upper bound for the number of pq -grams that contain v .

According to Theorem 6.1 the cost for changing a leaf node ($d = l$) is $q + 1$, i.e., depends only on q . For non-leaf nodes the impact of p is prevalent, and we can control the sensitivity of the pq -gram distance to structural changes by choosing the value for p .

The difference between non-leaf and leaf nodes is relevant for hierarchical data, where values higher up

in the hierarchy are more significant. For example, two streets with different house numbers (with subnumbers and apartment numbers) are considered more different than streets in which only apartment numbers differ.

We further investigate the case when part of a tree is missing, i.e., a subtree is deleted. The effect on the structure is limited as the remaining part of the tree is unchanged. An example of a subtree is a subnumber with all its apartment numbers. If it is missing in one address tree, a relatively high number of nodes changes. These changes should be weighted less than the same number of changes on different house numbers.

If a subtree is deleted, several modifications are applied within a small neighborhood. The affected sets of pq -grams overlap each other, and hence, these changes have less impact on the pq -gram distance than changes that are uniformly distributed over the tree. The following theorem gives the number of pq -grams that change with a subtree deletion.

Theorem 6.2 *Let \mathbf{S} be the subtree of \mathbf{T} consisting of $v \in V(\mathbf{T}) \setminus \{\text{root}(\mathbf{T})\}$ and all its descendants, and let l be the number of leaves of \mathbf{S} , and let i be the number of non-leaf nodes. If all nodes of \mathbf{S} are deleted or updated, then $2l + iq + q - 1$ pq -grams change.*

Proof 6.2 *All pq -grams of the subtree change. This are $2l + iq - 1$ pq -grams (Theorem 4.1). Further v appears as a sibling in q pq -grams. The sum is $2l + iq + q - 1$.*

Example 6.1 *We refer to Figure 13 and discuss the deletion of the subtree of \mathbf{T} that consists of the node with label \mathbf{e} (lets call the node v) and all its descendants. An effect of this operation is that the following nodes are deleted: v plus the nodes labeled \mathbf{h} , \mathbf{i} , and \mathbf{k} . The number of 2,3-grams that contain the node v is $\text{cnt}_{2,3}(\mathbf{T}, v) = 11$, and $q + 1 = 4$ for the three other nodes. If these nodes did not share any pq -grams, the total number of affected pq -grams would be $11 + 3 \times 4 = 23$. However, as the deleted nodes build a subtree with $l = 3$ leaves and $i = 1$ non-leaf nodes, they do share pq -grams, and the total number of changing 2,3-grams is only $2l + iq + q - 1 = 11$.*

7 Experiments

7.1 Scalability

We compare the scalability of our algorithm with the tree edit distance [24] and the tree embedding distance [9], and we investigate the influence of the parameters p and q on the scalability of the pq -gram distance.

As a test set we produce pairs of trees $(\mathbf{T}_1, \mathbf{T}_2)$ of size $|V(\mathbf{T}_1)| = |V(\mathbf{T}_2)| = n$, where n ranges from 3 to 2×10^6 nodes. The depth of the trees is $\log(n)$ and the labels for each tree are randomly chosen from a set of n different labels.

Figure 14(a) shows the runtimes of tree edit distance and 2,3-gram distance calculations for different tree sizes. For the tree edit distance we use the implementation of Zhang and Shasha², whereas for the pq -gram distance we use the relational implementation described in Section 5.2. For very small trees edit distance is faster than pq -gram distance. The reason being that our algorithm writes all intermediate results to the disk, while the edit distance algorithm runs in the main memory. Therefore the overhead for disk access in this range masks the actual computing time for the distance. This effect can easily be prevented by keeping all data in main memory. For large trees the computation time for the tree edit distance grows very fast. For trees of size 10,000 it is already more than 27 hours, therefore we could not run our experiment for even larger trees. For the pq -gram distance the computation time is almost linear in the tree size.

Figure 14(b) compares the pq -gram distance for varying parameters with the tree embedding distance. We use our own implementation for tree embedding distance according to the algorithm of Garofalakis and Kumar [9]. For the comparison both algorithms run in main memory. The pq -gram distance is slightly faster, and varying values for p and q have little impact on the scalability of the pq -gram distance calculation.

7.2 Sensitivity to Structural Changes

In Section 6 we point out that the pq -gram distance weights deletions of non-leaf nodes more than deletions of leaves, and the sensitivity to structural changes is controlled by the parameters p and q . We show this property in an experiment, where only non-leaf nodes or only leaf nodes are deleted for varying parameters, and calculate the pq -gram distance for both cases.

We create an artificial tree \mathbf{T} with 144 nodes, 102 leaves, and depth 6. Each non-leaf has a fanout of between 2 and 5. Figure 15 shows the pq -gram distance for different numbers of leaf and non-leaf deletions. Each value in Figure 15 is an average over 100 runs.

For leaf node deletions only q has an influence (see Figure 15(a)). For the deletion of non-leaf nodes q has a small impact compared to p (see Figure 15(b)). This confirms our analytical results. Sensitivity to changes in the leaves depends only on q , and we can emphasize structural sensitivity with higher values of p . For deletions of non-leaf nodes the pq -gram distance is longer than for deletions of leaf nodes.

We further investigate the difference in the pq -gram distance for deleting a subtree or the same number of nodes randomly distributed all over the tree. For the experiment we use the same tree \mathbf{T} as above. We randomly choose a node $v \in \mathbf{T} \setminus \{\text{root}(\mathbf{T})\}$ and delete v and all its descendants. The tree edit distance between \mathbf{T} and the resulting tree \mathbf{T}' is the number of nodes in

²<http://www.cs.nyu.edu/cs/faculty/shasha/papers/tree.html>

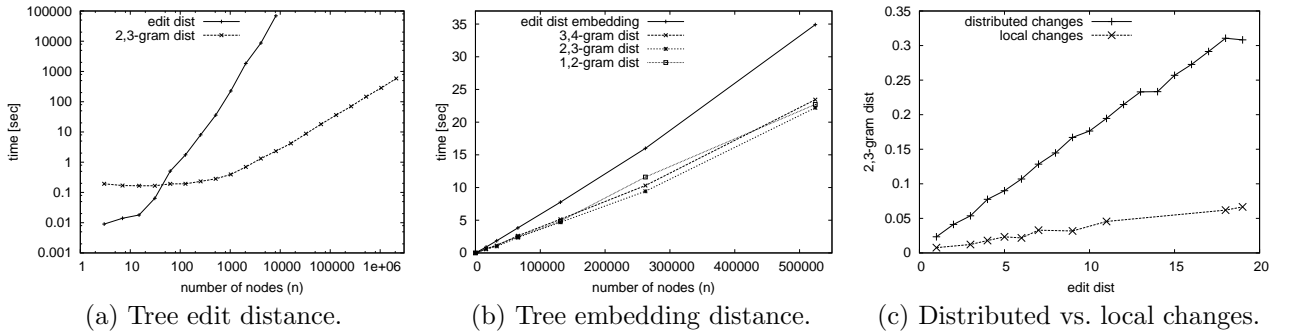


Figure 14: Scalability results and subtree deletions.

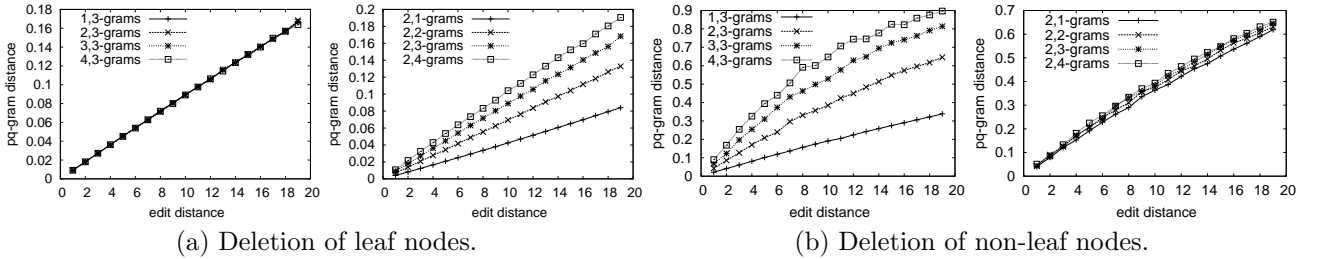


Figure 15: Properties of the pq -gram distance.

the deleted subtree. In Figure 14(c) we compare the results to distributed changes (average on 100 runs). We can see that local changes (subtree deletions) are cheaper than distributed changes.

7.3 Matchmaking with Real Data

To test the accuracy for real world data we use the address tables R0 and LR described in Section 2. We build the address trees for all streets in both tables and get the sets R and L. Each tree \mathbf{T} in one of the tree sets R and L represents a street with all the addresses in that street. Set R from R0 consists of 302 trees with 52,509 nodes in total, reflecting 43,187 addresses. Set L from table LR consists of 300 trees with 53,464 nodes and 44,447 addresses.

We say that two trees $\mathbf{T} \in F$ and $\mathbf{T}' \in F'$ *match* if \mathbf{T} has only one nearest neighbor in F' , namely \mathbf{T}' , and vice versa. For each distance function dist_x we compute a mapping $M_x \in F \times F'$ between all pairs of matching trees. Furthermore, we create a mapping, M_c , by hand with the correct pairs of trees, i.e., with all pairs of trees that represent the same street in the real world. We define the *accuracy* of M_x with respect to M_c as $a = \frac{|M_x \cap M_c|}{|M_c|}$. The false positives are computed as $M_x \setminus M_c$.

We compute a mapping for the tree edit distance dist_{ed} , the pq -gram distance $\Delta^{p,q}$, the tree embedding distance dist_{emb} , and the node intersection dist_i . The node intersection is a simple algorithm that completely ignores the structure of the tree. It is computed in the same way as the pq -gram distance, the only difference being that the profile of a tree consists of the bag of

all its node labels.

The results for the address tables R0 and LR are shown in Table 1. There are two streets in R0 that do not exist in LR, thus $|M_c| = 300$ for the calculation of the accuracy. The efficiency of the approximations is clearly greater than that of the tree edit distance: All of them can be computed within about five minutes, whereas the tree edit distance takes more than 52 hours.

	accuracy	correct	false pos.	runtime
dist_{ed}	82.7%	248	9	187,538s
$\Delta^{1,2}$	78.3%	235	5	181s
$\Delta^{2,3}$	77.3%	232	4	204s
$\Delta^{3,2}$	79.3%	238	2	180s
dist_{emb}	69.0%	207	8	313s
dist_i	66.3%	199	12	82s

Table 1: Accuracy of the tree edit distance and its approximations.

The pq -gram distance clearly outperforms the other approximations with respect to both, number of correct matches and number of false positives for all tested parameters. The number of false positives is even smaller than with the tree edit distance. The tree embedding distance does not perform much better than the simple node intersection. We will now briefly discuss how the tree embedding distance works, and why it performs poorly on typical address trees.

The tree embedding distance is computed by building a parsing hierarchy for a tree \mathbf{T} . In each phase i

a tree \mathbf{T}^i is obtained by nodes of the tree \mathbf{T}^{i-1} . The parsing procedure starts with the tree $\mathbf{T}^0 = \mathbf{T}$, and it stops if $|\mathbf{T}^i| = 1$. Figure 16 shows the parse trees \mathbf{T}^0 , \mathbf{T}^1 and \mathbf{T}^2 for an example tree \mathbf{T} that is shaped like a typical address tree. In our illustration we use different types of brackets to label the newly created nodes for the different situations in which nodes are merged:

- Contiguous sequences of children are split into blocks of length 2 and 3, and the blocks are contracted. The nodes 1, 3, 5, 10, 11 of \mathbf{T}^0 become the two new nodes (1, 3, 5) and (10, 11) of \mathbf{T}^1 .
- A lone leaf child is merged with the parent node if it is the leftmost lone leaf. The nodes SN and 3 in \mathbf{T}^0 become the new node {SN, 3} in \mathbf{T}^1 .
- Chains (paths of degree-two nodes) are split into blocks of length 2 and 3, and the blocks are contracted. The nodes 2, A, (1, 3, 7) of \mathbf{T}^1 become the new node [2, A, (1, 3, 7)] of \mathbf{T}^2 .

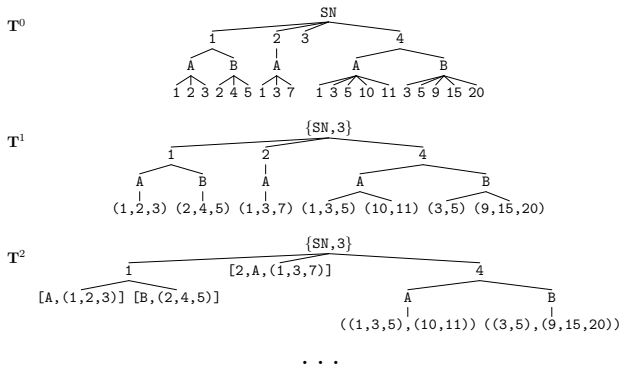


Figure 16: Parse trees for an example tree \mathbf{T} .

Each node in the parsing hierarchy corresponds to a set of nodes (“valid subtree”) in the original tree. The bag P of all valid subtrees corresponding to all nodes of the final hierarchical parsing structure (tagged with a phase label to distinguish between subtrees in different phases) is treated the same way we treat the pq -gram profile in order to calculate the distance.

The resulting bag P contains nodes corresponding to (1) single nodes, (2) node chains with parent-child relationship, (3) contiguous leaf children, and (4) subtrees. Single nodes contain no structural information, parent-child chains only vertical, leaf sequences contain only horizontal structure information. Only subtrees reflect both, horizontal and vertical structure. Table 2 gives an overview of how many nodes of each type are obtained in each phase for the example tree. We can see that 65% of all nodes are single nodes containing no structural information. Only 19% of nodes correspond to subtrees.

Trees with many leaves at the deepest level are parsed bottom-up, and the structure of the inner nodes

has less impact on the distance. For this reason the tree embedding distance performs only slightly better than a simple node intersection on our real world data.

phase	single node	chain	cont. leaf	subtree
0	29	-	-	-
1	8	1	7	-
2	4	1	2	3
3	2	-	-	4
4	1	-	-	3
5	-	-	-	2
6	-	-	-	1
total	44	2	9	13
	65%	3%	13%	19%

Table 2: Types of valid subtrees in the different phases.

8 Conclusions

Our work is motivated by a data integration scenario from the Municipality of Bozen, where data from different sources have to be integrated and no common keys exist. Data have to be joined over residential addresses, which in practice have some undesirable properties, and exact joins completely fail. To overcome these problems we introduced address trees as a representation of residential addresses. This reduces the integration to an approximate join on address trees.

We presented a new distance measure, the pq -gram distance, for ordered labeled trees as an effective and efficient approximation for the well known tree edit distance. We provided an algorithm for the computation of pq -grams in $O(n)$ time, where n is the number of tree nodes. Based on the profile the pq -gram distance can be computed in $O(n \log n)$ time. We discussed a scalable implementation using an interval representation of trees in a relational database.

The pq -gram distance behaves differently from the tree edit distance for structural and local changes. It gives more weight to edit operations that cause big changes in the tree structure. This property turned out to be relevant in our application domain.

Detailed experiments on real and synthetic data confirmed that the pq -gram distance is orders of magnitude faster than the tree edit distance for large trees. The accuracy of the pq -gram distance for real world data from the municipality domain turned out to be clearly better than other approximations of the tree edit distance.

In the future we will investigate additional application areas and apply the pq -gram distance for data cleaning and the comparison of XML data.

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