Temporal Knowledge and Ontologies

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Introduction: Constructors for Building Temporal Ontologies

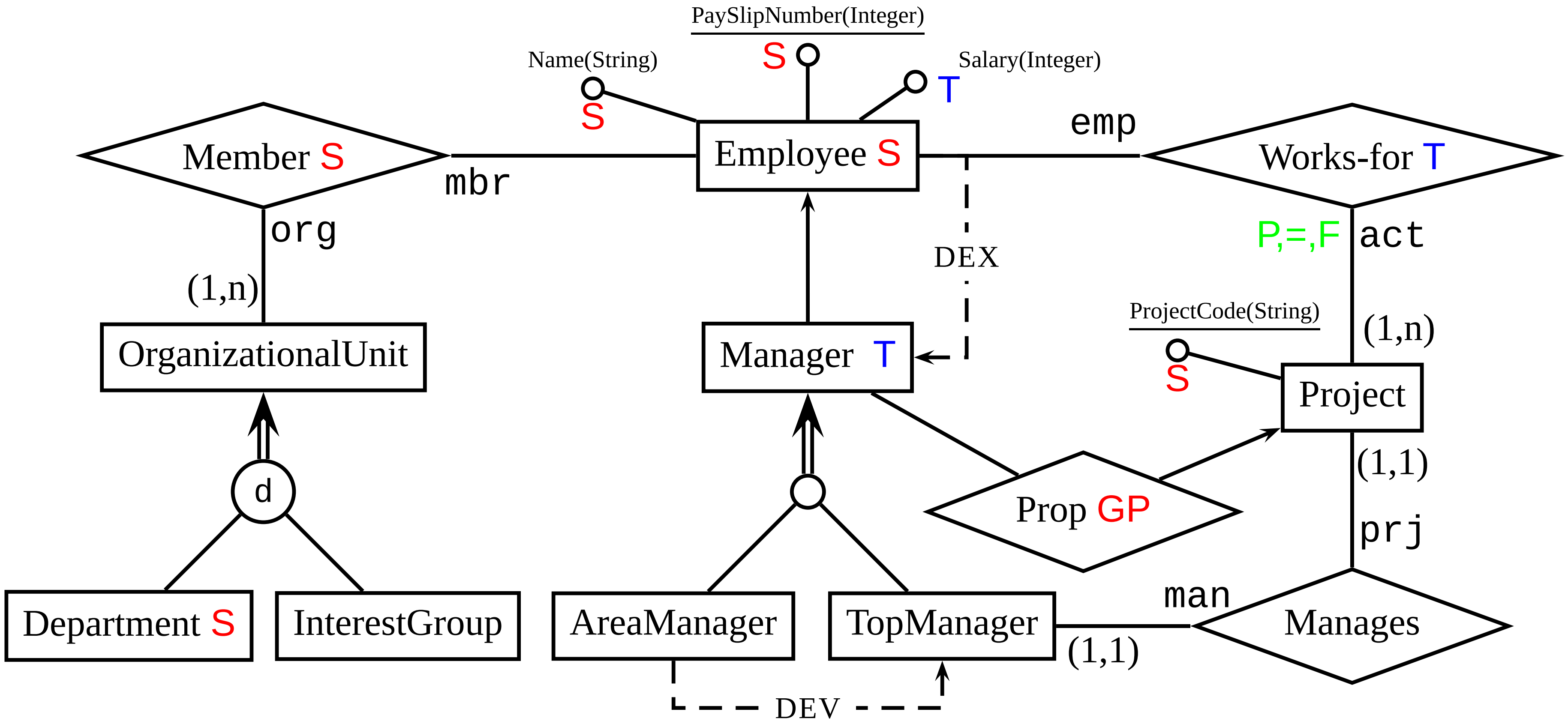
- Timestamping.

  The data model should distinguish between temporal and atemporal modeling constructs.

- Evolution Constraints.

  1. *Object Migration*: The possibility for an object to change its class membership;
  2. *Dynamic Relationships*: Either generate objects starting from other objects, or link objects existing at different times.

An Example: The $\mathcal{ER}_{VT}$ Company Schema



Introduction: Motivations

Give a formalization based on set-theory of the various temporal constructs used to model temporal information systems.

1. Clarify the meaning of the various temporal constructs;
2. Verify the validity of standard modeling requirements defined for temporal data models;
3. Give a formal definition of quality criteria: Entity/Relationships/Schema consistency, Entity/Relationships Subsumption, Logical Implication;
4. Investigate the complexity of automatically checking these quality criteria.

Introduction: Modeling Requirements in a Temporal Setting

- Orthogonality. Temporal constructs should be specified separately and independently for classes, relationships, and attributes.
- Upward Compatibility. Preserve the non-temporal semantics of legacy conceptual schemas when embedded into temporal schemas.
- Snapshot Reducibility. A snapshot of the temporal database is described by the same schema without temporal constructs interpreted atemporally.
  - We should be able to fully rebuild a temporal database by starting from the single temporal snapshots.

Outline

- The Temporal Ontological Language $\mathcal{ER}_{VT}$
- $\mathcal{DLR}_{\mathcal{US}}$: A Temporal Description Logic
- Modeling Timestamping
- Modeling Evolution Constraints
  - Status Classes
  - Transitions
  - Generation Relationships
  - Cross-Time Relationships
- Complexity Results
  - Undecidability Result

$\mathcal{ER}_{VT}$: The Proposed Temporal Conceptual Model

$\mathcal{ER}_{VT}$ is a temporal extended Entity-Relationship model able to capture Validity Time with the following features:

- it is equipped with both a linear and a graphical syntax;
- it has a model-theoretic semantics;
- it is a full-fledged conceptual model with constructors for representing both timestamping and evolution constraints.

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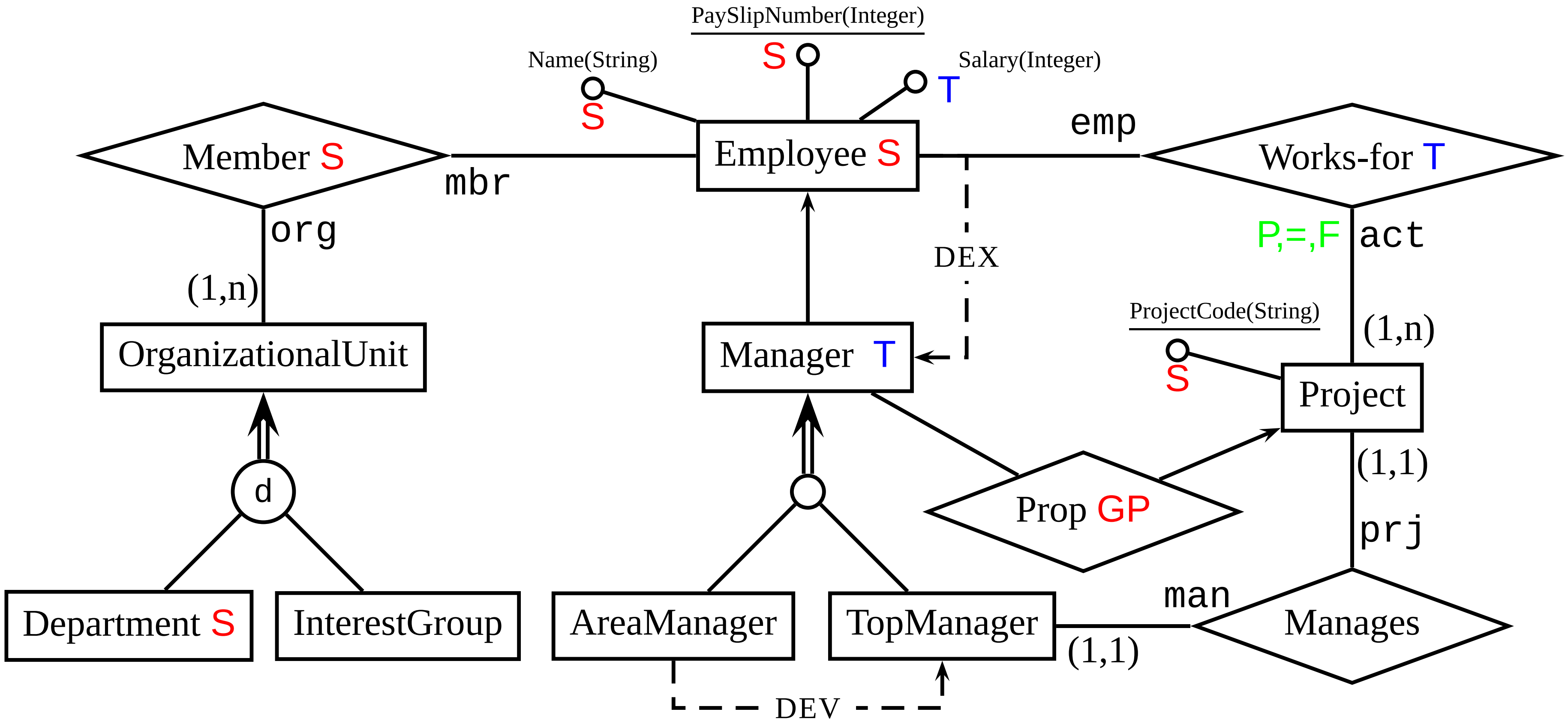

The Model-Theoretic Semantics for $\mathcal{ER}_{VT}$

An interpretation, called *temporal database state*, for an $\mathcal{ER}_{VT}$ schema Σ is a tuple $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_{D}, \cdot^{\mathcal{B}(t)})$:

- $\mathcal{T} = (\mathcal{T}_p, <)$, is the flow of time, where $\mathcal{T}_p$ is a set of time points (or chronons) and $<$ is a binary precedence relation on $\mathcal{T}_p$;
- $\Delta^{\mathcal{B}}$ is a nonempty set of abstract objects;
- $\Delta^{\mathcal{B}}_{D}$ is the set of basic domain values;
- $\cdot^{\mathcal{B}(t)}$ is a function that for each $t \in \mathcal{T}$ maps:
  - Every domain symbol D_i into a set $D_i^{\mathcal{B}(t)} = \Delta^{\mathcal{B}}_{D_i} \subseteq \Delta^{\mathcal{B}}_{D}$.
  - Every class C to a set $C^{\mathcal{B}(t)} \subseteq \Delta^{\mathcal{B}}$.
  - Every n-ary relationship R connecting the classes $C_1, \ldots, C_n$ to a set $R^{\mathcal{B}(t)}$, such that, $r \in R^{\mathcal{B}(t)} \rightarrow (r = \langle U_1 : o_1, \ldots, U_n : o_n \rangle \wedge \forall i \in \{1, \ldots, n\}. o_i \in C_i^{\mathcal{B}(t)})$.
  - Every attribute A to a set $A^{\mathcal{B}(t)} \subseteq \Delta^{\mathcal{B}} \times \Delta^{\mathcal{B}}_{D}$.

The Model-Theoretic Semantics for $\mathcal{ER}_{VT}$ (Cont.)

$\mathcal{B}$ is said a *legal temporal database state* if it satisfies all constraints expressed in the schema. Thus, for all $t \in \mathcal{T}$:

- If C_1 ISA C_2, then, $C_1^{\mathcal{B}(t)} \subseteq C_2^{\mathcal{B}(t)}$
- If R_1 ISA R_2, then, $R_1^{\mathcal{B}(t)} \subseteq R_2^{\mathcal{B}(t)}$
- If $\text{ATT}(C) = \langle A_1 : D_1, \ldots, A_h : D_h \rangle$, then:
  $o \in C^{\mathcal{B}(t)} \rightarrow (\forall i \in \{1, \ldots, h\}, \exists! a_i . \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)} \wedge \forall a_i . \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)} \rightarrow a_i \in \Delta_{D_i}^{\mathcal{B}})$
- For each cardinality constraint $\text{CARD}(C, R, U)$, then:
  $o \in C^{\mathcal{B}(t)} \rightarrow \text{CMIN}(C, R, U) \leq \#\{r \in R^{\mathcal{B}(t)} \mid r[U] = o\} \leq \text{CMAX}(C, R, U)$
- If $\{C_1, \ldots, C_n\}$ DISJ C, then:
  $\forall i \in \{1, \ldots, n\} . C_i \text{ ISA } C \wedge \forall j \in \{1, \ldots, n\}, j \neq i . C_i^{\mathcal{B}(t)} \cap C_j^{\mathcal{B}(t)} = \emptyset$
- If $\{C_1, \ldots, C_n\}$ COVER C, then:
  $\forall i \in \{1, \ldots, n\} . C_i \text{ ISA } C \wedge C^{\mathcal{B}(t)} = \bigcup_{i=1}^{n} C_i^{\mathcal{B}(t)}$

Quality Criteria $\mathcal{ER}_{VT}$

The following quality criteria can be defined:

1. C (R) is satisfiable if there exists a legal temporal database state $\mathcal{B}$ for Σ such that $C^{\mathcal{B}(t)} \neq \emptyset$ ($R^{\mathcal{B}(t)} \neq \emptyset$), for some $t \in \mathcal{T}$;
2. Σ is satisfiable if there exists a legal temporal database state $\mathcal{B}$ for Σ that satisfies at least one class in Σ ($\mathcal{B}$ is said a *model* for Σ);
3. C_1 (R_1) is subsumed by C_2 (R_2) in Σ if every legal temporal database state for Σ is also a legal temporal database state for C_1 ISA C_2 (R_1 ISA R_2);
4. A schema Σ' is logically implied by a schema Σ over the same signature if every legal temporal database state for Σ is also a legal temporal database state for Σ'.

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The $\mathcal{DLR}_{\mathcal{US}}$ Temporal Description Logic

$\mathcal{DLR}_{\mathcal{US}}$ is obtained by combining the propositional linear temporal logic with **Since** and **Until** and the description logic $\mathcal{DLR}$.

$$
\begin{array}{rcl}
C & \rightarrow & \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid \exists^{\leqslant k}[U_i]R \mid \\
& & \Diamond^+ C \mid \Diamond^- C \mid \Box^+ C \mid \Box^- C \mid \oplus C \mid \ominus C \mid C_1 \mathcal{U} C_2 \mid C_1 \mathcal{S} C_2 \\
R & \rightarrow & \top_n \mid RN \mid \neg R \mid R_1 \sqcap R_2 \mid U_i/n : C \mid \\
& & \Diamond^+ R \mid \Diamond^- R \mid \Box^+ R \mid \Box^- R \mid \oplus R \mid \ominus R \mid R_1 \mathcal{U} R_2 \mid R_1 \mathcal{S} R_2
\end{array}
$$

- $\mathcal{DLR}_{\mathcal{US}}$ Knowledge Base is a collection of axioms on relationship and entity expressions: $R_1 \sqsubseteq R_2$; $C_1 \sqsubseteq C_2$
- $\mathcal{DLR}_{\mathcal{US}}$ is a fragment of the first-order temporal logic $L^{\{\mathbf{since},\mathbf{until}\}}$

The $\mathcal{DLR}_{\mathcal{US}}$ Semantics

A *temporal interpretation* over $\mathcal{T}$ is a triple $\mathcal{I} = \langle \mathcal{T}, \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}(t)} \rangle$, where $\cdot^{\mathcal{I}(t)}$ is a function that for each $t \in \mathcal{T}$ maps:

- $CN^{\mathcal{I}(t)} \subseteq \Delta$
- $(\exists^{\leqslant k}[U_i]R)^{\mathcal{I}(t)} = \{d \in \Delta \mid \sharp\{\langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t)} \mid d_i = d\} \leqslant k\}$
- $(\diamond^+ C)^{\mathcal{I}(t)} = \{d \in \Delta \mid \exists v > t \mathbf{.}\, d \in C^{\mathcal{I}(v)}\}$
- $(C_1 \mathcal{U} C_2)^{\mathcal{I}(t)} = \{d \in \Delta \mid \exists v > t \mathbf{.}\, (d \in C_2^{\mathcal{I}(v)} \wedge \forall w \in (t, v) \mathbf{.}\, d \in C_1^{\mathcal{I}(w)})\}$
- $(C_1 \mathcal{S} C_2)^{\mathcal{I}(t)} = \{\, d \in \top^{\mathcal{I}(t)} \mid \exists v < t \mathbf{.}\, (d \in C_2^{\mathcal{I}(v)} \wedge \forall w \in (v, t) \mathbf{.}\, d \in C_1^{\mathcal{I}(w)})\}$

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- $(C_1 \mathcal{U} C_2)^{\mathcal{I}(t)} = \{d \in \Delta \mid \exists v > t.\, (d \in C_2^{\mathcal{I}(v)} \wedge \forall w \in (t, v).\, d \in C_1^{\mathcal{I}(w)})\}$
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- $RN^{\mathcal{I}(t)} \subseteq (\top_n)^{\mathcal{I}(t)} \subseteq (\Delta^{\mathcal{I}})^n$
- $(U_i/n : C)^{\mathcal{I}(t)} = \{\langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid d_i \in C^{\mathcal{I}(t)}\}$
- $(\oplus\, R)^{\mathcal{I}(t)} = \{\langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t+1)}\}$
- $(\ominus\, R)^{\mathcal{I}(t)} = \{\langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t-1)}\}$

Temporal Operators: 'Until'

Given a Temporal Interpretation, $\mathcal{T}$, a time, t, an object, $o \in \Delta$, and two concept expressions C, D:

$$\langle \mathcal{I}, t, o \rangle \models C \mathcal{U} D \text{ iff there exists } t' \; s.t. \; (t' > t) \wedge \langle \mathcal{I}, t', o \rangle \models D \wedge$$
$$\text{for all } t'' \; s.t. \; (t < t'' < t') \rightarrow \langle \mathcal{I}, t'', o \rangle \models C$$

Examples:

$$\begin{aligned} \texttt{Start\_Lecture} &\sqsubseteq \texttt{Talk}\, \mathcal{U}\, \texttt{End\_Lecture} \\ \texttt{Born} &\sqsubseteq \texttt{Alive}\, \mathcal{U}\, \texttt{Dead} \\ \texttt{Request} &\sqsubseteq \texttt{Reply}\, \mathcal{U}\, \texttt{Acknowledgement} \end{aligned}$$

Equivalences in $\mathcal{DLR}_{\mathcal{US}}$

The temporal operators $\Diamond^+$ ($\Diamond^-$) and $\Box^+$ ($\Box^-$) are duals (for concept expressions):

$$\neg\Box^+ C \equiv \Diamond^+ \neg C$$

$\Diamond^+$ (and then $\Box^+$) can be rewritten in terms of $\mathcal{U}$

$$\Diamond^+ C \equiv \top \mathcal{U} C$$

$\Diamond^-$ (and then $\Box^-$) can be rewritten in terms of $\mathcal{S}$

$$\Diamond^- C \equiv \top \mathcal{S} C$$

$\oplus$ can be rewritten in terms of $\mathcal{U}$

$$\oplus\, C \equiv \bot \mathcal{U} C$$

$\ominus$ can be rewritten in terms of $\mathcal{S}$

$$\ominus\, C \equiv \bot \mathcal{S} C$$

Interpretation of $\mathcal{DLR}_{\mathcal{US}}$ Knowledge Bases

- An interpretation $\mathcal{I}$ satisfies an axiom $C_1 \sqsubseteq C_2$ iff:
  $C_1^{\mathcal{I}(t)} \subseteq C_2^{\mathcal{I}(t)}$, for all $t \in \mathcal{T}$.
- An interpretation $\mathcal{I}$ satisfies an axiom $R_1 \sqsubseteq R_2$ iff:
  $R_1^{\mathcal{I}(t)} \subseteq R_2^{\mathcal{I}(t)}$, for all $t \in \mathcal{T}$.
- A knowledge base, Σ, is satisfiable if there is an interpretation that satisfies all the axioms in Σ (in symbols, $\mathcal{I} \models \Sigma$).

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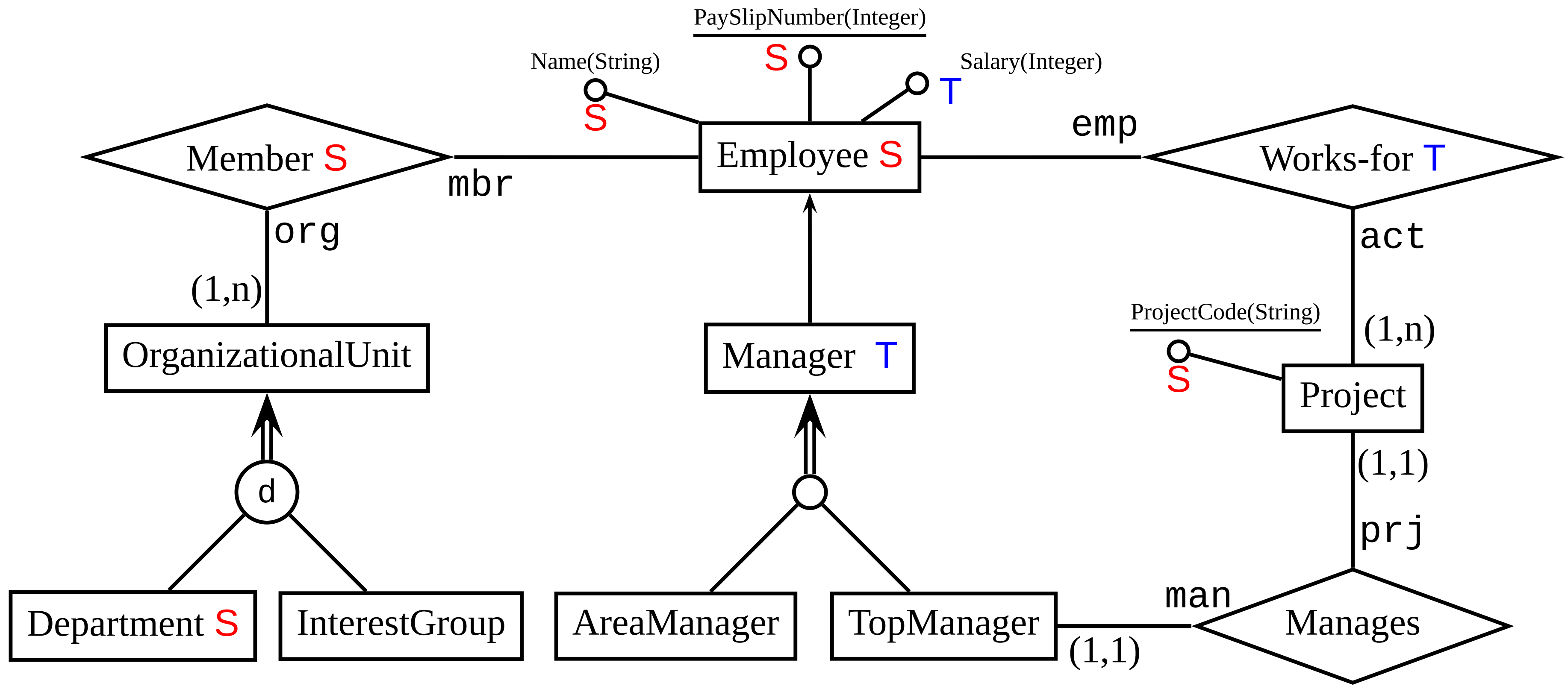
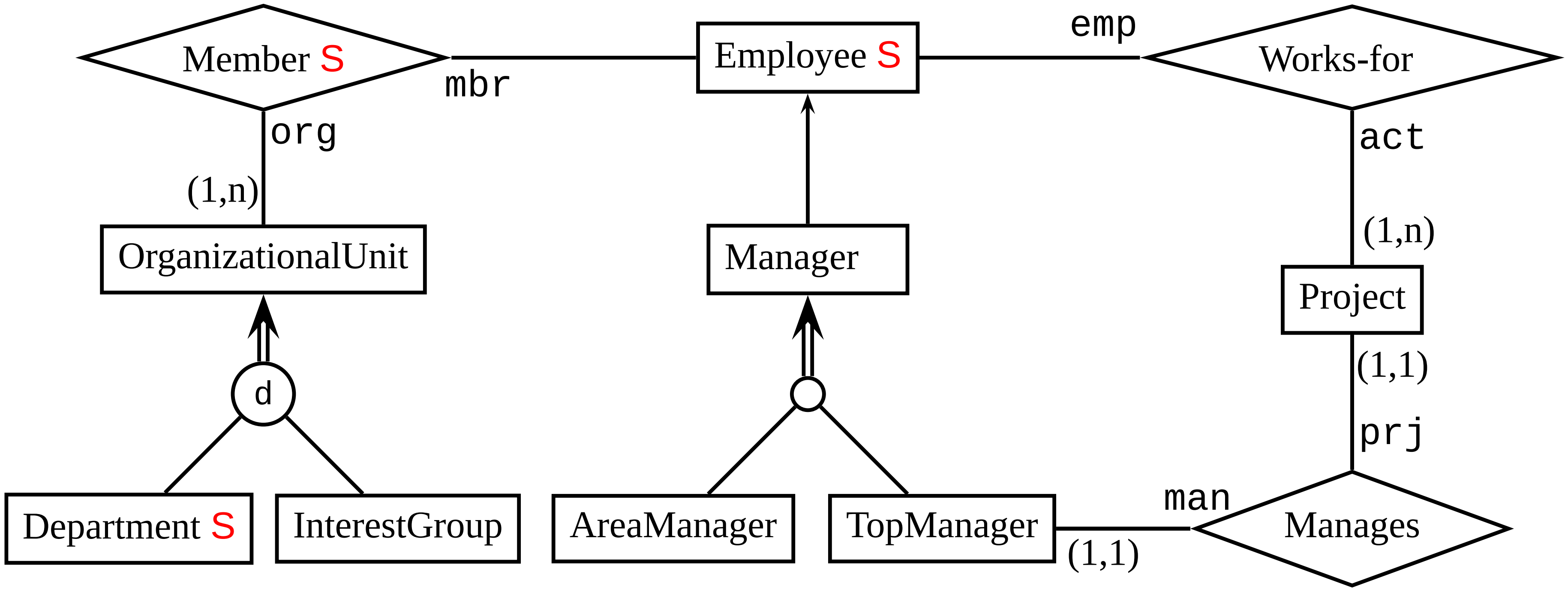
$\mathcal{ER}_{VT}$ & Timestamping



- At the syntactical level, $\mathcal{ER}_{VT}$ supports timestamping of entities, relationships, and attributes using two different marks:
  - S, for **Snapshot** constructs: Each of their instances has a global lifetime;
  - T, for **Temporary** constructs: Each of their instances has a limited lifetime.

A Semantics for Timestamps



- $o \in C^{\mathcal{B}(t)} \rightarrow \forall t' \in \mathcal{T}.\, o \in C^{\mathcal{B}(t')}$

  $\texttt{Employee} \sqsubseteq (\Box^{+}\texttt{Employee}) \sqcap (\Box^{-}\texttt{Employee})$

- $r \in R^{\mathcal{B}(t)} \rightarrow \forall t' \in \mathcal{T}.\, r \in R^{\mathcal{B}(t')}$

  $\texttt{Member} \sqsubseteq (\Box^{+}\texttt{Member}) \sqcap (\Box^{-}\texttt{Member})$

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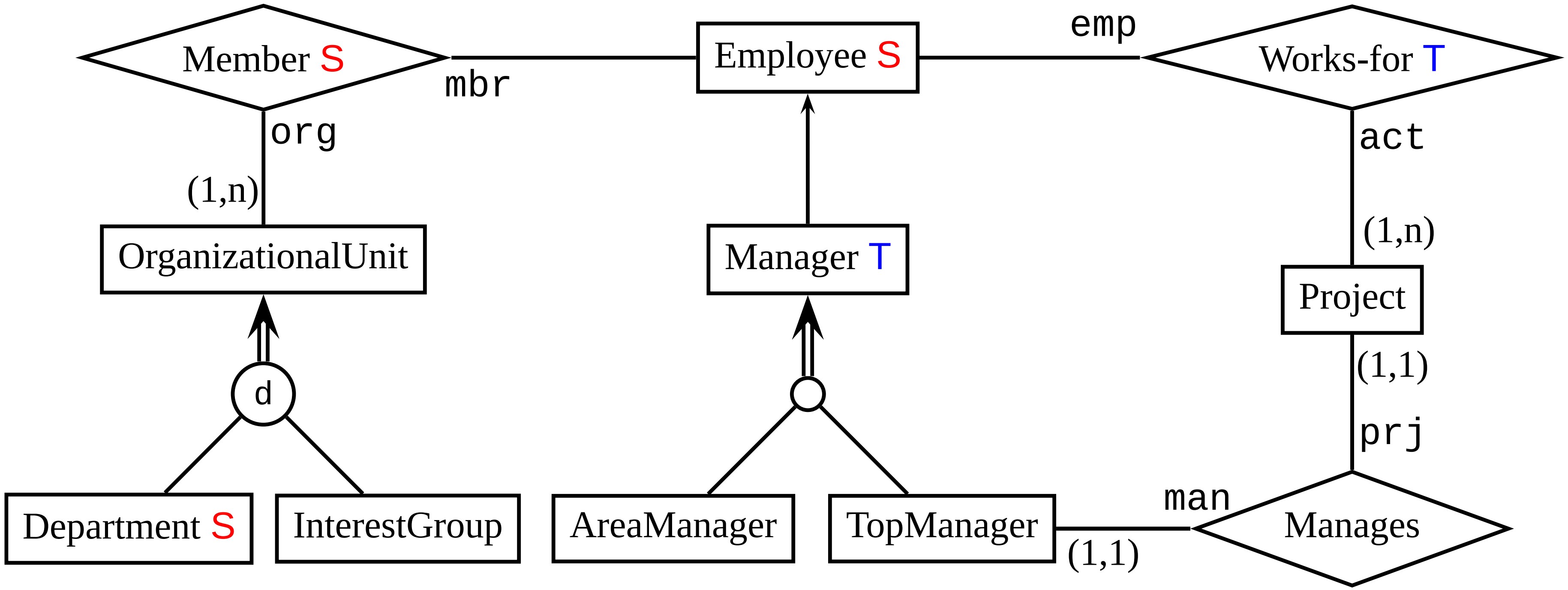

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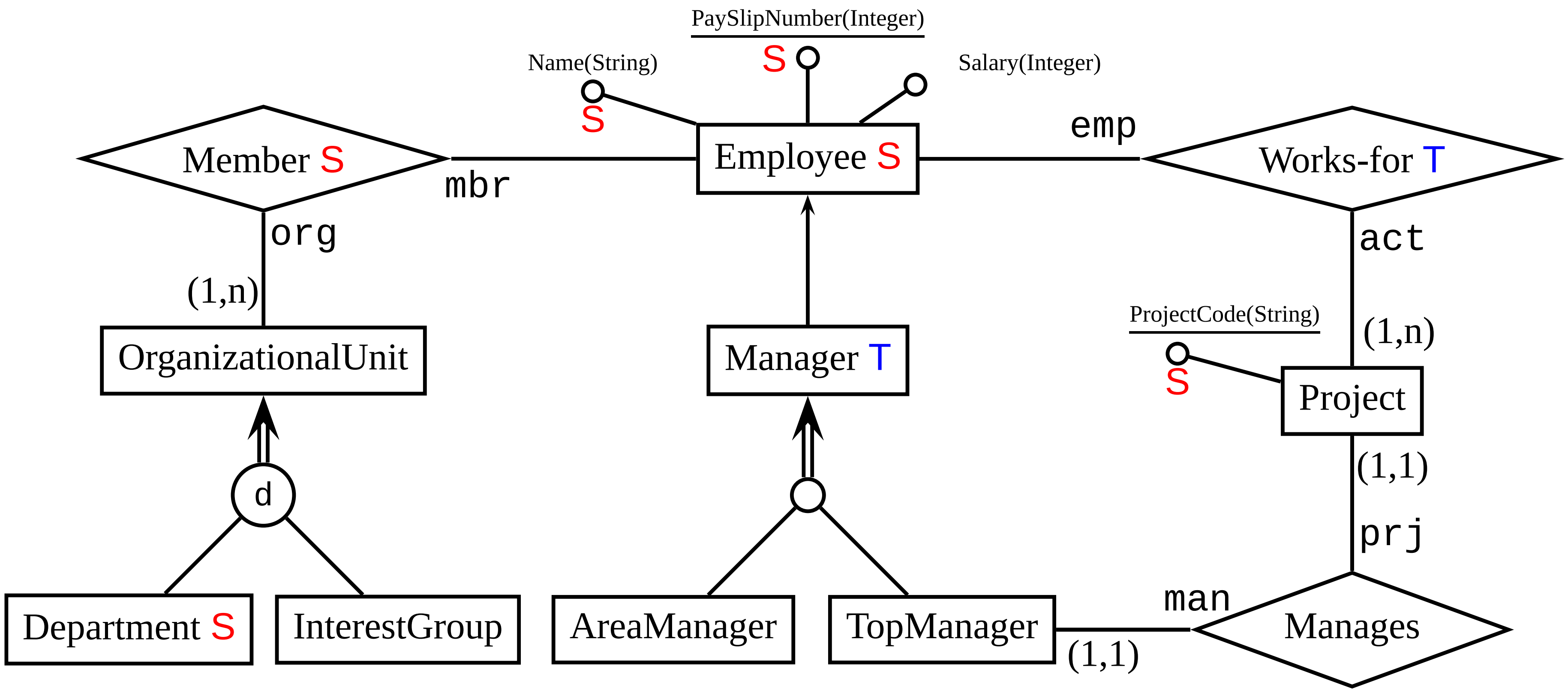
- $o \in C^{\mathcal{B}(t)} \rightarrow \exists t' \neq t. o \notin C^{\mathcal{B}(t')}$

  $\texttt{Manager} \sqsubseteq (\Diamond^+ \neg \texttt{Manager}) \sqcup (\Diamond^- \neg \texttt{Manager})$

- $r \in R^{\mathcal{B}(t)} \rightarrow \exists t' \neq t. r \notin R^{\mathcal{B}(t')}$

  $\texttt{Works-for} \sqsubseteq (\Diamond^+ \neg \texttt{Works-for}) \sqcup (\Diamond^- \neg \texttt{Works-for})$

Timestamping Attributes



- $(o \in C^{\mathcal{B}(t)} \wedge \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)}) \rightarrow \forall t' \in \mathcal{T}. \langle o, a_i \rangle \in A_i^{\mathcal{B}(t')}$

  $\texttt{Employee} \sqsubseteq \exists^{=1}[\texttt{From}](\texttt{Name} \sqcap \texttt{To}/2 : \texttt{String}) \sqcap \exists^{=1}[\texttt{From}]\Box^{*}\texttt{Name}$

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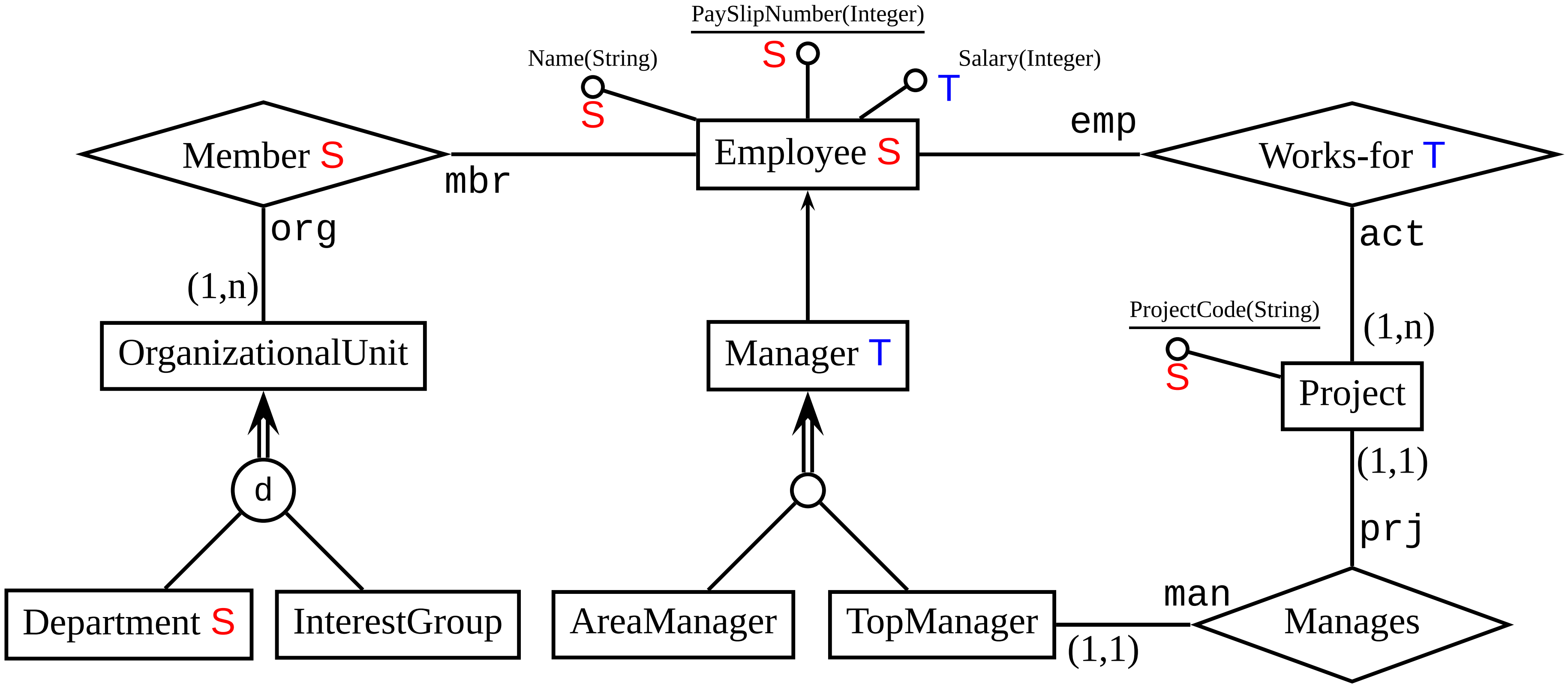

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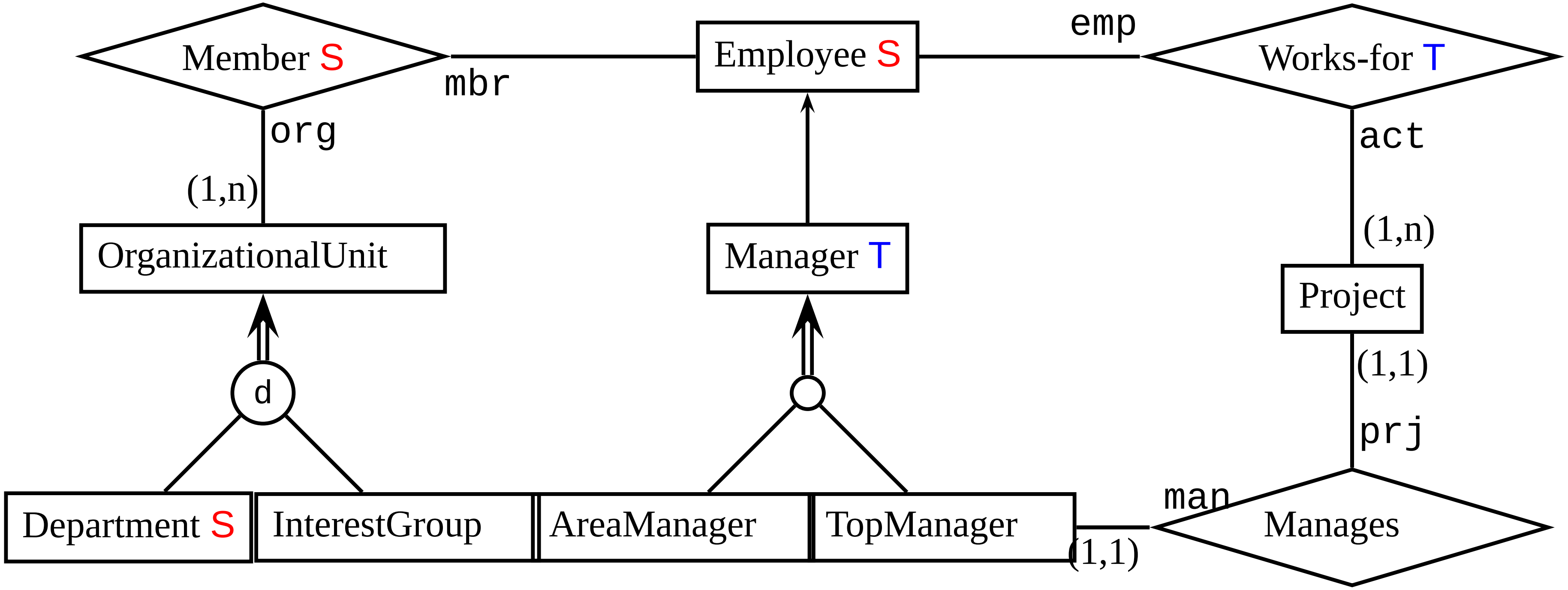
- $(o \in C^{\mathcal{B}(t)} \wedge \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)}) \rightarrow \exists t' \neq t.\, \langle o, a_i \rangle \notin A_i^{\mathcal{B}(t')}$

  $\texttt{Employee} \sqsubseteq \exists^{=1}[\texttt{From}](\texttt{Salary} \sqcap \texttt{To}/2 : \texttt{Integer}) \sqcap$

  $\exists^{=1}[\texttt{From}](\texttt{Salary} \sqcap (\Diamond^{+}\neg\texttt{Salary} \sqcup \Diamond^{-}\neg\texttt{Salary}))$

Logical Consequences Involving Timestamps



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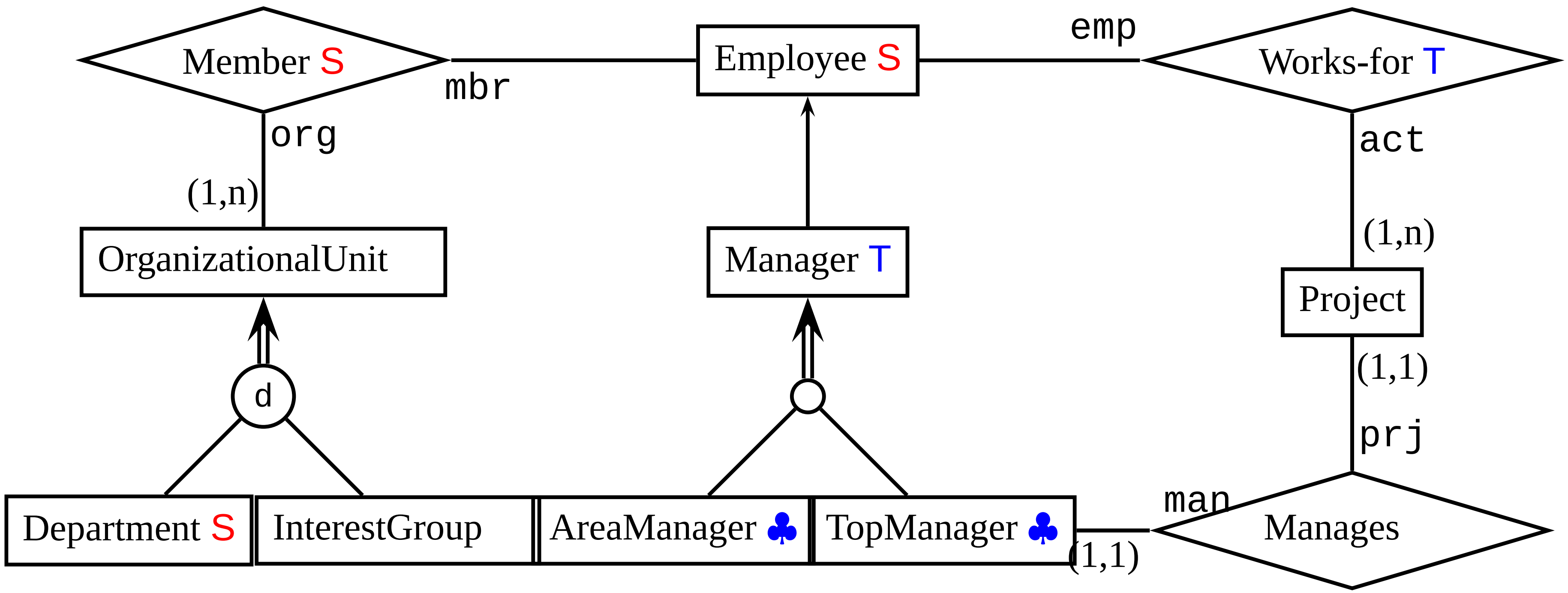

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- Sub-entities of temporary entities must be temporary.

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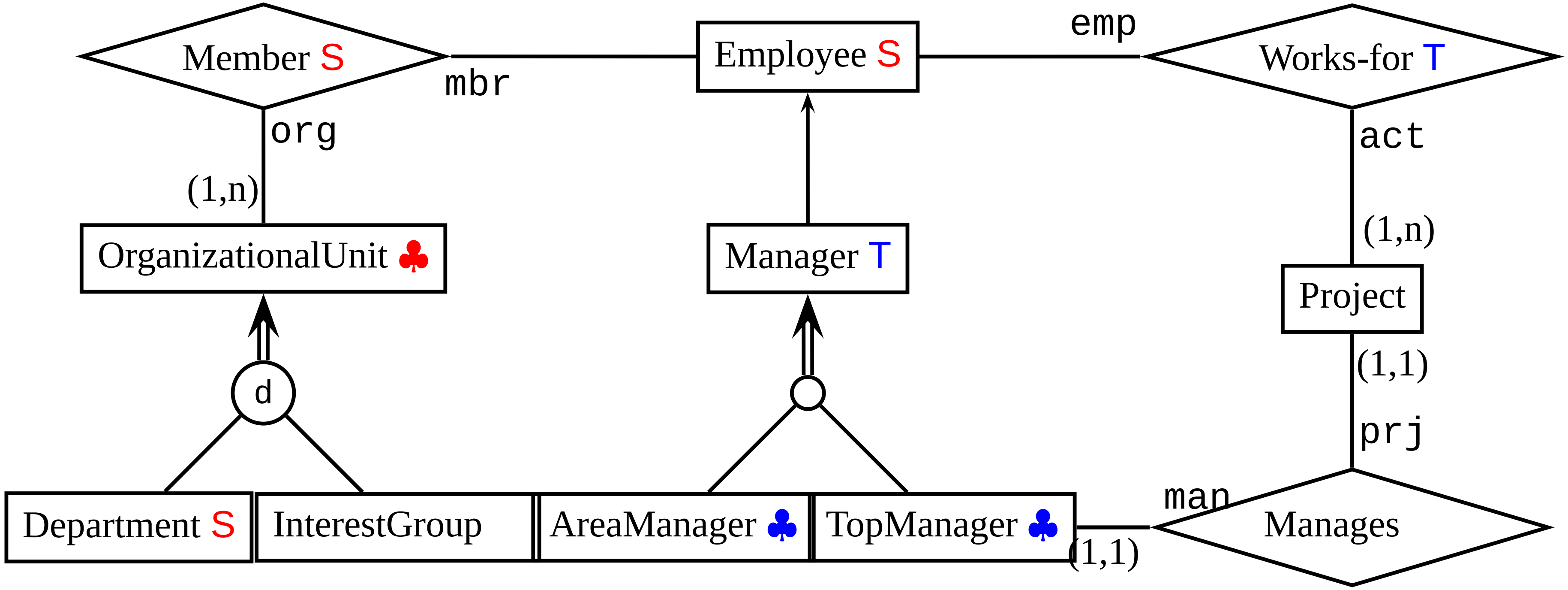

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- Participants of snapshot relationships must be snapshot entities when they participate at least once.

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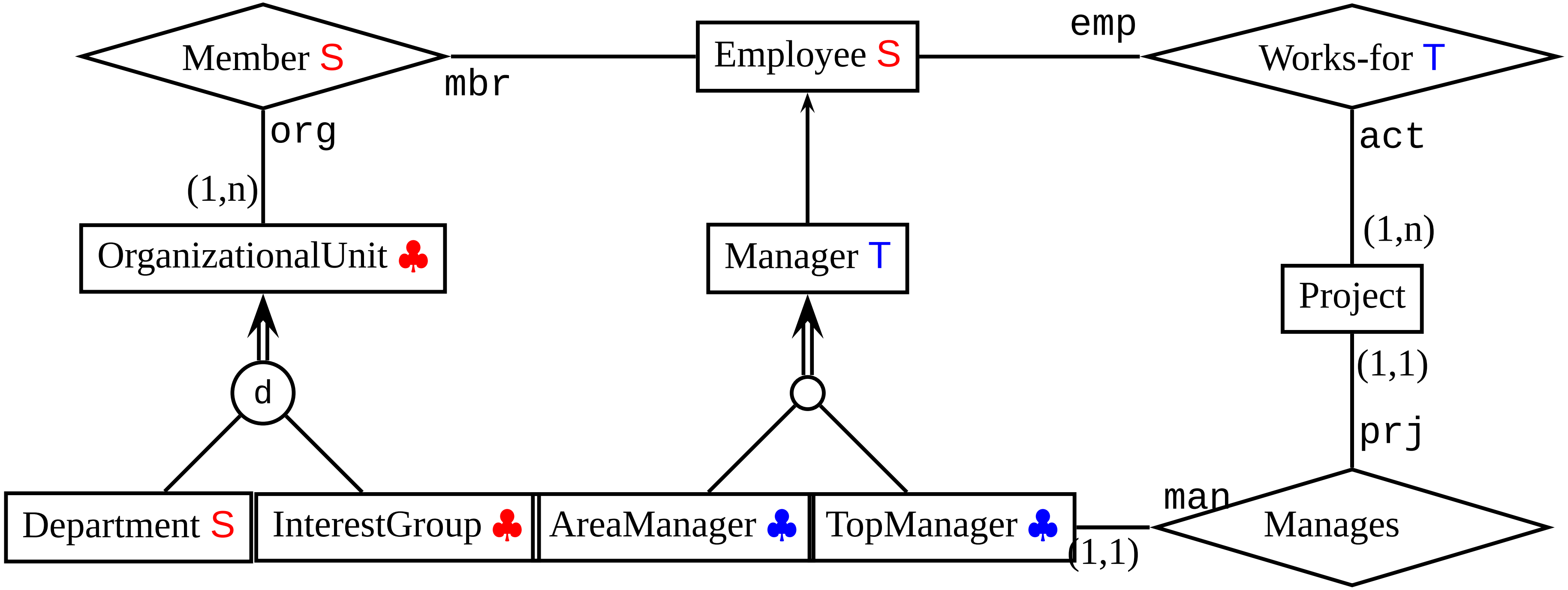

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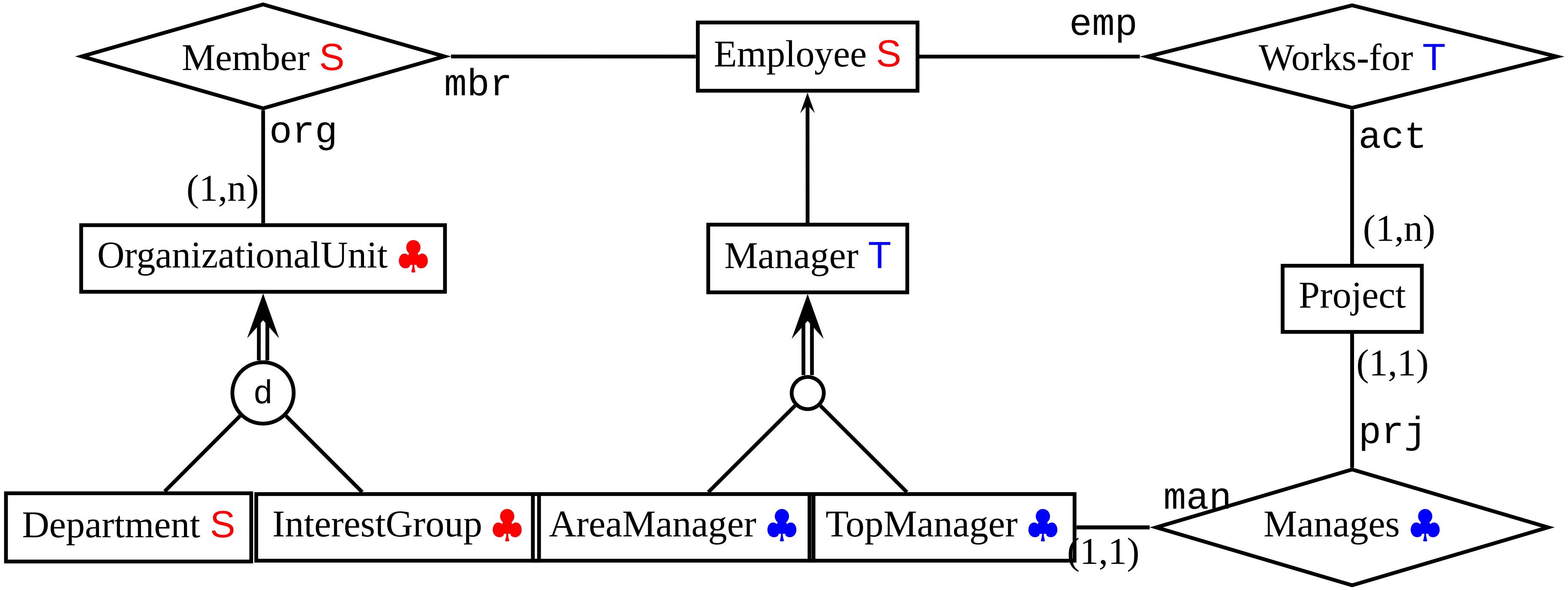

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- Participants of snapshot relationships must be snapshot entities when they participate at least once.
- A schema is inconsistent if exactly one of a whole set of snapshot partitioning sub-entities is temporary.
- A relationship is temporary if one of the participating entities is temporary.

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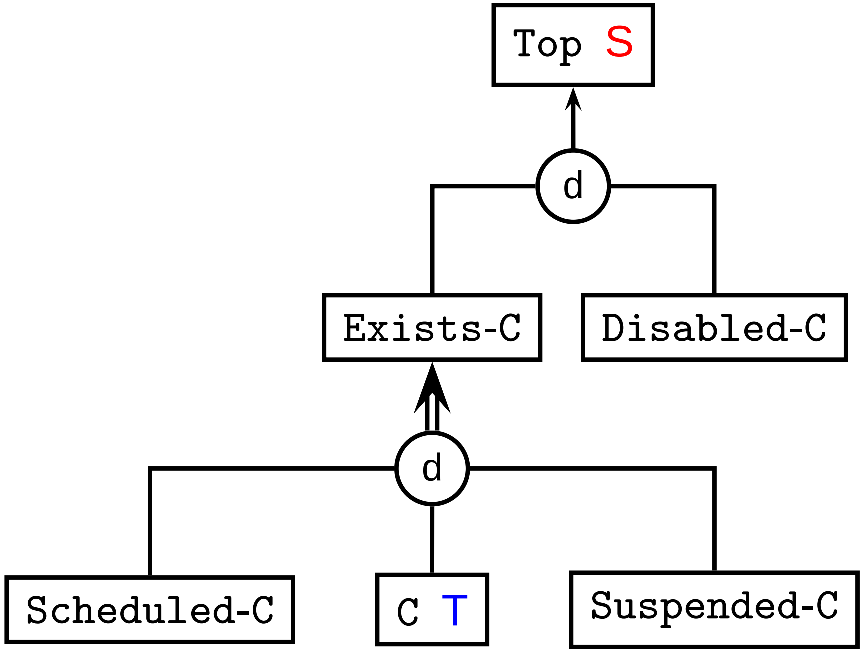
Evolution Constraints: Status Classes

Describe the evolving status of membership of each object in the class. Four different statuses can be specified, together with precise transitions between them:

- Scheduled. An object is scheduled if its existence within the class is known but its membership in the class will only become effective some time later.
- Active. The status of an object is active if the object is a full member of the class.
- Suspended. This status qualifies objects that exist as members of the class, but are to be seen as inactive members of the class.
- Disabled. It is used to model expired objects in a class.

Constraints and Semantics for Status Classes



(EXISTS) *Existence persists until Disabled.*

$$o \in \texttt{Exists-C}^{\mathcal{B}(t)} \rightarrow \forall t' > t. (o \in \texttt{Exists-C}^{\mathcal{B}(t')} \vee o \in \texttt{Disabled-C}^{\mathcal{B}(t')})$$

$$\texttt{Exists-C} \sqsubseteq \Box^{+}(\texttt{Exists-C} \sqcup \texttt{Disabled-C})$$

(DISAB1) *Disabled persists.*

$$o \in \texttt{Disabled-C}^{\mathcal{B}(t)} \rightarrow \forall t' > t. o \in \texttt{Disabled-C}^{\mathcal{B}(t')}$$

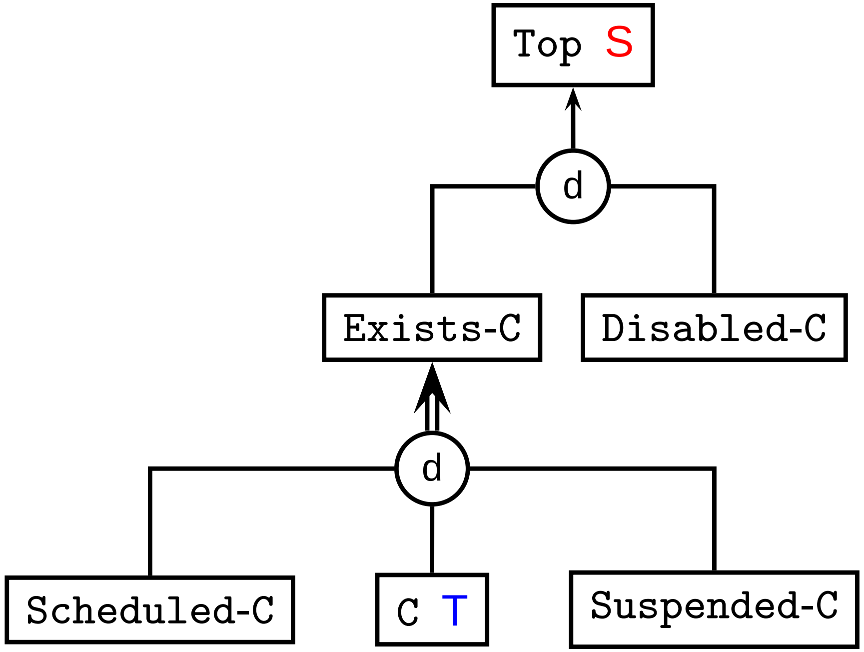
$$\texttt{Disabled-C} \sqsubseteq \Box^{+}\texttt{Disabled-C}$$

(DISAB2) *Disabled was Active in the past.*

$$o \in \texttt{Disabled-C}^{\mathcal{B}(t)} \rightarrow \exists t' < t. o \in \texttt{C}^{\mathcal{B}(t')}$$

$$\texttt{Disabled-C} \sqsubseteq \Diamond^{-}\texttt{C}$$

Constraints and Semantics for Status Classes (Cont.)



(SUSP) *Suspended was Active in the past.*

$$o \in \texttt{Suspended-C}^{\mathcal{B}(t)} \rightarrow \exists t' < t. o \in \texttt{C}^{\mathcal{B}(t')}$$

$$\texttt{Suspended-C} \sqsubseteq \Diamond^{-}\texttt{C}$$

(SCH1) *Scheduled will eventually become Active.*

$$o \in \texttt{Scheduled-C}^{\mathcal{B}(t)} \rightarrow \exists t' > t. o \in \texttt{C}^{\mathcal{B}(t')}$$

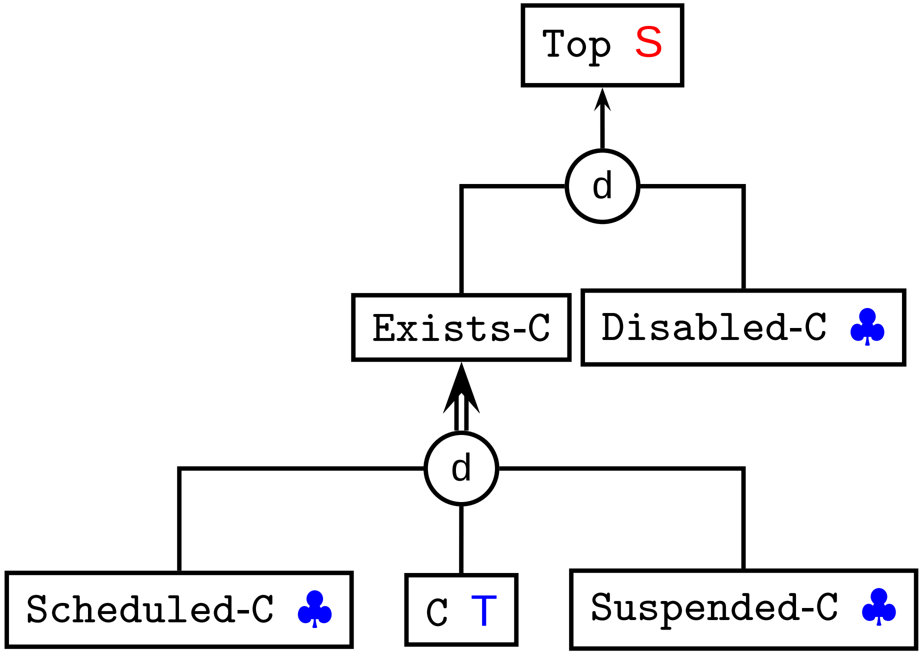
$$\texttt{Scheduled-C} \sqsubseteq \Diamond^{+}\texttt{C}$$

(SCH2) *Scheduled can never follow Active.*

$$o \in \texttt{C}^{\mathcal{B}(t)} \rightarrow \forall t' > t. o \notin \texttt{Scheduled-C}^{\mathcal{B}(t')}$$

$$\texttt{C} \sqsubseteq \Box^{+}\neg\texttt{Scheduled-C}$$

Logical Consequences from Status Classes



(TEMP) *Scheduled, Suspended and Disabled are temporary classes.*

(SCH3) *Scheduled persists until active.*

$$\texttt{Scheduled-C} \sqsubseteq \texttt{Scheduled-C}\,\mathcal{U}\,\texttt{C}.$$

(SCH4) *Scheduled cannot evolve directly to Disabled*

$$\texttt{Scheduled-C} \sqsubseteq \oplus\neg\texttt{Disbled-C}.$$

(DISAB3) *Disabled was active but it will never become active anymore*

$$\texttt{Disabled-C} \sqsubseteq \Diamond^{-}(\texttt{C} \sqcap \Box^{+}\neg\texttt{C}).$$

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Evolution Constraints: Transitions

Dynamic Transitions between classes model the notion of object migration from a source to a target class.

1. Dynamic Evolution, when an object ceases to be an instance of a source class.
   - **Example.** "An area manger can become a top manger while ceasing to be an area manager."

2. Dynamic Extension, when an object is still allowed to belong to the source.
   - **Example.** "An employee can become a manger."

Constraints and Semantics for Transitions

Specifying a transition between two classes means that:

1. We want to keep track of such migration;
2. Not necessarily all the objects in the source participate in the migration;
3. When the source class is a temporal class, migration involves only objects active or suspended.

Constraints and Semantics for Transitions (Cont.)

We introduce two classes denoted by either DEX_{C_1,C_2} or DEV_{C_1,C_2} to store the migration of objects from C_1 to C_2.

- Semantics for dynamic extension between classes C_1, C_2.
  $o \in \text{DEX}^{\mathcal{B}(t)}_{C_1,C_2} \rightarrow \left(o \in \left(\texttt{Suspended-C}_1{}^{\mathcal{B}(t)} \cup \texttt{C}_1{}^{\mathcal{B}(t)}\right) \wedge o \notin \texttt{C}_2{}^{\mathcal{B}(t)} \wedge o \in C_2^{\mathcal{B}(t+1)}\right)$
  $\text{DEX}_{C_1,C_2} \sqsubseteq \left(\texttt{Suspended-C}_1 \sqcup \texttt{C}_1\right) \sqcap \neg\texttt{C}_2 \sqcap \bigoplus C_2.$

Constraints and Semantics for Transitions (Cont.)

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Logical Consequences from Transitions

1. The classes DEX_{C_1,C_2} and DEV_{C_1,C_2} are temporary classes (actually, they are instantaneous).
2. Objects in the classes DEX_{C_1,C_2} and DEV_{C_1,C_2} cannot be disabled as C_2.
3. The target class C_2 cannot be snapshot (it becomes temporary if all of its members are involved in the migration).
4. The source class C_1 cannot be snapshot when it is involved into a dynamic evolution (it becomes temporary if all of its members are involved in the migration).
5. Dynamic evolution cannot involve sub-classes (Note: this implication doesn't hold for dynamic extension).
6. Dynamic extension between disjoint classes logically implies Dynamic evolution.

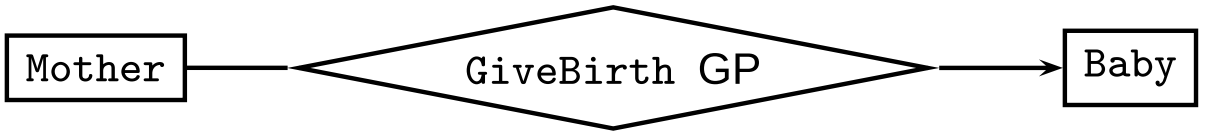
Outline

- The Temporal Ontological Language $\mathcal{ER}_{VT}$
- Modeling Timestamping
- Modeling Evolution Constraints
  - Status Classes
  - Transitions
  - Generation Relationships
  - Cross-Time Relationships
- Complexity Results
  - Undecidability Result

Evolution Constraints: Generation Relationships

Generation relationships represent processes that lead to the emergence of new instances starting from a set of instances.

1. Production Relationships, when the source objects survive the generation process (GP marked).



2. Transformation Relationships, when all the instances involved in the process are consumed (GT marked).

A Semantics for Generation Relationships

We model generation as binary relationships connecting a source class to a target one:

$$\text{REL}(R) = \langle \texttt{source} : C_1, \texttt{target} : \texttt{Scheduled-C}_2 \rangle$$

- Semantics for Production Relationships

  $\langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow (o_1 \in C_1^{\mathcal{B}(t)} \wedge o_2 \in \texttt{Scheduled-C}_2{}^{\mathcal{B}(t)} \wedge o_2 \in C_2^{\mathcal{B}(t+1)})$

  $R \sqsubseteq \texttt{source} : C_1 \sqcap \texttt{target} : (\texttt{Scheduled-C}_2 \sqcap \oplus C_2)$

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- Semantics for Transformation Relationships

  $\langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow (o_1 \in C_1^{\mathcal{B}(t)} \wedge o_1 \in \texttt{Disabled-C}_1{}^{\mathcal{B}(t+1)} \wedge$
  $o_2 \in \texttt{Scheduled-C}_2{}^{\mathcal{B}(t)} \wedge o_2 \in C_2^{\mathcal{B}(t+1)})$

  $R \sqsubseteq \texttt{source} : (C_1 \sqcap \oplus\texttt{Disabled-C}_1) \sqcap \texttt{target} : (\texttt{Scheduled-C}_2 \sqcap \oplus C_2)$

Logical Consequences from Generation Relationships

1. A generation relationship, R, is temporary (actually, it is instantaneous).
2. The target class, C_2, cannot be snapshot (it becomes temporary if total participation is specified).
3. The target class, C_2, cannot be disabled.
4. If R is a transformation relationship, then, C_1 cannot be snapshot.

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Evolution Constraints: Cross-Time Relationships

- Cross-time relationships relate objects that are members of the participating classes at different times.
- We formalize cross-time relationships with the aim of preserving the snapshot reducibility.
- **Example:**
  - $\texttt{Biography} \subseteq \texttt{Author} \times \texttt{Person}$
  - $\texttt{bio} = \langle \texttt{Tulard}, \texttt{Napoleon} \rangle$ and $\texttt{bio} \in \texttt{Biography}^{\mathcal{B}(\texttt{1984})}$

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  - $\texttt{Tulard} \in \texttt{Author}^{\mathcal{B}(1984)}$;
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- **Example:**
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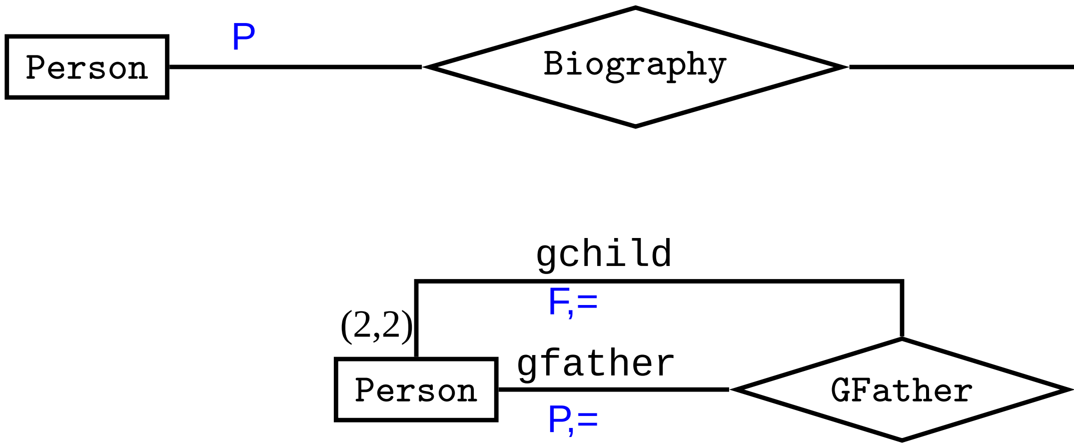
- Snapshot Reducibility would imply the following constraints:
  - $\texttt{Tulard} \in \texttt{Author}^{\mathcal{B}(1984)}$;
  - $\texttt{Napoleon} \in \texttt{Person}^{\mathcal{B}(1984)}$
- **Solution.** Use status classes to preserve snapshot reducibility.
  - `Napoleon` is a member of the `Disabled-Person` class in 1984.

A Semantics for Cross-Time Relationships



(b)

- Strictly Past (P).

  $r = \langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow o_1 \in \texttt{Disabled-C}_1{}^{\mathcal{B}(t)}$

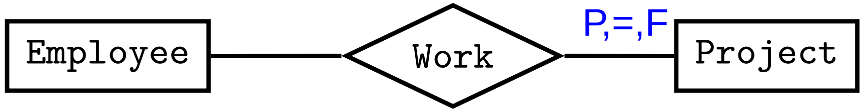
  $\texttt{R} \sqsubseteq \texttt{U}_1 : \texttt{Disabled-C}_1.$

- Past (P,=)

  $r = \langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow o_1 \in (C_1 \sqcup \texttt{Disabled-C}_1)^{\mathcal{B}(t)}$

  $\texttt{R} \sqsubseteq \texttt{U}_1 : (\texttt{C}_1 \sqcup \texttt{Disabled-C}_1).$

A Semantics for Cross-Time Relationships (Cont.)



- Strictly Future (F)

  $r = \langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow o_1 \in \texttt{Scheduled-C}_1{}^{\mathcal{B}(t)}$

  $\texttt{R} \sqsubseteq \texttt{U}_1 : \texttt{Scheduled-C}_1$.

- Future (F,=)

  $r = \langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow o_1 \in (C_1 \sqcup \texttt{Scheduled-C}_1)^{\mathcal{B}(t)}$

  $\texttt{R} \sqsubseteq \texttt{U}_1 : (\texttt{C}_1 \sqcup \texttt{Scheduled-C}_1)$.

- Full-Cross (P,=,F)

  $r = \langle o_1, o_2 \rangle \in R^{\mathcal{B}(t)} \rightarrow o_1 \in (C_1 \sqcup \texttt{Scheduled-C}_1 \sqcup \texttt{Disabled-C}_1)^{\mathcal{B}(t)}$

  $\texttt{R} \sqsubseteq \texttt{U}_1 : (\texttt{C}_1 \sqcup \texttt{Scheduled-C}_1 \sqcup \texttt{Disabled-C}_1)$.

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Correctness of the Encoding

Theorem. An $\mathcal{ER}_{VT}$ schema can be correctly encoded into a $\mathcal{DLR}_{\mathcal{US}}$ theory—i.e., to each temporal legal database of an $\mathcal{ER}_{VT}$ schema corresponds a model of the resulting $\mathcal{DLR}_{\mathcal{US}}$ theory and viceversa. [Artale,Franconi:ER99]

Corollary. Reasoning over $\mathcal{ER}_{VT}$ schemas can be reduced to reasoning over the $\mathcal{DLR}_{\mathcal{US}}$ encoding.

Computational Properties of $\mathcal{DLR}_{\mathcal{US}}$: Two theorems

Theorem. Logical implication in $\mathcal{DLR}_{\mathcal{US}}$ over a linear, unbounded, discrete temporal structure is *undecidable*. [Artale-et-al:JELIA-02]

Computational Properties of $\mathcal{DLR}_{\mathcal{US}}$: Two theorems

Theorem. Logical implication in $\mathcal{DLR}_{\mathcal{US}}$ over a linear, unbounded, discrete temporal structure is *undecidable*. [Artale-et-al:JELIA-02]

- The *maximal* decidable fragment of $\mathcal{DLR}_{\mathcal{US}}$ is the monodic fragment $\mathcal{DLR}^{-}_{\mathcal{US}}$:

$$
\begin{array}{rcl}
R & \to & \top_n \mid RN \mid \neg R \mid R_1 \sqcap R_2 \mid U_i/n : C \mid \\
 & & \sout{\Diamond^+ R \mid \Diamond^- R \mid \Box^+ R \mid \Box^- R \mid \oplus R \mid \ominus R \mid R_1 \mathcal{U} R_2 \mid R_1 \mathcal{S} R_2} \\
C & \to & \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid \exists^{\leqslant k}[U_i]R \mid \\
 & & \Diamond^+ C \mid \Diamond^- C \mid \Box^+ C \mid \Box^- C \mid \oplus C \mid \ominus C \mid C_1 \mathcal{U} C_2 \mid C_1 \mathcal{S} C_2
\end{array}
$$

Theorem. Logical implication in the monodic fragment of $\mathcal{DLR}_{\mathcal{US}}$ over a linear, unbounded, discrete temporal structure is *EXPTIME-complete*.

Decidability Results for $\mathcal{ER}_{VT}$

[QUESTION:] Does the $\mathcal{DLR}_{\mathcal{US}}$ undecidability result transfers to $\mathcal{ER}_{VT}$, too?

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[QUESTION:] Does the $\mathcal{DLR}_{\mathcal{US}}$ undecidability result transfers to $\mathcal{ER}_{VT}$, too?

- [ANSWER 1:] YES! As far as $\mathcal{ER}_{VT}$ uses both timestamping and evolution constructs.
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping and evolution constraints is undecidable. [Artale:TIME-04]

- [ANSWER 2:] Open Problem! As far as $\mathcal{ER}_{VT}$ uses just timestamping.

Decidability Results for $\mathcal{ER}_{VT}$ (Cont.)

[QUESTION:] Does the *EXPTIME-complete* result for $\mathcal{DLR}^{-}_{\mathcal{US}}$ transfers to $\mathcal{ER}_{VT}$ as well?

Decidability Results for $\mathcal{ER}_{VT}$ (Cont.)

[QUESTION:] Does the *EXPTIME-complete* result for $\mathcal{DLR}^{-}_{\mathcal{US}}$ transfers to $\mathcal{ER}_{VT}$ as well?

- [ANSWER:] YES! As far as $\mathcal{ER}_{VT}$ does not use temporal constructs over relationships and attributes.
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping just over Classes and evolution constraints is complete for *EXPTIME*. [Artale-et-al:FoIKS-06]

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$\mathcal{ER}_{VT}$ Undecidability Proof

The proof is based on a reduction from the undecidable **Halting Problem** to the **Entity Satisfiability Problem w.r.t. an $\mathcal{ER}_{VT}$ Schema**. We proceed as follows:

1. Reduction of the Halting Problem to Concept Satisfiability Problem w.r.t. an $\mathcal{ALC}_{\mathsf{F}}$ KB (ideas similar to [Gabbay:Kurucz:Wolter:Zakharyaschev:03]);
2. Reduction of Concept Satisfiability w.r.t. an $\mathcal{ALC}_{\mathsf{F}}$ KB to Entity Satisfiability w.r.t. an $\mathcal{ER}_{VT}$ Schema.

Remark. $\mathcal{ALC}_{\mathsf{F}}$ is a tense-logical extension of $\mathcal{ALC}$: $\diamond^+ C$ (sometime in the future), $\Box^+ C$ (always in the future), and possibly *Global Roles*.

Halting Problem

- *Single-tape right-infinite deterministic Turing machine* $\mathbf{M}$: $\langle A, S, \rho \rangle$, where:
  - A is the *tape alphabet* ($b \in A$ stands for blank);
  - S is a finite set of *states* with the *initial state*, s_0, and the *final state*, s_1;
  - ρ is the *transition function*, $\rho : (S - \{s_1\}) \times A \to S \times (A \cup \{\mathtt{L}, \mathtt{R}\})$.
- *Configuration* of $\mathbf{M}$ is an infinite sequence: $\langle £, a_1, \ldots, a_{i-1}, \langle s_i, a_i \rangle, \ldots, a_n, b, \ldots \rangle$;

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- *Configuration* of $\mathbf{M}$ is an infinite sequence: $\langle \pounds, a_1, \ldots, a_{i-1}, \langle s_i, a_i \rangle, \ldots, a_n, b, \ldots \rangle$;
- Since a transition function can only modify the active cell and its neighbors we introduce the *instruction function*, δ:

$$\delta(a_i, \langle s, a_j \rangle, a_k) = \begin{cases} \langle a_i, \langle s', a'_j \rangle, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', a'_j \rangle \\ \langle \langle s', a_i \rangle, a_j, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', \mathtt{L} \rangle \text{ and } a_i \neq \pounds \\ \langle \pounds, \langle s', a_j \rangle, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', \mathtt{L} \rangle \text{ and } a_i = \pounds \\ \langle a_i, a_j, \langle s', a_k \rangle \rangle, & \text{if } \rho(s, a_j) = \langle s', \mathtt{R} \rangle \end{cases}$$

- A sequence $\langle c_0, c_1, \ldots, c_k, c_{k+1}, \ldots \rangle$ of configurations is said a *Computation* of $\mathbf{M}$.

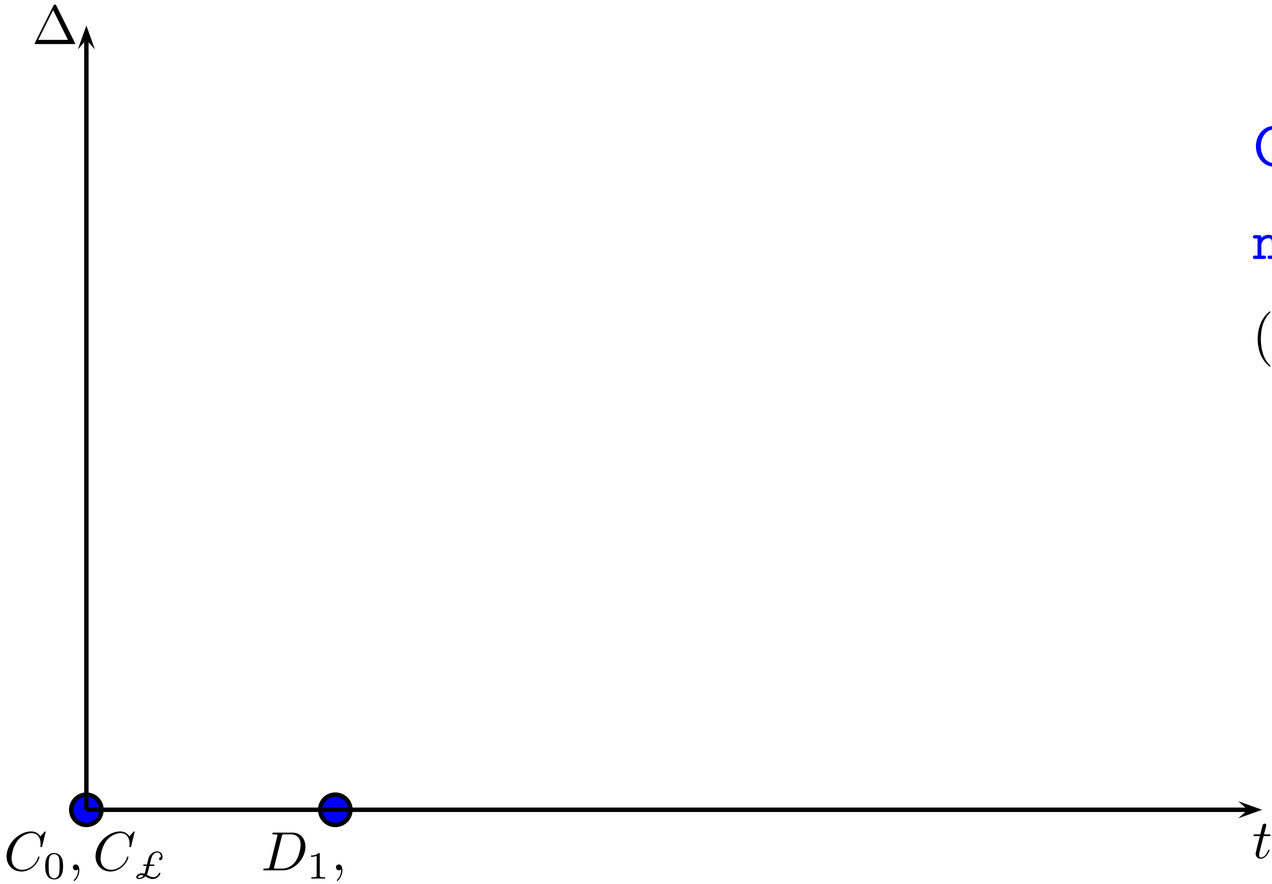
Halting Problem: Definition

We say that $\mathbf{M}$ *halts*, starting with the empty tape—i.e. with starting configuration: $\langle \pounds, \langle s_0, b \rangle, b, \ldots, b, \ldots \rangle$—if there is a finite computation, $\langle c_0, c_1, \ldots, c_k \rangle$, such that the state of c_k is s_1 (the final state).

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$

$$\mathsf{C}_0 \sqsubseteq \mathsf{C}_{\pounds} \sqcap \Diamond^{+}\mathsf{C}_{\langle \mathsf{s}_0, \mathsf{b} \rangle}$$

Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$

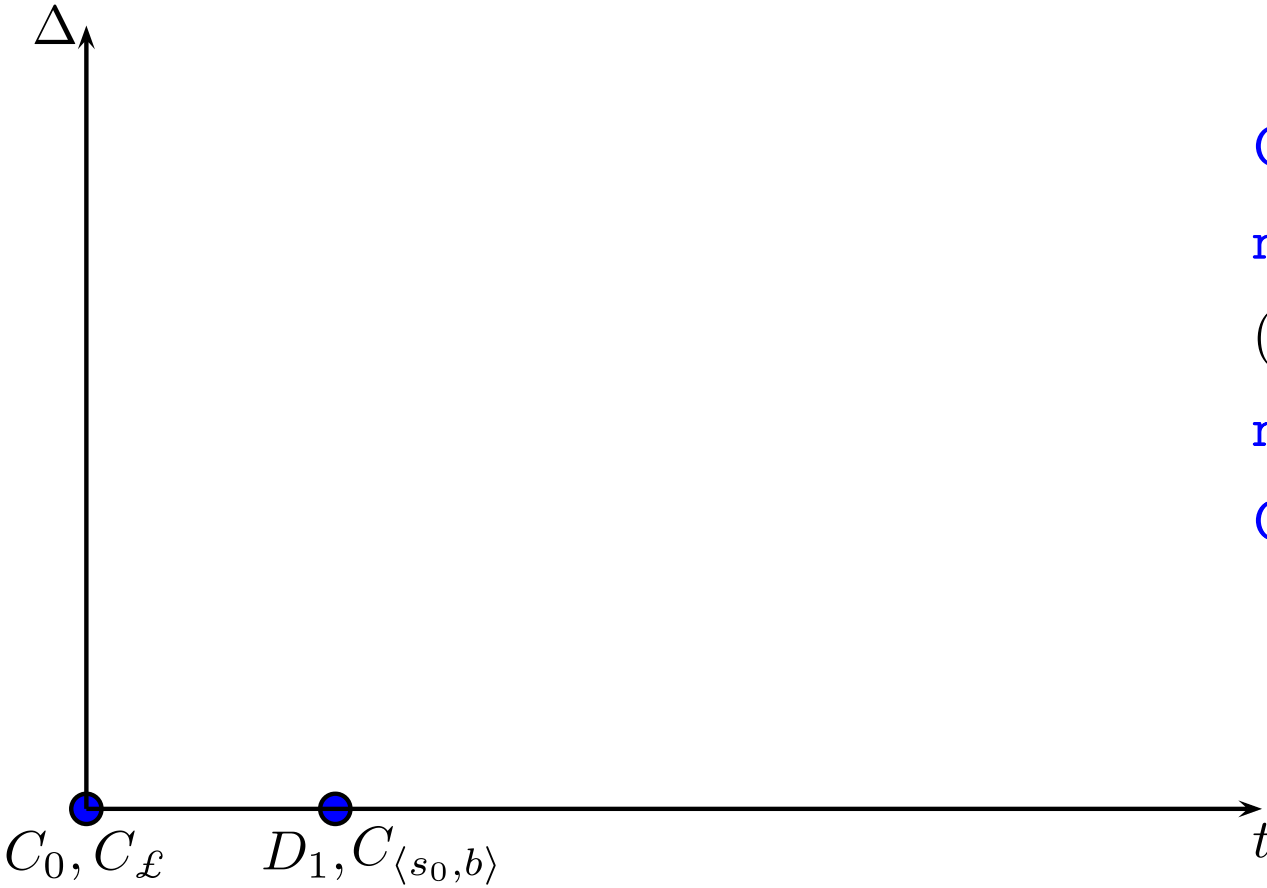


$\mathsf{C}_0 \sqsubseteq \mathsf{C}_{\pounds} \sqcap \Diamond^+ \mathsf{C}_{\langle \mathsf{s}_0, \mathsf{b} \rangle}$

$\texttt{next}(\mathsf{C}_{\pounds}, \mathsf{D}_1)$

$(C_{\pounds} \sqsubseteq \Diamond^+ D_1 \sqcap \neg \Diamond^+ \Diamond^+ D_1)$

Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$



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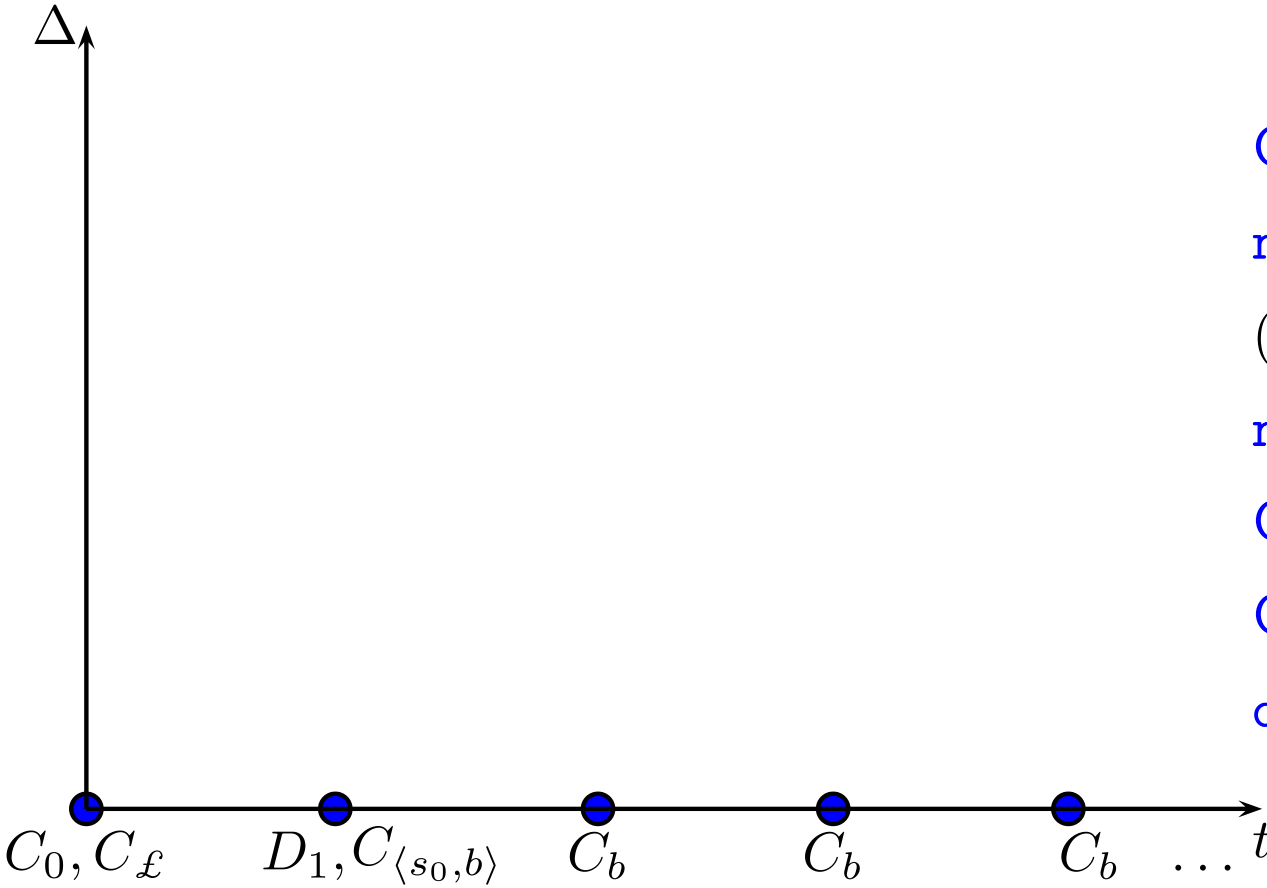
$\mathtt{next}(\mathsf{C}_\pounds, \mathsf{D_1})$

$(C_\pounds \sqsubseteq \Diamond^+ D_1 \sqcap \neg \Diamond^+ \Diamond^+ D_1)$

$\mathtt{next}(\mathsf{D_1}, \mathsf{D_2})$

$\mathsf{C}_{\langle \mathsf{s_0}, \mathsf{b}\rangle} \sqsubseteq \mathsf{D_1}$

Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$



$\mathsf{C}_0 \sqsubseteq \mathsf{C}_£ \sqcap \Diamond^+ \mathsf{C}_{\langle \mathsf{s}_0, \mathsf{b}\rangle}$

$\mathtt{next}(\mathsf{C}_£, \mathsf{D}_1)$

$(C_£ \sqsubseteq \Diamond^+ D_1 \sqcap \neg \Diamond^+ \Diamond^+ D_1)$

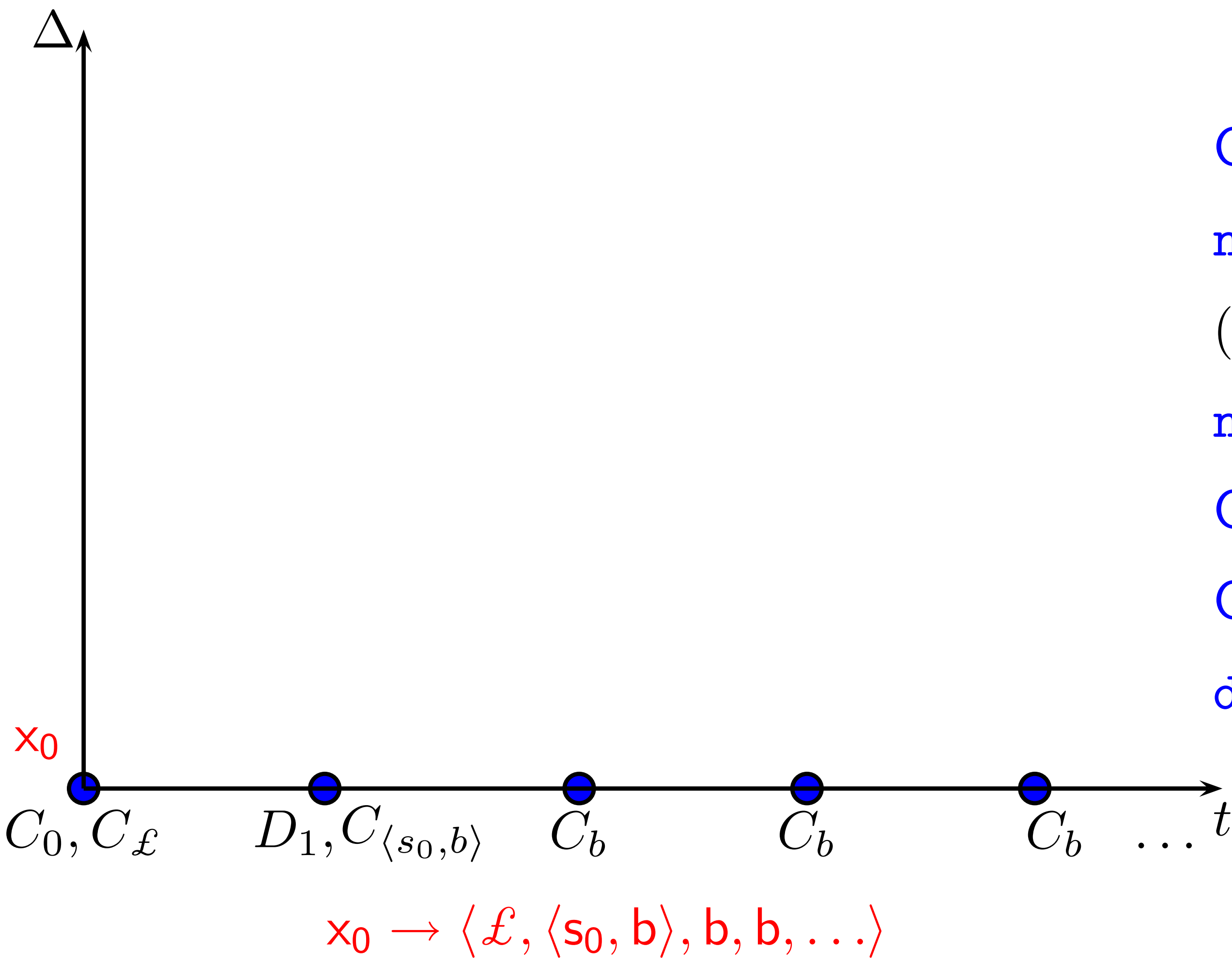
$\mathtt{next}(\mathsf{D}_1, \mathsf{D}_2)$

$\mathsf{C}_{\langle \mathsf{s}_0, \mathsf{b}\rangle} \sqsubseteq \mathsf{D}_1$

$\mathsf{C}_{\langle \mathsf{s}_0, \mathsf{b}\rangle} \sqsubseteq \Box^+ \mathsf{C}_\mathsf{b}$

$\mathtt{discover}(\mathsf{C}, \{\mathsf{C}_\mathsf{x} \mid \mathsf{x} \in \mathsf{A} \cup \{£\} \cup (\mathsf{S} \times \mathsf{A})\})$

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$



$\mathsf{C}_0 \sqsubseteq \mathsf{C}_£ \sqcap \Diamond^+ \mathsf{C}_{\langle s_0, b\rangle}$

$\mathtt{next}(\mathsf{C}_£, \mathsf{D}_1)$

$(C_£ \sqsubseteq \Diamond^+ D_1 \sqcap \neg\Diamond^+\Diamond^+ D_1)$

$\mathtt{next}(\mathsf{D}_1, \mathsf{D}_2)$

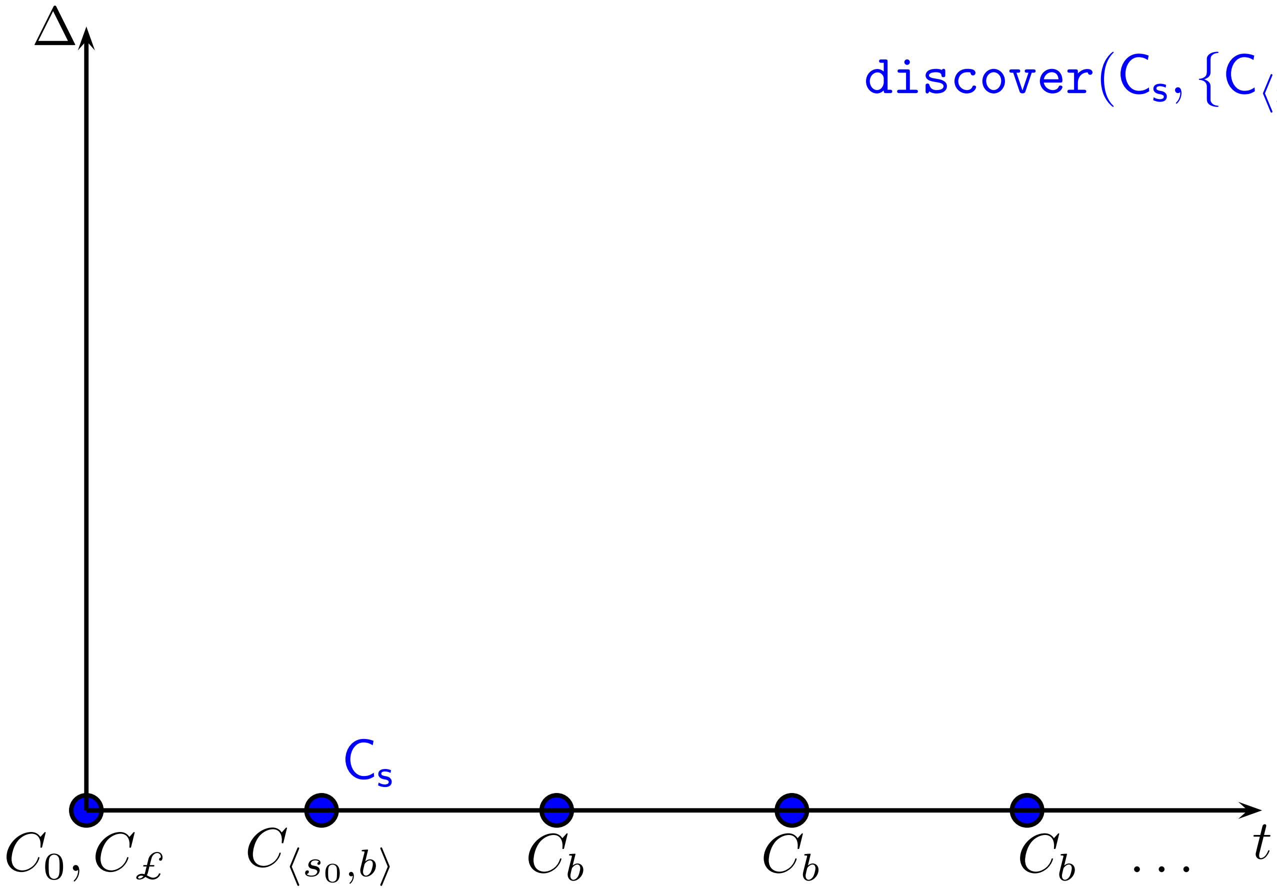
$\mathsf{C}_{\langle s_0, b\rangle} \sqsubseteq \mathsf{D}_1$

$\mathsf{C}_{\langle s_0, b\rangle} \sqsubseteq \Box^+ \mathsf{C}_b$

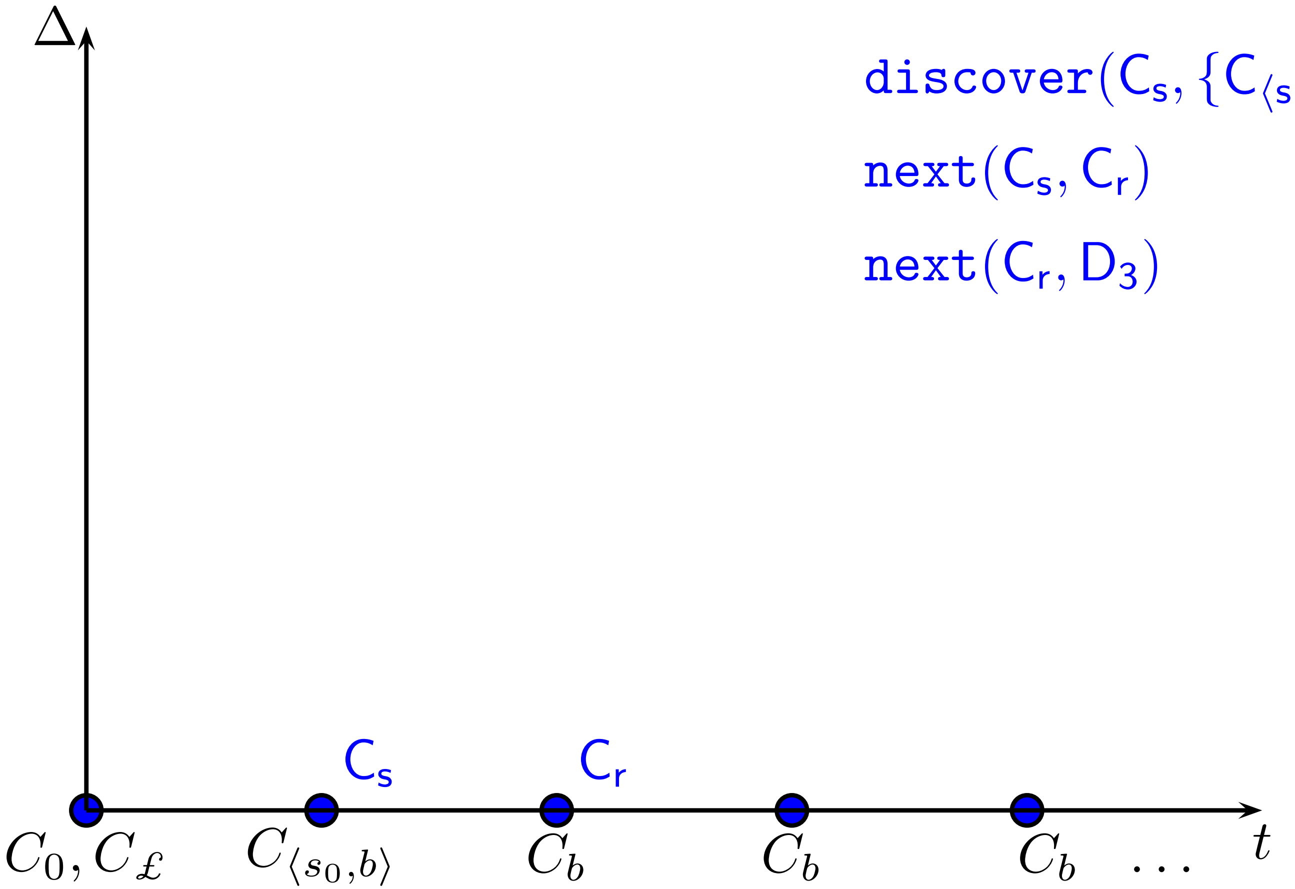
$\mathtt{discover}(\mathsf{C}, \{\mathsf{C}_x \mid x \in A \cup \{£\} \cup (S \times A)\})$

Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$ (Cont.)

$\mathtt{discover}(\mathsf{C_s}, \{\mathsf{C}_{\langle \mathsf{s},\mathsf{a}\rangle} \mid \langle \mathsf{s}, \mathsf{a}\rangle \in \mathsf{S} \times \mathsf{A}\})$



Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$ (Cont.)

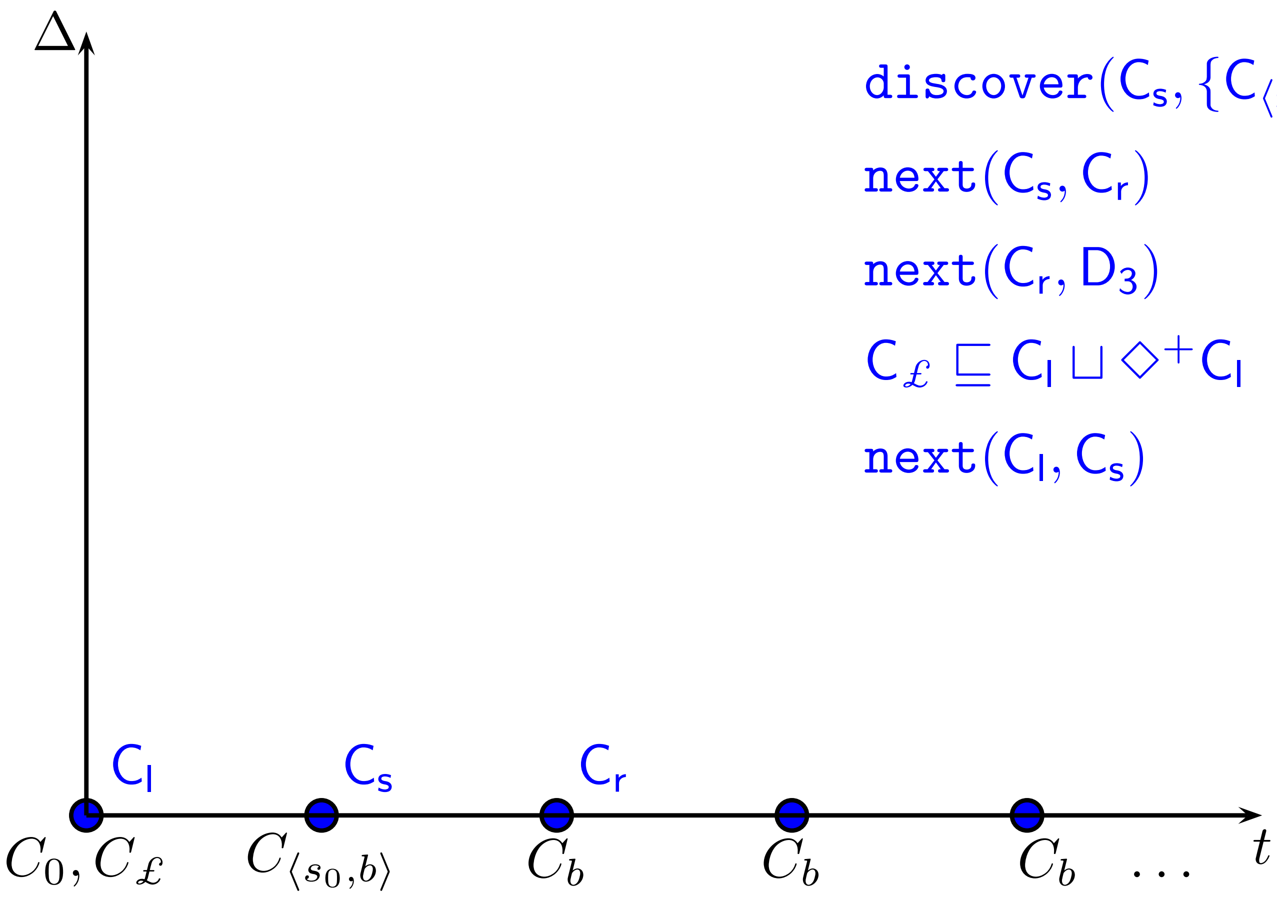


$\texttt{discover}(\mathsf{C_s}, \{\mathsf{C}_{\langle \mathsf{s},\mathsf{a}\rangle} \mid \langle \mathsf{s}, \mathsf{a}\rangle \in \mathsf{S} \times \mathsf{A}\})$

$\texttt{next}(\mathsf{C_s}, \mathsf{C_r})$

$\texttt{next}(\mathsf{C_r}, \mathsf{D_3})$

Encoding the Halting Problem with $\mathcal{ALC}_\mathsf{F}$ (Cont.)



$\mathtt{discover}(\mathsf{C_s}, \{\mathsf{C}_{\langle \mathsf{s,a}\rangle} \mid \langle \mathsf{s,a}\rangle \in \mathsf{S} \times \mathsf{A}\})$

$\mathtt{next}(\mathsf{C_s}, \mathsf{C_r})$

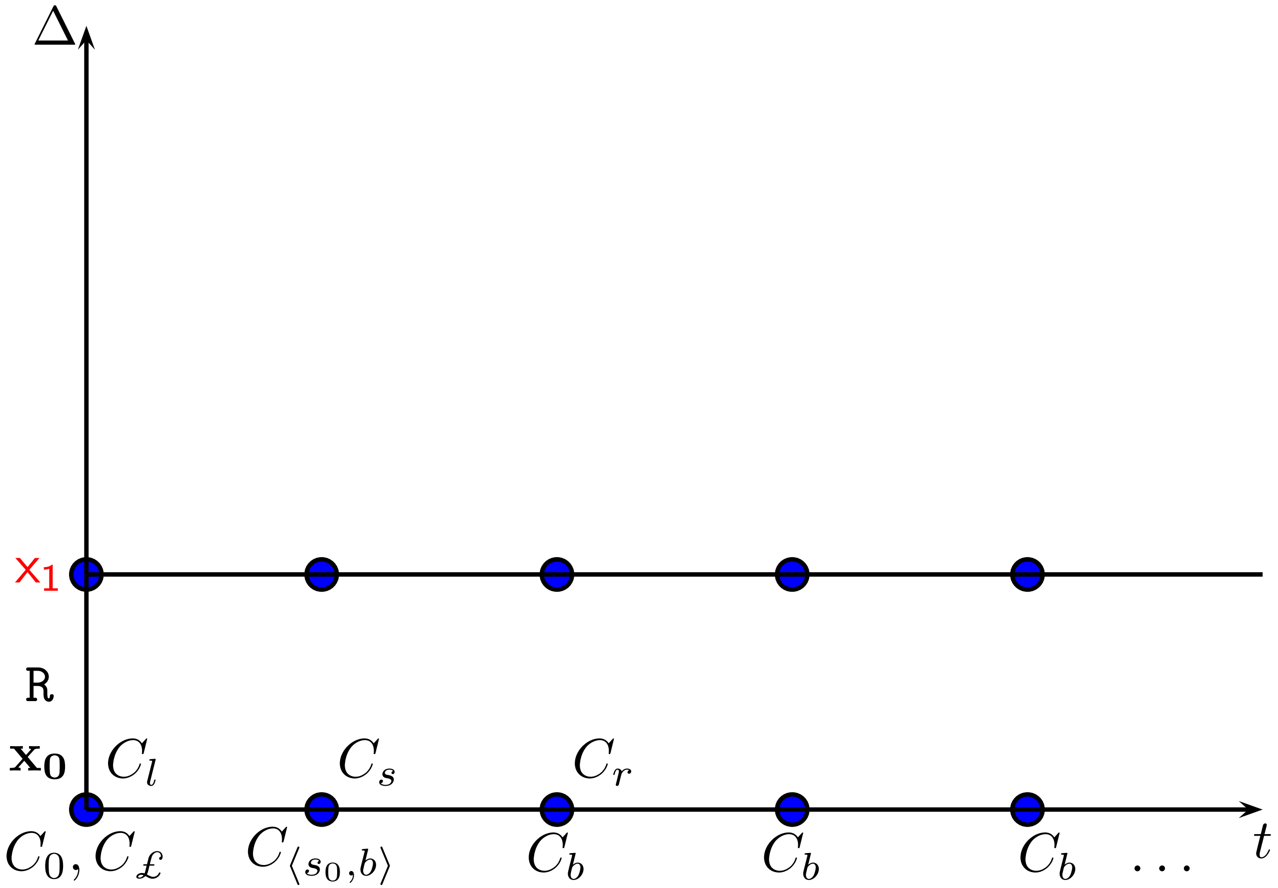
$\mathtt{next}(\mathsf{C_r}, \mathsf{D_3})$

$\mathsf{C}_\pounds \sqsubseteq \mathsf{C_l} \sqcup \Diamond^+ \mathsf{C_l}$

$\mathtt{next}(\mathsf{C_l}, \mathsf{C_s})$

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$ (Cont.)

$\top \sqsubseteq \exists \mathsf{R}.\top$ (with R global)

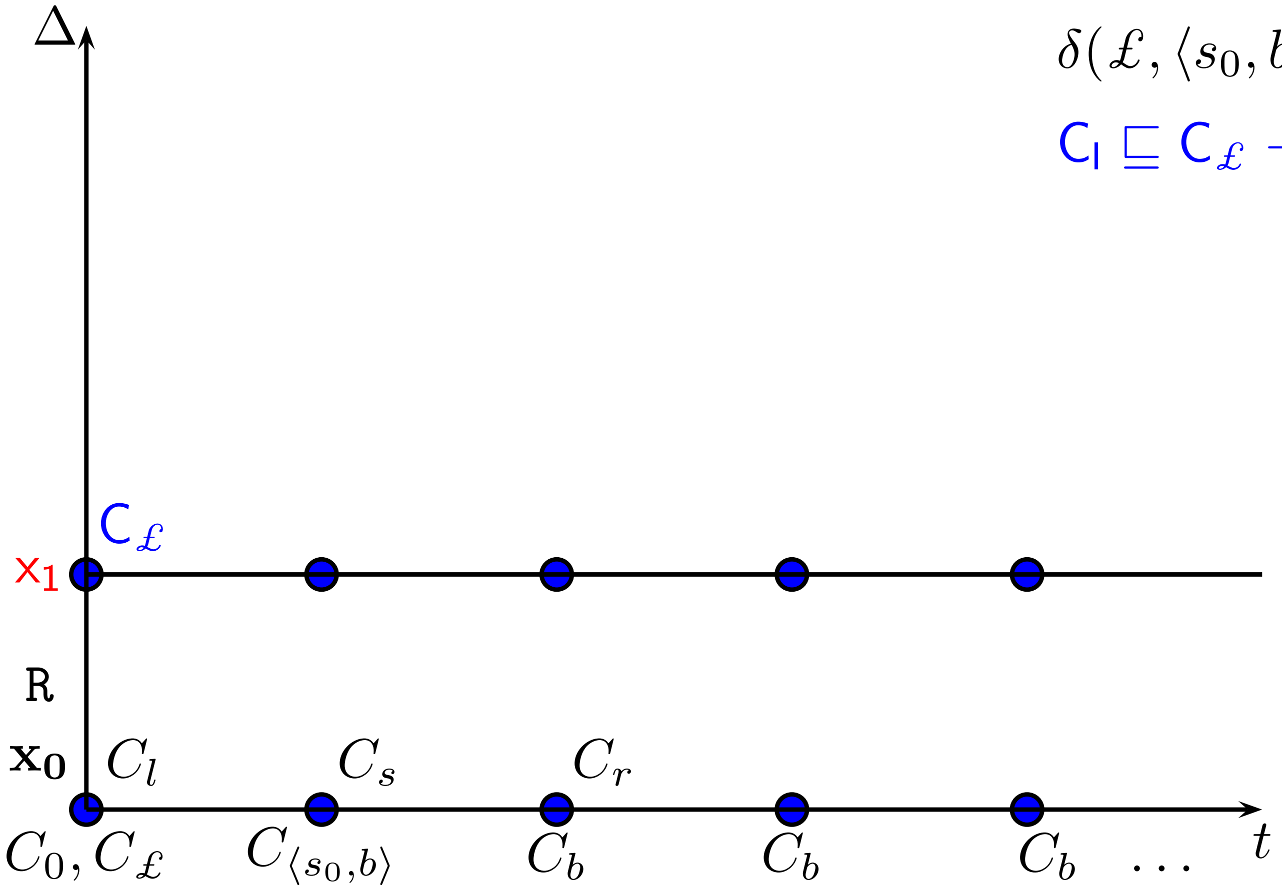


Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$ (Cont.)

$\top \sqsubseteq \exists \mathsf{R}.\top$ (with R global)

$\delta(\pounds, \langle s_0, b\rangle, b) = \langle \pounds, b, \langle s', b\rangle\rangle$

$\mathsf{C_l} \sqsubseteq \mathsf{C}_\pounds \rightarrow \forall \mathsf{R}.\mathsf{C}_\pounds$



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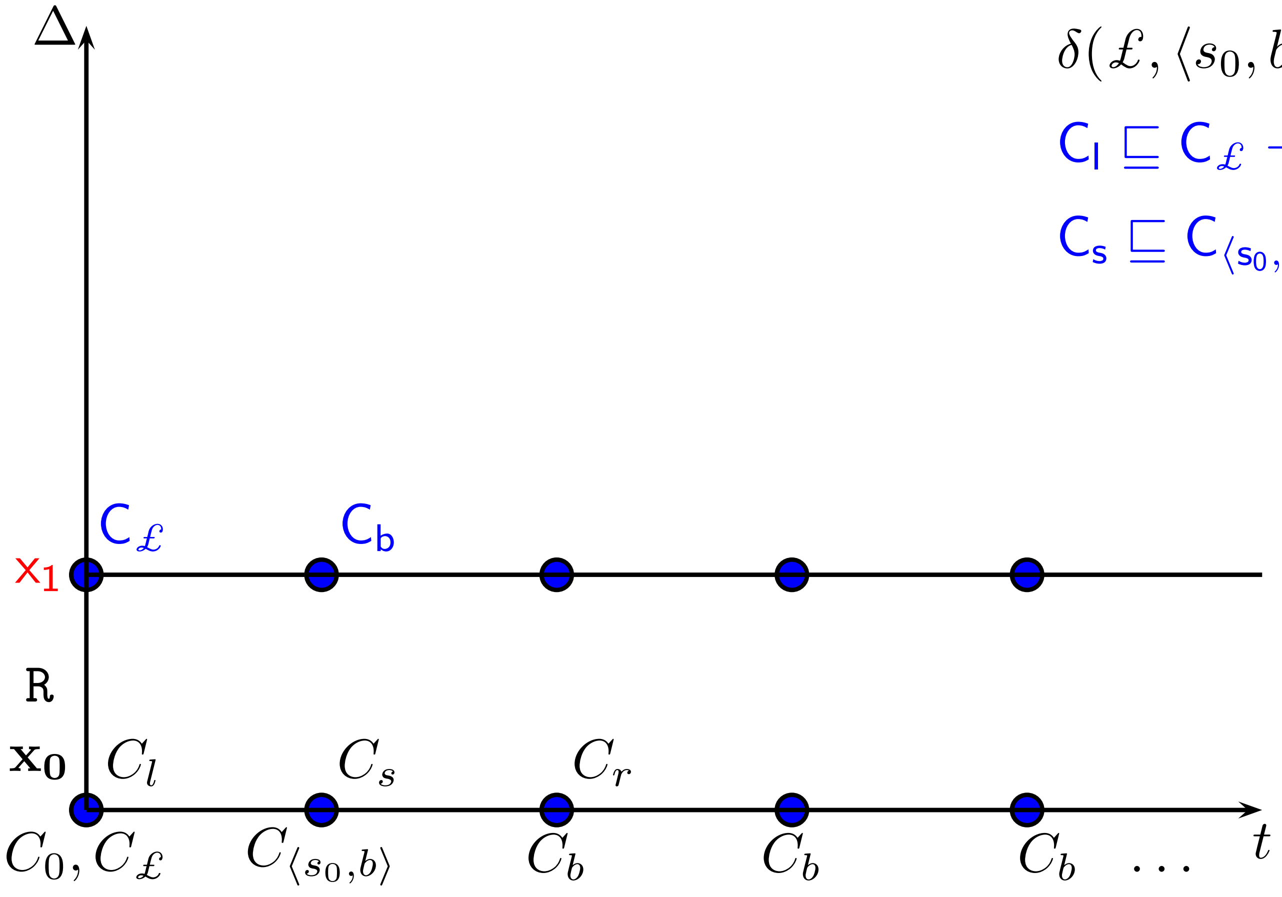
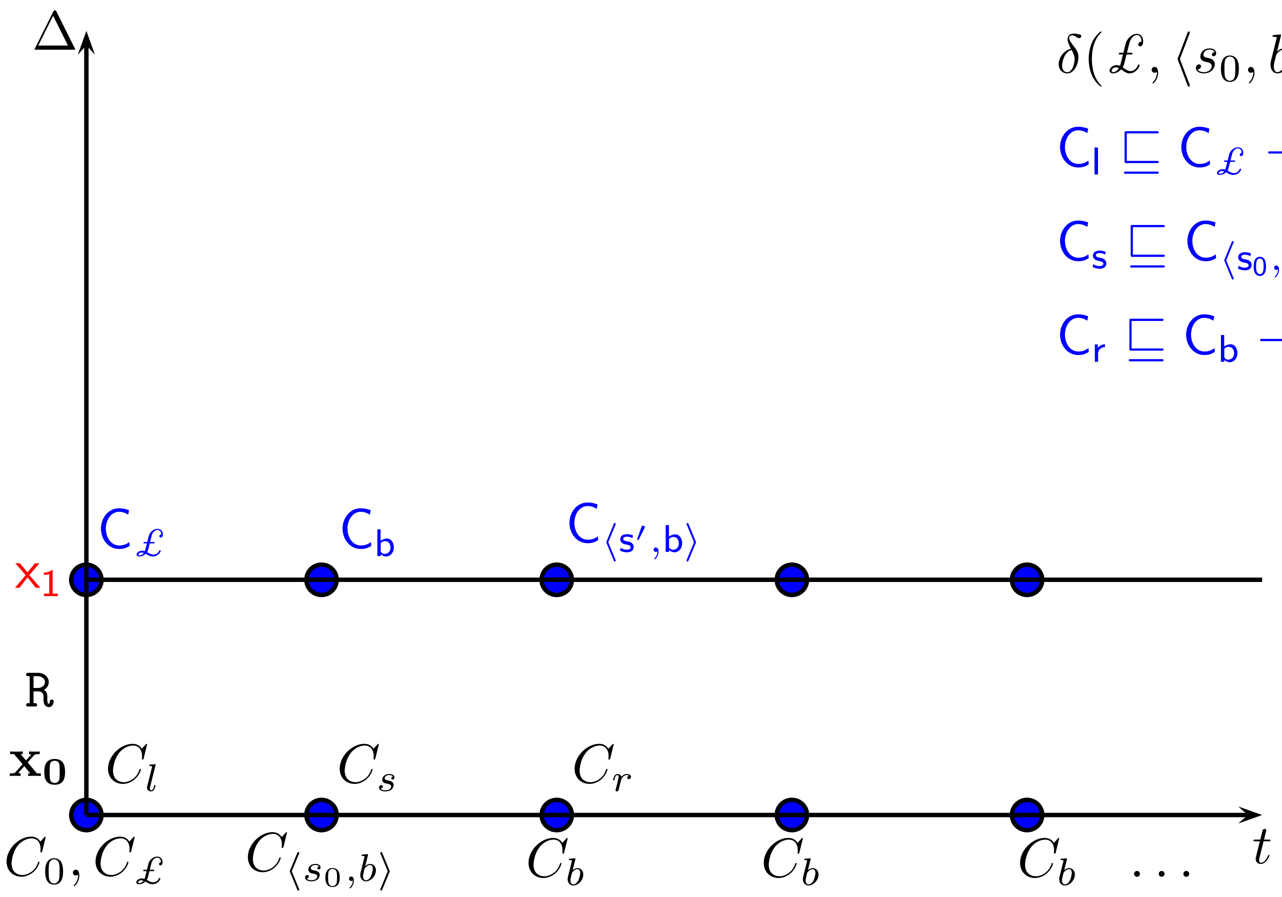
$\mathsf{C_l} \sqsubseteq \mathsf{C}_\pounds \rightarrow \forall \mathsf{R}.\mathsf{C}_\pounds$

$\mathsf{C_s} \sqsubseteq \mathsf{C}_{\langle \mathsf{s_0}, \mathsf{b}\rangle} \rightarrow \forall \mathsf{R}.\mathsf{C_b}$



Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)



$\top \sqsubseteq \exists R.\top$ (with R global)

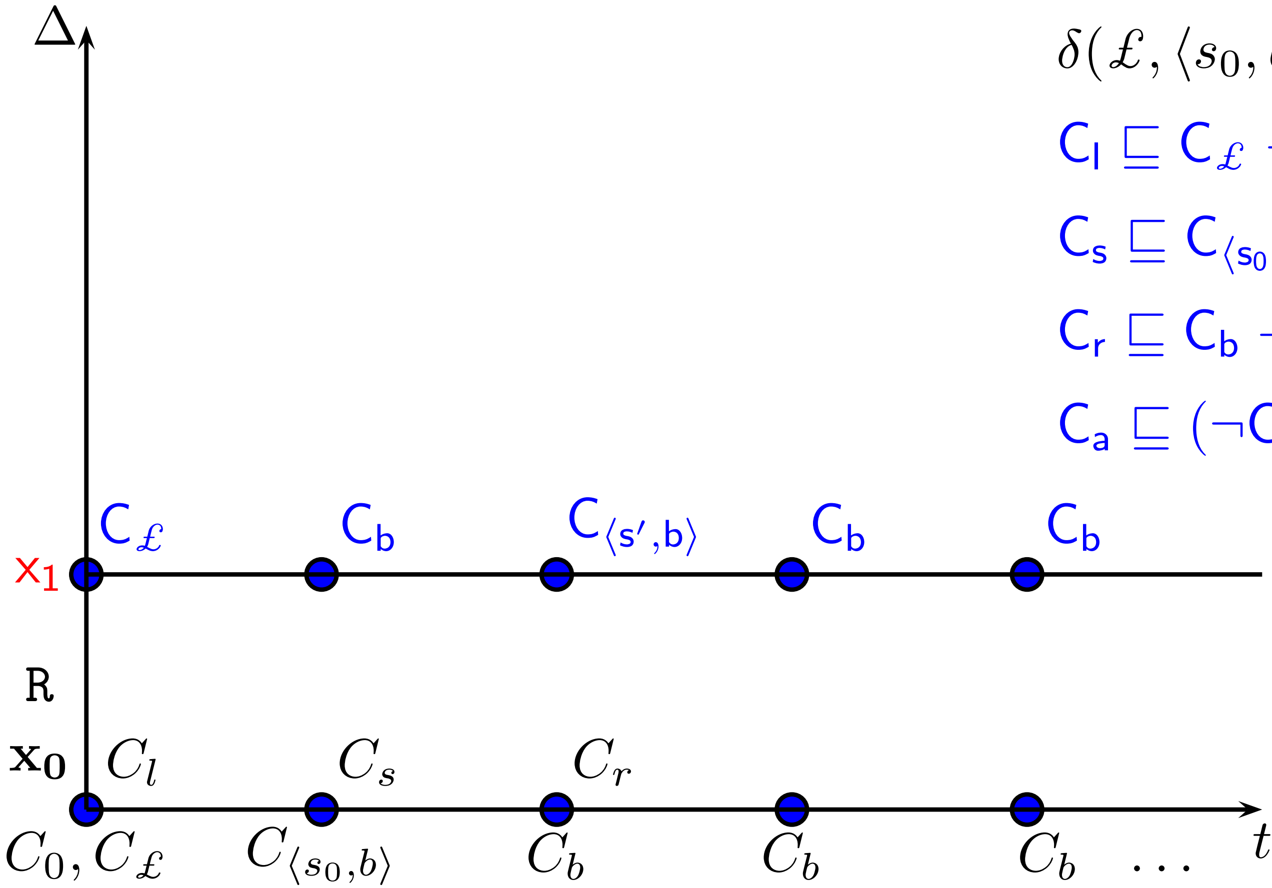
$\delta(£, \langle s_0, b\rangle, b) = \langle £, b, \langle s', b\rangle\rangle$

$C_l \sqsubseteq C_{£} \rightarrow \forall R.C_{£}$

$C_s \sqsubseteq C_{\langle s_0,b\rangle} \rightarrow \forall R.C_b$

$C_r \sqsubseteq C_b \rightarrow \forall R.C_{\langle s',b\rangle}$

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$ (Cont.)



$\top \sqsubseteq \exists R.\top$ (with R global)

$\delta(£, \langle s_0, b\rangle, b) = \langle £, b, \langle s', b\rangle\rangle$

$C_l \sqsubseteq C_{£} \to \forall R.C_{£}$

$C_s \sqsubseteq C_{\langle s_0,b\rangle} \to \forall R.C_b$

$C_r \sqsubseteq C_b \to \forall R.C_{\langle s',b\rangle}$

$C_a \sqsubseteq (\neg C_l \sqcap \neg C_s \sqcap \neg C_r) \to \forall R.C_a$

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$ (Cont.)

$\top \sqsubseteq \exists \mathsf{R}.\top$ (with R global)

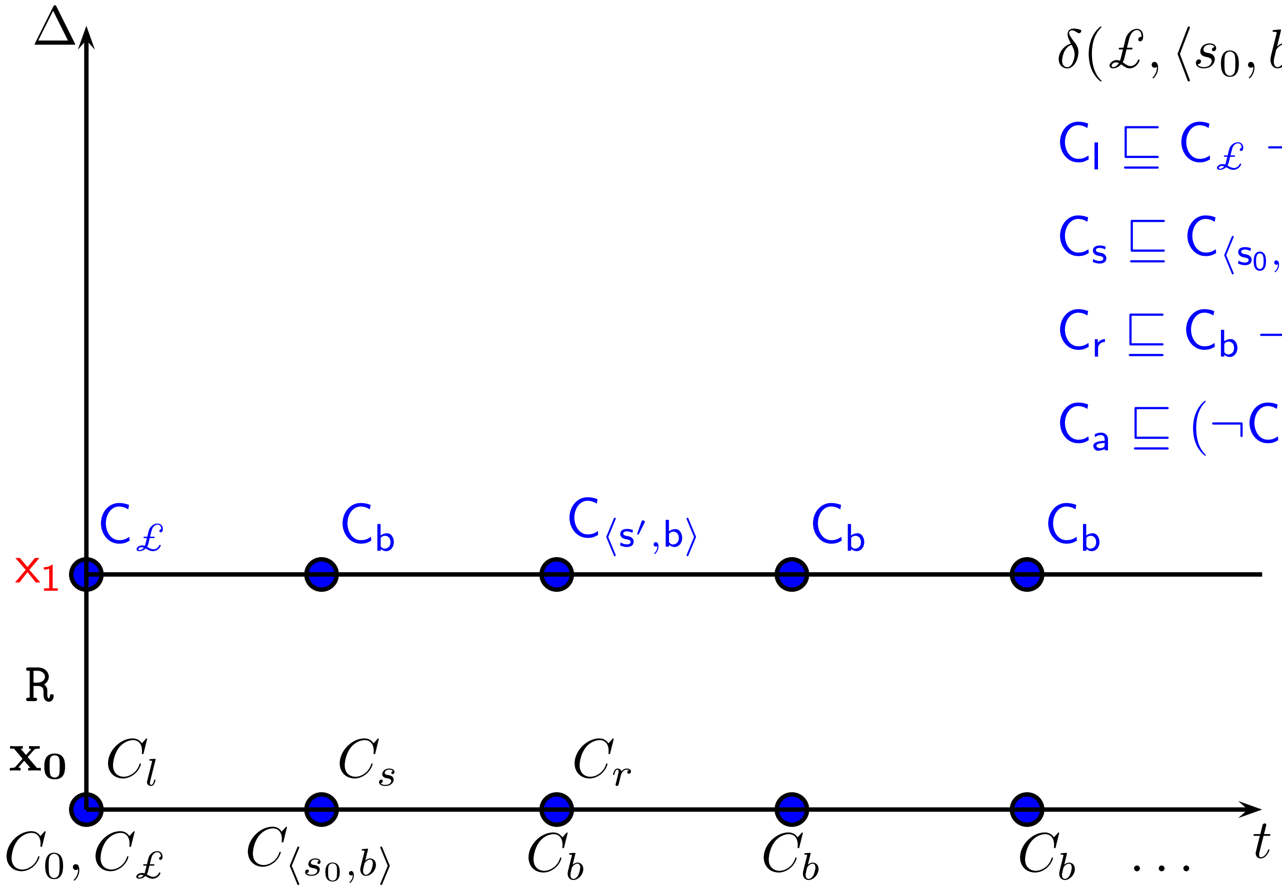
$\delta(\pounds, \langle s_0, b\rangle, b) = \langle \pounds, b, \langle s', b\rangle\rangle$

$\mathsf{C_l} \sqsubseteq \mathsf{C}_\pounds \rightarrow \forall \mathsf{R}.\mathsf{C}_\pounds$

$\mathsf{C_s} \sqsubseteq \mathsf{C}_{\langle \mathsf{s_0}, \mathsf{b}\rangle} \rightarrow \forall \mathsf{R}.\mathsf{C_b}$

$\mathsf{C_r} \sqsubseteq \mathsf{C_b} \rightarrow \forall \mathsf{R}.\mathsf{C}_{\langle \mathsf{s'}, \mathsf{b}\rangle}$

$\mathsf{C_a} \sqsubseteq (\neg \mathsf{C_l} \sqcap \neg \mathsf{C_s} \sqcap \neg \mathsf{C_r}) \rightarrow \forall \mathsf{R}.\mathsf{C_a}$



$\mathsf{x_1} \rightarrow \langle \pounds, \mathsf{b}, \langle \mathsf{s'}, \mathsf{b}\rangle, \mathsf{b}, \mathsf{b}, \ldots\rangle$

Encoding the Halting Problem with $\mathcal{ALC}_{\mathsf{F}}$ (Cont.)

- The chain of R-successor, $\langle \mathsf{x}_0, \mathsf{x}_1, \mathsf{x}_2, \ldots \rangle$, represents a computation of **M**;
- The following axioms:

  $\texttt{discover}(\mathsf{C_s}, \{\mathsf{C}_{\langle \mathsf{s},\mathsf{a} \rangle} \mid \langle \mathsf{s}, \mathsf{a} \rangle \in \mathsf{S} \times \mathsf{A}\})$

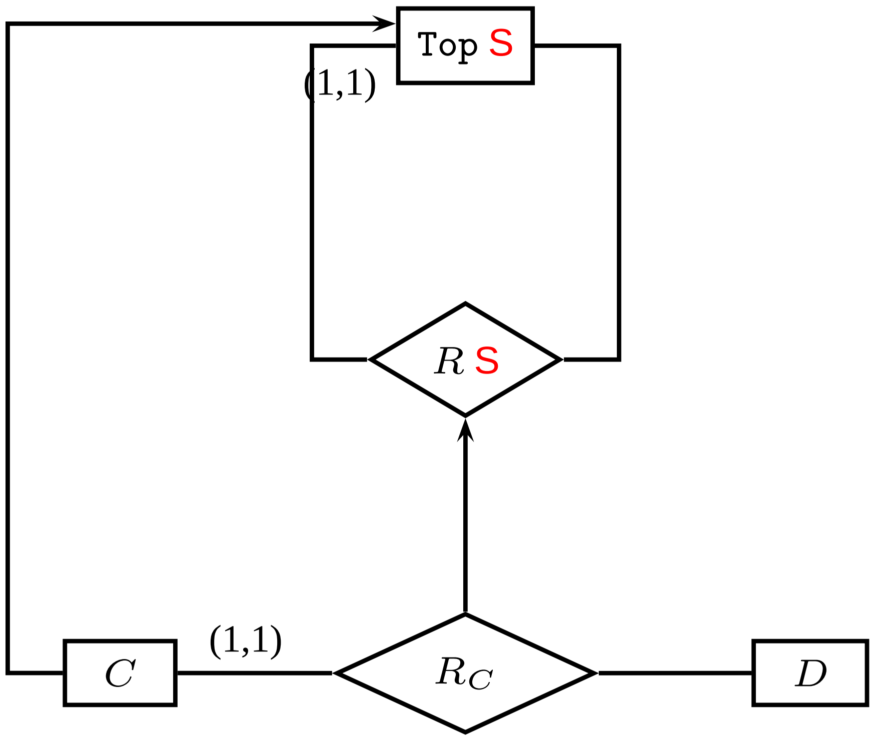
  $\texttt{discover}(\mathsf{S1}, \{\mathsf{C}_{\langle \mathsf{s}_1,\mathsf{a} \rangle} \mid \mathsf{a} \in \mathsf{A} \cup \{\pounds\}\})$

  $\mathsf{C_s} \sqsubseteq \neg \mathsf{S1}$

  Guarantee that **M** does not halt.

Reducing $\mathcal{ALC}_{\mathsf{F}}$ Axioms to $\mathcal{ER}_{VT}$ Schema

- To capture standard $\mathcal{ALC}$ axioms we use the translation presented in [Berardi:Calì:Calvanese:DeGiacomo:03] apart from axioms of the form $\mathsf{C} \sqsubseteq \forall \mathsf{R}.\mathsf{C}$ and $\top \sqsubseteq \exists \mathsf{R}.\mathsf{C}$:

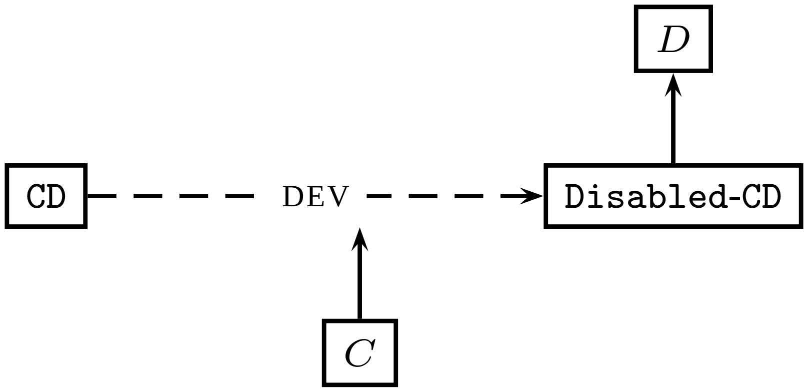


(a)

Reducing $\mathcal{ALC}_\mathsf{F}$ Axioms to $\mathcal{ER}_{VT}$ Schema (Cont.)

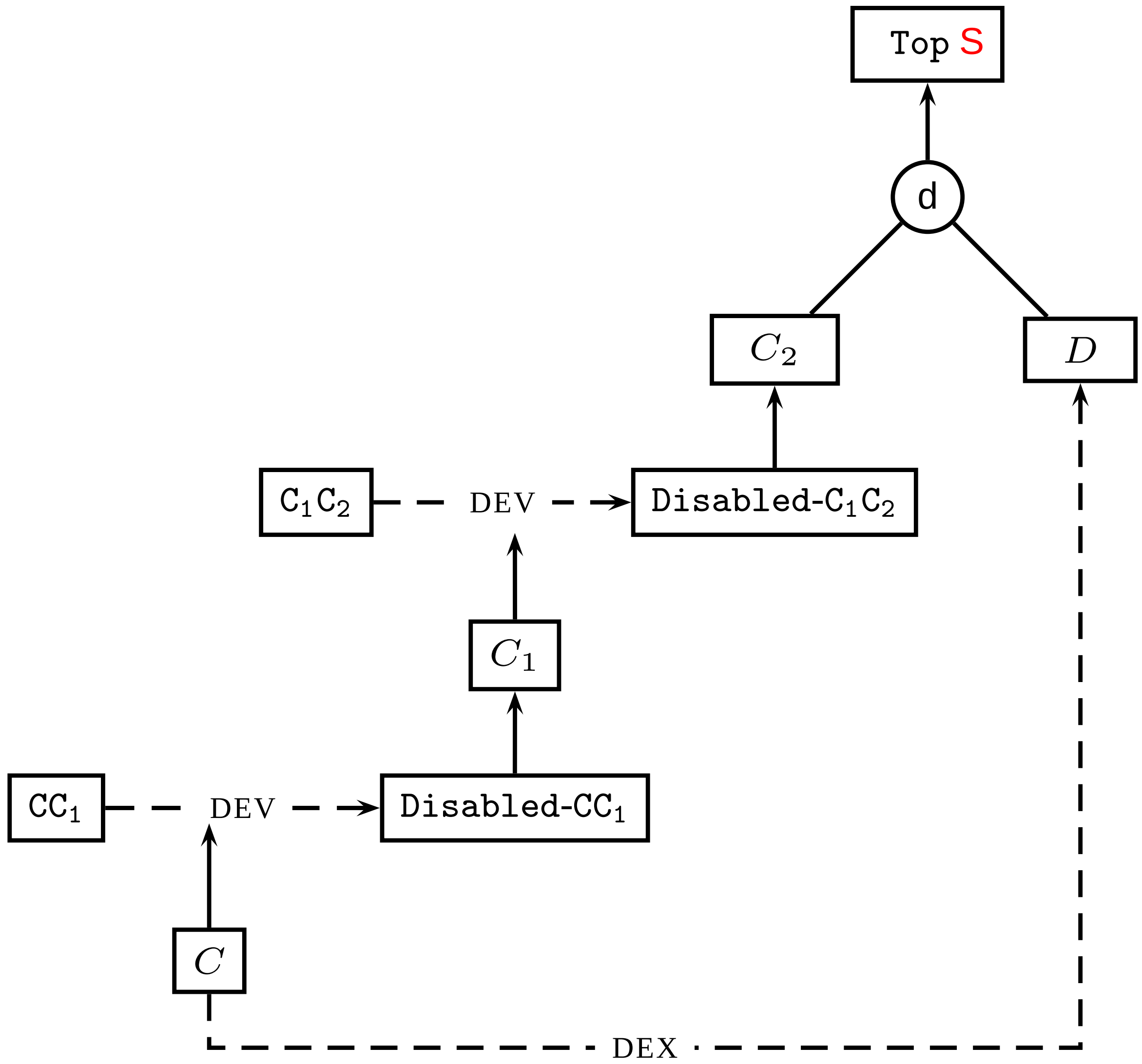
- Axioms of the form $\mathsf{C} \sqsubseteq \Diamond^+\mathsf{D}$ are captured using total dynamic extension:

- Axioms of the form $\mathsf{C} \sqsubseteq \Box^+\mathsf{D}$ are captured using dynamic evolution and status classes (in particular, disabled status):



Reducing $\mathcal{ALC}_\mathsf{F}$ Axioms to $\mathcal{ER}_{VT}$ Schema (Cont.)

- Axioms of the form $\mathtt{next}(\mathsf{C}, \mathsf{D}) \equiv \diamond^+ \mathsf{D} \sqcap \Box^+ \Box^+ \neg \mathsf{D}$ are mapped by using the dynamic constraints:



Conclusions

- We presented the temporal data model $\mathcal{ER}_{VT}$ which combines a linear and visual syntax with a rigorous set-theoretic semantics.
- $\mathcal{ER}_{VT}$ captures both timestamping and evolution constraints.
- The formalization of each construct gives rise to a set of constraints as a logical consequence of its semantics.
- Quality criteria as schema consistency and logical implication of implicit constraints have been semantically defined.
- Using a description logic translation reasoning over $\mathcal{ER}_{VT}$ has been showed decidable (if we give up temporal relationships).

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- Using a description logic translation reasoning over $\mathcal{ER}_{VT}$ has been showed decidable (if we give up temporal relationships).
- Open Problem. Does reasoning on $\mathcal{ER}_{VT}$ with full timestamping but without evolution constraints become decidable?
  - *Hint.* Check the decidability of the epistemic description logic $S5 \times \mathcal{DLR}$.