Temporal Knowledge and Ontologies

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Introduction: Constructors for Building Temporal Ontologies

- **Timestamping.**
  The data model should distinguish between temporal and atemporal modeling constructs.

- **Evolution Constraints.**
  1. *Object Migration*: The possibility for an object to change its class membership;
  2. *Dynamic Relationships*: Either generate objects starting from other objects, or link objects existing at different times.
An Example: The $\mathcal{ER}_{VT}$ Company Schema

- **Member**: $S$ (1,n) to **OrganizationalUnit**
- **Employee**: $S$ (1,1) to **Manager**
- **Manager**: $T$ (1,1) to **TopManager**
- **OrganizationalUnit**: $S$
- **Department**: $S$
- **InterestGroup**: $S$
- **AreaManager**: $T$
- **TopManager**: $T$
- **Manager**: $T$
- **Project**: $S$ (1,n) to **Works-for**: $T$
- **Prop GP**: $S$ (1,1) to **Manages**: $T$
- **PaySlipNumber**: $S$
- **Name**: $S$
- **Salary**: $S$
- **ProjectCode**: $S$
Introduction: Motivations

Give a formalization based on set-theory of the various temporal constructs used to model temporal information systems.

1. Clarify the meaning of the various temporal constructs;

2. Verify the validity of standard modeling requirements defined for temporal data models;


4. Investigate the complexity of automatically checking these quality criteria.
Introduction: Modeling Requirements in a Temporal Setting

- **Orthogonality.** Temporal constructs should be specified separately and independently for classes, relationships, and attributes.

- **Upward Compatibility.** Preserve the non-temporal semantics of legacy conceptual schemas when embedded into temporal schemas.

- **Snapshot Reducibility.** A snapshot of the temporal database is described by the same schema without temporal constructs interpreted atemporally.
  - We should be able to fully rebuild a temporal database by starting from the single temporal snapshots.
Outline

• The Temporal Ontological Language $\mathcal{ER}_{VT}$
• $\mathcal{DLR}_{US}$: A Temporal Description Logic
• Modeling Timestamping
• Modeling Evolution Constraints
  – Status Classes
  – Transitions
  – Generation Relationships
  – Cross-Time Relationships
• Complexity Results
  – Undecidability Result
\( \mathcal{E}R_{VT} \): The Proposed Temporal Conceptual Model

\( \mathcal{E}R_{VT} \) is a temporal extended Entity-Relationship model able to capture Validity Time with the following features:

- it is equipped with both a linear and a graphical syntax;
- it has a model-theoretic semantics;
- it is a full-fledged conceptual model with constructors for representing both timestamping and evolution constraints.
An Example: The $\mathcal{ER}_{VT}$ Company Schema

- **Member $S$**
  - $\text{org}$
  - $\text{mbr}$
  - $(1,n)$
  - **OrganizationalUnit**
    - **Department $S$**
    - **InterestGroup**
  - **Manager $T$**
  - **Employee $S$**
    - **PaySlipNumber(Integer)**
    - **Salary(Integer)**
    - **Name(String)**
  - **Works-for $T$**
    - **Project $S$**
      - **ProjectCode(String)**
      - $(1,n)$
    - **Prop $GP$**
      - $(1,1)$
    - **Manages $T$**
      - **Prop $GP$**
      - $(1,1)$
      - **Manages**
        - **TopManager**
        - **AreaManager**
        - **Manager $T$**
The Model-Theoretic Semantics for $\mathcal{ER}_{VT}$

An interpretation, called *temporal database state*, for an $\mathcal{ER}_{VT}$ schema $\Sigma$ is a tuple $\mathcal{B} = (\mathcal{T}, \Delta^\mathcal{B}, \Delta^\mathcal{B}_D, .^\mathcal{B}(t))$:

- $\mathcal{T} = (\mathcal{Tp}, <)$, is the flow of time, where $\mathcal{Tp}$ is a set of time points (or chronons) and $<$ is a binary precedence relation on $\mathcal{Tp}$;
- $\Delta^\mathcal{B}$ is a nonempty set of abstract objects;
- $\Delta^\mathcal{B}_D$ is the set of basic domain values;
- $.^\mathcal{B}(t)$ is a function that for each $t \in \mathcal{T}$ maps:
  - Every domain symbol $D_i$ into a set $D_i^\mathcal{B}(t) = \Delta^\mathcal{B}_{D_i} \subseteq \Delta^\mathcal{B}_D$.
  - Every class $C$ to a set $C^\mathcal{B}(t) \subseteq \Delta^\mathcal{B}$.
  - Every n-ary relationship $R$ connecting the classes $C_1, \ldots, C_n$ to a set $R^\mathcal{B}(t)$, such that, $r \in R^\mathcal{B}(t) \rightarrow (r = \langle U_1 : o_1, \ldots, U_n : o_n \rangle \land \forall i \in \{1, \ldots, n\}. o_i \in C_i^\mathcal{B}(t))$.
  - Every attribute $A$ to a set $A^\mathcal{B}(t) \subseteq \Delta^\mathcal{B} \times \Delta^\mathcal{B}_D$. 

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The Model-Theoretic Semantics for $\mathcal{ER}_{VT}$ (Cont.)

$\mathcal{B}$ is said a legal temporal database state if it satisfies all constraints expressed in the schema. Thus, for all $t \in \mathcal{T}$:

- If $C_1$ ISA $C_2$, then, $C_1^{\mathcal{B}(t)} \subseteq C_2^{\mathcal{B}(t)}$
- If $R_1$ ISA $R_2$, then, $R_1^{\mathcal{B}(t)} \subseteq R_2^{\mathcal{B}(t)}$
- If $\text{ATT}(C) = \langle A_1 : D_1, \ldots, A_h : D_h \rangle$, then:
  $$o \in C^{\mathcal{B}(t)} \to (\forall i \in \{1, \ldots, h\}, \exists!a_i. \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)} \land \forall a_i. \langle o, a_i \rangle \in A_i^{\mathcal{B}(t)} \to a_i \in \Delta_{D_i}^{\mathcal{B}})$$
- For each cardinality constraint $\text{CARD}(C, R, U)$, then:
  $$o \in C^{\mathcal{B}(t)} \to \text{CMIN}(C, R, U) \leq \#\{r \in R^{\mathcal{B}(t)} | r[U] = o\} \leq \text{CMAX}(C, R, U)$$
- If $\{C_1, \ldots, C_n\}$ DISJ $C$, then:
  $$\forall i, j \in \{1, \ldots, n\}, C_i \text{ ISA } C \land C_i^{\mathcal{B}(t)} \cap C_j^{\mathcal{B}(t)} = \emptyset$$
- If $\{C_1, \ldots, C_n\}$ COVER $C$, then:
  $$\forall i \in \{1, \ldots, n\}. C_i \text{ ISA } C \land C^{\mathcal{B}(t)} = \bigcup_{i=1}^n C_i^{\mathcal{B}(t)}$$
Quality Criteria $\mathcal{ER}_{VT}$

The following quality criteria can be defined:

1. **$C$ ($R$) is satisfiable** if there exists a legal temporal database state $\mathcal{B}$ for $\Sigma$ such that $C^{\mathcal{B}(t)} \neq \emptyset$ ($R^{\mathcal{B}(t)} \neq \emptyset$), for some $t \in T$;

2. **$\Sigma$ is satisfiable** if there exists a legal temporal database state $\mathcal{B}$ for $\Sigma$ that satisfies at least one class in $\Sigma$ ($\mathcal{B}$ is said a model for $\Sigma$);

3. **$C_1$ ($R_1$) is subsumed** by $C_2$ ($R_2$) in $\Sigma$ if every legal temporal database state for $\Sigma$ is also a legal temporal database state for $C_1$ ISA $C_2$ ($R_1$ ISA $R_2$);

4. A schema $\Sigma'$ is **logically implied** by a schema $\Sigma$ over the same signature if every legal temporal database state for $\Sigma$ is also a legal temporal database state for $\Sigma'$. 

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The $\mathcal{DLR}_{US}$ Temporal Description Logic

$\mathcal{DLR}_{US}$ is obtained by combining the propositional linear temporal logic with \textbf{Since} and \textbf{Until} and the description logic $\mathcal{DLR}$.

\[
C \rightarrow \top \mid \bot \mid CN \mid \neg C \mid C_1 \sqcap C_2 \mid \exists^{\leq k}[U_i]R \\
\Diamond^+ C \mid \Diamond^- C \mid \Box^+ C \mid \Box^- C \mid \oplus C \mid \ominus C \mid C_1 \cup C_2 \mid C_1 \sqcup C_2
\]

\[
R \rightarrow \top_n \mid RN \mid \neg R \mid R_1 \sqcap R_2 \mid U_i/n : C \\
\Diamond^+ R \mid \Diamond^- R \mid \Box^+ R \mid \Box^- R \mid \oplus R \mid \ominus R \mid R_1 \cup R_2 \mid R_1 \sqcup R_2
\]

- $\mathcal{DLR}_{US}$ Knowledge Base is a collection of axioms on relationship and entity expressions: $R_1 \sqsubseteq R_2; \quad C_1 \sqsubseteq C_2$

- $\mathcal{DLR}_{US}$ is a fragment of the first-order temporal logic $L^{\{\text{since,until}\}}$
The $\mathcal{DLR}_US$ Semantics

A temporal interpretation over $\mathcal{T}$ is a triple $\mathcal{I} = \langle \mathcal{T}, \Delta^{\mathcal{I}}, \cdot^{\mathcal{I}}(t) \rangle$, where $\cdot^{\mathcal{I}}(t)$ is a function that for each $t \in \mathcal{T}$ maps:

- $CN^{\mathcal{I}(t)} \subseteq \Delta$
- $(\exists \leq k [U_i]R)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \#\langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t)} \mid d_i = d \} \leq k$
- $(\Diamond^+ C)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \exists v > t. d \in C^{\mathcal{I}(v)} \}$
- $(C_1 UC_2)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \exists v > t. (d \in C_2^{\mathcal{I}(v)} \land \forall w \in (t,v). d \in C_1^{\mathcal{I}(w)}) \}$
- $(C_1 SC_2)^{\mathcal{I}(t)} = \{ d \in \top^{\mathcal{I}(t)} \mid \exists v < t. (d \in C_2^{\mathcal{I}(v)} \land \forall w \in (v,t). d \in C_1^{\mathcal{I}(w)}) \}$
The $\mathcal{DLR}_{US}$ Semantics

A temporal interpretation over $\mathcal{T}$ is a triple $\mathcal{I} = \langle \mathcal{T}, \Delta^\mathcal{I}, \mathcal{I}(t) \rangle$, where $\mathcal{I}(t)$ is a function that for each $t \in \mathcal{T}$ maps:

- $CN^{\mathcal{I}(t)} \subseteq \Delta$
- $(\exists \leq^k [U_i] R)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \#\{d_1, \ldots, d_n\} \in R^{\mathcal{I}(t)} \mid d_i = d \} \leq k$
- $(\Diamond^+ C)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \exists v > t. d \in C^{\mathcal{I}(v)} \}$
- $(C_1 U C_2)^{\mathcal{I}(t)} = \{ d \in \Delta \mid \exists v > t. (d \in C_2^{\mathcal{I}(v)} \land \forall w \in (t, v). d \in C_1^{\mathcal{I}(w)}) \}$
- $(C_1 S C_2)^{\mathcal{I}(t)} = \{ d \in \top^{\mathcal{I}(t)} \mid \exists v < t. (d \in C_2^{\mathcal{I}(v)} \land \forall w \in (v, t). d \in C_1^{\mathcal{I}(w)}) \}$
- $R_N^{\mathcal{I}(t)} \subseteq (\top_n)^{\mathcal{I}(t)} \subseteq (\Delta^\mathcal{I})^n$
- $(U_i/n : C)^{\mathcal{I}(t)} = \{ \langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid d_i \in C^{\mathcal{I}(t)} \}$
- $(\oplus R)^{\mathcal{I}(t)} = \{ \langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t+1)} \}$
- $(\ominus R)^{\mathcal{I}(t)} = \{ \langle d_1, \ldots, d_n \rangle \in (\top_n)^{\mathcal{I}(t)} \mid \langle d_1, \ldots, d_n \rangle \in R^{\mathcal{I}(t-1)} \}$

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Temporal Operators: ‘Until’

Given a Temporal Interpretation, $\mathcal{I}$, a time, $t$, an object, $o \in \Delta$, and two concept expressions $C, D$:

\[
\langle \mathcal{I}, t, o \rangle \models C \text{U} D \text{ iff there exists } t' \text{ s.t. } (t' > t) \land \langle \mathcal{I}, t', o \rangle \models D \land \\
\text{for all } t'' \text{ s.t. } (t < t'' < t') \rightarrow \langle \mathcal{I}, t'', o \rangle \models C
\]

Examples:

\begin{align*}
\text{Start Lecture} & \sqsubseteq \text{Talk} \text{U} \text{End Lecture} \\
\text{Born} & \sqsubseteq \text{Alive} \text{U} \text{Dead} \\
\text{Request} & \sqsubseteq \text{Reply} \text{U} \text{Acknowledgement}
\end{align*}
Equivalences in $\mathcal{DLR}_{US}$

The temporal operators $\Diamond^+$ ($\Diamond^-$) and $\Box^+$ ($\Box^-$) are duals (for concept expressions):

$\neg \Box^+ C \equiv \Diamond^+ \neg C$

$\Diamond^+$ (and then $\Box^+$) can be rewritten in terms of $U$

$\Diamond^+ C \equiv \top U C$

$\Diamond^-$ (and then $\Box^-$) can be rewritten in terms of $S$

$\Diamond^- C \equiv \top S C$

$\oplus$ can be rewritten in terms of $U$

$\oplus C \equiv \bot U C$

$\ominus$ can be rewritten in terms of $S$

$\ominus C \equiv \bot S C$
Interpretation of $\mathcal{DLR}_{US}$ Knowledge Bases

- An interpretation $\mathcal{I}$ satisfies an axiom $C_1 \sqsubseteq C_2$ iff:
  $C_1^{\mathcal{I}(t)} \subseteq C_2^{\mathcal{I}(t)}$, for all $t \in \mathcal{T}$.

- An interpretation $\mathcal{I}$ satisfies an axiom $R_1 \sqsubseteq R_2$ iff:
  $R_1^{\mathcal{I}(t)} \subseteq R_2^{\mathcal{I}(t)}$, for all $t \in \mathcal{T}$.

- A knowledge base, $\Sigma$, is **satisfiable** if there is an interpretation that satisfies all the axioms in $\Sigma$ (in symbols, $\mathcal{I} \models \Sigma$).
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At the syntactical level, $\mathcal{ER}_{VT}$ supports timestamping of entities, relationships, and attributes using two different marks:

- $S$, for **Snapshot** constructs: Each of their instances has a global lifetime;
- $T$, for **Temporary** constructs: Each of their instances has a limited lifetime.
A Semantics for Timestamps

\[ o \in C^B(t) \rightarrow \forall t' \in T. o \in C^B(t') \]

Employer \( \sqsubseteq (\square^+ \text{Employee}) \cap (\square^- \text{Employee}) \)

\[ r \in R^B(t) \rightarrow \forall t' \in T. r \in R^B(t') \]

Member \( \sqsubseteq (\square^+ \text{Member}) \cap (\square^- \text{Member}) \)
• \( o \in C^B(t) \rightarrow \forall t' \in T. o \in C^B(t') \)
  \[ \text{Employee } \sqsubseteq (\Box^+ \text{Employee}) \sqcap (\Box^- \text{Employee}) \]

• \( r \in R^B(t) \rightarrow \forall t' \in T. r \in R^B(t') \)
  \[ \text{Member } \sqsubseteq (\Box^+ \text{Member}) \sqcap (\Box^- \text{Member}) \]

• \( o \in C^B(t) \rightarrow \exists t' \neq t. o \notin C^B(t') \)
  \[ \text{Manager } \sqsubseteq (\Diamond^+ \neg \text{Manager}) \sqcup (\Diamond^- \neg \text{Manager}) \]

• \( r \in R^B(t) \rightarrow \exists t' \neq t. r \notin R^B(t') \)
  \[ \text{Works-for } \sqsubseteq (\Diamond^+ \neg \text{Works-for}) \sqcup (\Diamond^- \neg \text{Works-for}) \]
• \((o \in C^{B(t)} \land \langle o, a_i \rangle \in A^{B(t)}_i) \rightarrow \forall t' \in T. \langle o, a_i \rangle \in A^{B(t')}_i\)

Employee \(\sqsubseteq \exists^{-1}[\text{From}](\text{Name} \sqcap \text{To}/2 : \text{String}) \sqcap \exists^{-1}[\text{From}]\Box^*\text{Name}\)
Timestamping Attributes

- \( (o \in C^B(t) \land \langle o, a_i \rangle \in A^B_i(t)) \rightarrow \forall t' \in T. \langle o, a_i \rangle \in A^B_i(t') \)
  
  Employee \( \sqsubseteq \exists^=1[From](Name \sqcap \text{To}/2 : \text{String}) \sqcap \exists^=1[From] \square^* \text{Name} \)

- \( (o \in C^B(t) \land \langle o, a_i \rangle \in A^B_i(t)) \rightarrow \exists t' \neq t. \langle o, a_i \rangle \not\in A^B_i(t') \)
  
  Employee \( \sqsubseteq \exists^=1[From](Salary \sqcap \text{To}/2 : \text{Integer}) \sqcap \exists^=1[From](Salary \sqcap (\diamond^+ \neg \text{Salary} \sqcup \diamond^- \neg \text{Salary})) \)
The following are some of the classical cases of logical implications found in the literature and captured by the $\mathcal{ER}_{VT}$ semantics:
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- Sub-entities of temporary entities must be temporary.
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- Participants of snapshot relationships must be snapshot entities when they participate at least once.
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- Sub-entities of temporary entities must be temporary.
- Participants of snapshot relationships must be snapshot entities when they participate at least once.
- A schema is inconsistent if exactly one of a whole set of snapshot partitioning sub-entities is temporary.
The following are some of the classical cases of logical implications found in the literature and captured by the $\mathcal{ER}_{VT}$ semantics:

- Sub-entities of temporary entities must be temporary.
- Participants of snapshot relationships must be snapshot entities when they participate at least once.
- A schema is inconsistent if exactly one of a whole set of snapshot partitioning sub-entities is temporary.
- A relationship is temporary if one of the participating entities is temporary.
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Evolution Constraints: Status Classes

Describe the evolving status of membership of each object in the class. Four different statuses can be specified, together with precise transitions between them:

- **Scheduled.** An object is scheduled if its existence within the class is known but its membership in the class will only become effective some time later.

- **Active.** The status of an object is active if the object is a full member of the class.

- **Suspended.** This status qualifies objects that exist as members of the class, but are to be seen as inactive members of the class.

- **Disabled.** It is used to model expired objects in a class.
(EXISTS) *Existence persists until Disabled.*
\[
o \in \text{Exists-C}^B(t) \rightarrow \forall t' > t. (o \in \text{Exists-C}^B(t') \lor o \in \text{Disabled-C}^B(t'))
\]
\[
\text{Exists-C} \sqsubseteq \square^+ (\text{Exists-C} \sqcup \text{Disabled-C})
\]

(DISAB1) *Disabled persists.*
\[
o \in \text{Disabled-C}^B(t) \rightarrow \forall t' > t. o \in \text{Disabled-C}^B(t')
\]
\[
\text{Disabled-C} \sqsubseteq \square^+ \text{Disabled-C}
\]

(DISAB2) *Disabled was Active in the past.*
\[
o \in \text{Disabled-C}^B(t) \rightarrow \exists t' < t. o \in \text{C}^B(t')
\]
\[
\text{Disabled-C} \sqsubseteq \Diamond \neg \text{C}
\]
(SUSP) *Suspended was Active in the past.*
\[ o \in \text{Scheduled-}C^B(t) \rightarrow \exists t' < t. o \in C^B(t') \]
\[ \text{Suspended-}C \sqsubseteq \Diamond \neg C \]

(SCH1) *Scheduled will eventually become Active.*
\[ o \in \text{Scheduled-}C^B(t) \rightarrow \exists t' > t. o \in C^B(t') \]
\[ \text{Scheduled-}C \sqsubseteq \Diamond^+ C \]

(SCH2) *Scheduled can never follow Active.*
\[ o \in C^B(t) \rightarrow \forall t' > t. o \not\in \text{Scheduled-}C^B(t') \]
\[ C \sqsubseteq \square^+ \neg \text{Scheduled-}C \]
Logical Consequences from Status Classes

(TEMP) *Scheduled, Suspended and Disabled are temporary classes.*

(SCH3) *Scheduled persists until active.*

Scheduled-C ⊑ Scheduled-C ∪ C.

(SCH4) *Scheduled cannot evolve directly to Disabled*

Scheduled-C ⊑ ⊕ ¬Disabled-C.

(DISAB3) *Disabled was active but it will never become active anymore*

Disabled-C ⊑ ◊¬(C ∩ ♠⁺¬C).

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Evolution Constraints: Transitions

Dynamic Transitions between classes model the notion of object migration from a source to a target class.

1. **Dynamic Evolution**, when an object ceases to be an instance of a source class.
   - **Example.** “An area manager can become a top manager while ceasing to be an area manager.”
   
   ![Diagram](AreaManager -- DEV -- TopManager)

2. **Dynamic Extension**, when an object is still allowed to belong to the source.
   - **Example.** “An employee can become a manager.”
   
   ![Diagram](Employee -- DEX -- Manger)
Constraints and Semantics for Transitions

Specifying a transition between two classes means that:

1. We want to keep track of such migration;

2. Not necessarily all the objects in the source participate in the migration;

3. When the source class is a temporal class, migration involves only objects active or suspended.
Constraints and Semantics for Transitions (Cont.)

We introduce two classes denoted by either $\text{DEX}_{C_1,C_2}$ or $\text{DEV}_{C_1,C_2}$ to store the migration of objects from $C_1$ to $C_2$.

- Semantics for dynamic extension between classes $C_1, C_2$.

\[
o \in \text{DEX}^{B(t)}_{C_1,C_2} \rightarrow (o \in (\text{Suspended}-C_1^{B(t)} \cup C_1^{B(t)})) \land o \notin C_2^{B(t)} \land o \in C_2^{B(t+1)}
\]

\[
\text{DEX}_{C_1,C_2} \sqsubseteq (\text{Suspended}-C_1 \sqcup C_1) \sqcap \neg C_2 \sqcap \bigoplus C_2.
\]
Constraints and Semantics for Transitions (Cont.)

We introduce two classes denoted by either $\text{DEX}_{C_1,C_2}$ or $\text{DEV}_{C_1,C_2}$ to store the migration of objects from $C_1$ to $C_2$.

- **Semantics for dynamic extension between classes $C_1, C_2$.**
  \[
  o \in \text{DEX}^{B(t)}_{C_1,C_2} \rightarrow (o \in (\text{Suspended-}C_1^B(t) \cup C_1^B(t)) \land o \notin C_2^B(t) \land o \in C_2^{B(t+1)})
  \]
  
  \[
  \text{DEX}_{C_1,C_2} \sqsubseteq (\text{Suspended-}C_1 \sqcup C_1) \sqcap \neg C_2 \sqcap \oplus C_2.
  \]

- **Semantics for dynamic evolution between classes $C_1, C_2$.**
  \[
  o \in \text{DEV}^{B(t)}_{C_1,C_2} \rightarrow (o \in (\text{Suspended-}C_1^B(t) \cup C_1^B(t)) \land o \notin C_2^B(t) \land
  \]
  \[
  o \in C_2^{B(t+1)} \land o \notin C_1^{B(t+1)})
  \]
  
  \[
  \text{DEV}_{C_1,C_2} \sqsubseteq (\text{Suspended-}C_1 \sqcup C_1) \sqcap \neg C_2 \sqcap \oplus (C_2 \sqcap \neg C_1)
  \]
Logical Consequences from Transitions

1. The classes $\text{DEX}_{C_1,C_2}$ and $\text{DEV}_{C_1,C_2}$ are temporary classes (actually, they are instantaneous).

2. Objects in the classes $\text{DEX}_{C_1,C_2}$ and $\text{DEV}_{C_1,C_2}$ cannot be disabled as $C_2$.

3. The target class $C_2$ cannot be snapshot (it becomes temporary if all of its members are involved in the migration).

4. The source class $C_1$ cannot be snapshot when it is involved into a dynamic evolution (it becomes temporary if all of its members are involved in the migration).

5. Dynamic evolution cannot involve sub-classes (Note: this implication doesn’t hold for dynamic extension).

6. Dynamic extension between disjoint classes logically implies Dynamic evolution.
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Evolution Constraints: Generation Relationships

Generation relationships represent processes that lead to the emergence of new instances starting from a set of instances.

1. **Production Relationships**, when the source objects survive the generation process (GP marked).

   ![Production Relationship Diagram]

   - Mother
   - GiveBirth GP
   - Baby

2. **Transformation Relationships**, when all the instances involved in the process are consumed (GT marked).

   ![Transformation Relationship Diagram]

   - Orange
   - Give GT
   - Juice
A Semantics for Generation Relationships

We model generation as binary relationships connecting a source class to a target one:

\[ \text{REL}(R) = \langle \text{source} : C_1, \text{target} : \text{Scheduled-C}_2 \rangle \]

- **Semantics for Production Relationships**
  \[
  \langle o_1, o_2 \rangle \in R^B(t) \rightarrow (o_1 \in C_1^B(t) \land o_2 \in \text{Scheduled-C}_2^B(t) \land o_2 \in C_2^B(t+1))
  \]
  \[
  R \sqsubseteq \text{source} : C_1 \sqcap \text{target} : (\text{Scheduled-C}_2 \sqcap \oplus C_2)
  \]
A Semantics for Generation Relationships

We model generation as binary relationships connecting a source class to a target one:

\[
\text{REL}(R) = \langle \text{source} : C_1, \text{target} : \text{Scheduled}-C_2 \rangle
\]

- **Semantics for Production Relationships**
  \[
  \langle o_1, o_2 \rangle \in R^B(t) \rightarrow (o_1 \in C_1^B(t) \land o_2 \in \text{Scheduled}-C_2^B(t) \land o_2 \in C_2^B(t+1))
  \]
  \[
  R \sqsubseteq \text{source} : C_1 \sqinter \text{target} : (\text{Scheduled}-C_2 \sqinter \oplus C_2)
  \]

- **Semantics for Transformation Relationships**
  \[
  \langle o_1, o_2 \rangle \in R^B(t) \rightarrow (o_1 \in C_1^B(t) \land o_1 \in \text{Disabled}-C_1^B(t+1) \land
  o_2 \in \text{Scheduled}-C_2^B(t) \land o_2 \in C_2^B(t+1))
  \]
  \[
  R \sqsubseteq \text{source} : (C_1 \sqinter \oplus \text{Disabled}-C_1) \sqinter \text{target} : (\text{Scheduled}-C_2 \sqinter \oplus C_2)
  \]
Logical Consequences from Generation Relationships

1. A generation relationship, $R$, is temporary (actually, it is instantaneous).

2. The target class, $C_2$, cannot be snapshot (it becomes temporary if total participation is specified).

3. The target class, $C_2$, cannot be disabled.

4. If $R$ is a transformation relationship, then, $C_1$ cannot be snapshot.
Outline

• The Temporal Ontological Language $\mathcal{ER}_{VT}$

• $\mathcal{DLR}_{US}$: A Temporal Description Logic

• Modeling Timestamping

• **Modeling Evolution Constraints**
  – Status Classes
  – Transitions
  – Generation Relationships
  – Cross-Time Relationships

• Complexity Results
  – Undecidability Result
Evolution Constraints: Cross-Time Relationships

- Cross-time relationships relate objects that are members of the participating classes at different times.
- We formalize cross-time relationships with the aim of preserving the snapshot reducibility.
- Example:
  - Biography \subseteq Author \times Person
  - bio = \langle Tulard, Napoleon \rangle and bio \in Biography^B(1984)
Evolution Constraints: Cross-Time Relationships

- **Cross-time relationships** relate objects that are members of the participating classes at different times.

- We formalize cross-time relationships with the aim of preserving the snapshot reducibility.

- **Example:**
  - Biography $\subseteq$ Author $\times$ Person
  - $\text{bio} = \langle \text{Tulard}, \text{Napoleon} \rangle$ and $\text{bio} \in \text{Biography}^B(1984)$

- Snapshot Reducibility would imply the following constraints:
  - $\text{Tulard} \in \text{Author}^B(1984)$;
  - $\text{Napoleon} \in \text{Person}^B(1984)$
Evolution Constraints: Cross-Time Relationships

- **Cross-time relationships** relate objects that are members of the participating classes at different times.

- We formalize cross-time relationships with the aim of preserving the snapshot reducibility.

- **Example:**
  - Biography ⊆ Author × Person
  - bio = ⟨Tulard, Napoleon⟩ and bio ∈ Biography\(^B(1984)\)

- **Snapshot Reducibility** would imply the following constraints:
  - Tulard ∈ Author\(^B(1984)\);
  - Napoleon ∈ Person\(^B(1984)\)

- **Solution.** Use status classes to preserve snapshot reducibility.
  - Napoleon is a member of the Disabled-Person class in 1984.
A Semantics for Cross-Time Relationships

- Strictly Past ($P$).
  \[ r = \langle o_1, o_2 \rangle \in R^B(t) \rightarrow o_1 \in Disabled-C_1^B(t) \]
  \[ R \sqsubseteq U_1 : Disabled-C_1. \]

- Past ($P,=)$
  \[ r = \langle o_1, o_2 \rangle \in R^B(t) \rightarrow o_1 \in (C_1 \sqcup Disabled-C_1)^B(t) \]
  \[ R \sqsubseteq U_1 : (C_1 \sqcup Disabled-C_1). \]
A Semantics for Cross-Time Relationships (Cont.)

- **Strictly Future (F)**
  \[ r = \langle o_1, o_2 \rangle \in R^B(t) \rightarrow o_1 \in \text{Scheduled-C}_1^B(t) \]
  \[ R \subseteq U_1 : \text{Scheduled-C}_1. \]

- **Future (F,=)**
  \[ r = \langle o_1, o_2 \rangle \in R^B(t) \rightarrow o_1 \in (C_1 \sqcup \text{Scheduled-C}_1)^B(t) \]
  \[ R \subseteq U_1 : (C_1 \sqcup \text{Scheduled-C}_1). \]

- **Full-Cross (P,=,F)**
  \[ r = \langle o_1, o_2 \rangle \in R^B(t) \rightarrow o_1 \in (C_1 \sqcup \text{Scheduled-C}_1 \sqcup \text{Disabled-C}_1)^B(t) \]
  \[ R \subseteq U_1 : (C_1 \sqcup \text{Scheduled-C}_1 \sqcup \text{Disabled-C}_1). \]
Outline

- The Temporal Ontological Language $ER_{VT}$
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- **Complexity Results**
  - Undecidability Result
Correctness of the Encoding

**Theorem.** An \( \mathcal{ER}_{VT} \) schema can be correctly encoded into a \( \mathcal{DLR}_{US} \) theory—i.e., to each temporal legal database of an \( \mathcal{ER}_{VT} \) schema corresponds a model of the resulting \( \mathcal{DLR}_{US} \) theory and vice versa. [Artale,Franconi:ER99]

**Corollary.** Reasoning over \( \mathcal{ER}_{VT} \) schemas can be reduced to reasoning over the \( \mathcal{DLR}_{US} \) encoding.
Computational Properties of $\mathcal{DLR}_{US}$: Two theorems

**Theorem.** Logical implication in $\mathcal{DLR}_{US}$ over a linear, unbounded, discrete temporal structure is *undecidable*. [Artale-et-al:JELIA-02]
Computational Properties of $\mathcal{DLR}_{US}$: Two theorems

**Theorem.** Logical implication in $\mathcal{DLR}_{US}$ over a linear, unbounded, discrete temporal structure is *undecidable*. [Artale-et-al:JELIA-02]

- The maximal decidable fragment of $\mathcal{DLR}_{US}$ is the monodic fragment $\mathcal{DLR}_{US}^{-}$:

  $\begin{align*}
  R & \rightarrow \top_n | RN | \neg R | R_1 \sqcap R_2 | U_i/n : C | \\
  & | \diamond^+ R | \diamond^- R | \Box^+ R | \Box^- R | \bigcirc R | R_1 \cup R_2 | R_1 \sqcup R_2 \\
  C & \rightarrow \top | \bot | CN | \neg C | C_1 \sqcap C_2 | \exists^k [U_i] R | \\
  & | \diamond^+ C | \diamond^- C | \Box^+ C | \Box^- C | \bigcirc C | C_1 \cup C_2 | C_1 \sqcup C_2
  \end{align*}$

**Theorem.** Logical implication in the monodic fragment of $\mathcal{DLR}_{US}$ over a linear, unbounded, discrete temporal structure is *EXPTIME-complete*.
Decidability Results for $\mathcal{ER}_{VT}$

[QUESTION:] Does the $\mathcal{DLR}_{US}$ undecidability result transfers to $\mathcal{ER}_{VT}$, too?
Decidability Results for $\mathcal{ER}_{VT}$

[QUESTION:] Does the $\mathcal{DLR}_{US}$ undecidability result transfer to $\mathcal{ER}_{VT}$, too?

- [ANSWER 1:] YES! As far as $\mathcal{ER}_{VT}$ uses both timestamping and evolution constructs.
  
  - **Theorem.** Reasoning in $\mathcal{ER}_{VT}$ using both timestamping and evolution constraints is undecidable. [Artale:TIME-04]

- [ANSWER 2:] Open Problem! As far as $\mathcal{ER}_{VT}$ uses just timestamping.
Decidability Results for $\mathcal{ER}_{VT}$ (Cont.)

[QUESTION:] Does the $EXPTIME$-complete result for $\mathcal{DLR}_{US}^-$ transfers to $\mathcal{ER}_{VT}$ as well?
Decidability Results for $\mathcal{ER}_{VT}$ (Cont.)

[QUESTION:] Does the $EXPTIME$-complete result for $\mathcal{DLR}_{US}$ transfers to $\mathcal{ER}_{VT}$ as well?

- [ANSWER:] YES! As far as $\mathcal{ER}_{VT}$ does not use temporal constructs over relationships and attributes.
  
  - Theorem. Reasoning in $\mathcal{ER}_{VT}$ using both timestamping just over Classes and evolution constraints is complete for $EXPTIME$. [Artale-et-al:FoIKS-06]
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**ERVT Undecidability Proof**

The proof is based on a reduction from the undecidable **Halting Problem** to the **Entity Satisfiability Problem w.r.t. an ERVT Schema**. We proceed as follows:

1. Reduction of the Halting Problem to Concept Satisfiability Problem w.r.t. an $\mathcal{ALC}_F$ KB (ideas similar to [Gabbay:Kurucz:Wolter:Zakharyaschev:03]);

2. Reduction of Concept Satisfiability w.r.t. an $\mathcal{ALC}_F$ KB to Entity Satisfiability w.r.t. an $\mathcal{ER}_{VT}$ Schema.

**Remark.** $\mathcal{ALC}_F$ is a tense-logical extension of $\mathcal{ALC}$: $\Diamond^+ C$ (sometime in the future), $\Box^+ C$ (always in the future), and possibly **Global Roles**.
Halting Problem

- **Single-tape right-infinite deterministic Turing machine** \( \mathcal{M} : \langle A, S, \rho \rangle \), where:
  - \( A \) is the *tape alphabet* (\( b \in A \) stands for blank);
  - \( S \) is a finite set of *states* with the *initial state*, \( s_0 \), and the *final state*, \( s_1 \);
  - \( \rho \) is the *transition function*, \( \rho : (S - \{s_1\}) \times A \to S \times (A \cup \{L, R\}) \).

- **Configuration** of \( \mathcal{M} \) is an infinite sequence: \( \langle \mathcal{L}, a_1, \ldots, a_{i-1}, \langle s_i, a_i \rangle, \ldots, a_n, b, \ldots \rangle \);
Halting Problem

• Single-tape right-infinite deterministic Turing machine $M$: $\langle A, S, \rho \rangle$, where:
  – $A$ is the tape alphabet ($b \in A$ stands for blank);
  – $S$ is a finite set of states with the initial state, $s_0$, and the final state, $s_1$;
  – $\rho$ is the transition function, $\rho: (S - \{s_1\}) \times A \rightarrow S \times (A \cup \{L, R\})$.

• Configuration of $M$ is an infinite sequence: $\langle \mathcal{L}, a_1, \ldots, a_{i-1}, \langle s_i, a_i \rangle, \ldots, a_n, b, \ldots \rangle$;

• Since a transition function can only modify the active cell and its neighbors we introduce the instruction function, $\delta$:

$$
\delta(a_i, \langle s, a_j \rangle, a_k) = \begin{cases} 
\langle a_i, \langle s', a'_j \rangle, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', a'_j \rangle \\
\langle \langle s', a_i \rangle, a_j, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', L \rangle \text{ and } a_i \neq \mathcal{L} \\
\langle \mathcal{L}, \langle s', a_j \rangle, a_k \rangle, & \text{if } \rho(s, a_j) = \langle s', L \rangle \text{ and } a_i = \mathcal{L} \\
\langle a_i, a_j, \langle s', a_k \rangle \rangle, & \text{if } \rho(s, a_j) = \langle s', R \rangle 
\end{cases}
$$

• A sequence $\langle c_0, c_1, \ldots, c_k, c_{k+1}, \ldots \rangle$ of configurations is said a Computation of $M$. 
Halting Problem: Definition

We say that $M$ halts, starting with the empty tape—i.e. with starting configuration: $\langle \mathcal{L}, \langle s_0, b \rangle, b, \ldots, b, \ldots \rangle$—if there is a finite computation, $\langle c_0, c_1, \ldots, c_k \rangle$, such that the state of $c_k$ is $s_1$ (the final state).
Encoding the Halting Problem with $\mathcal{ALC}_F$

\[ C_0 \sqsubseteq C_L \sqcap \Diamond^+ C_{\langle s_0, b \rangle} \]
Encoding the Halting Problem with $\mathcal{ALC}_F$

\[
C_0 \subseteq C_L \cap \Diamond^+ C_{\langle s_0, b \rangle} \\
\text{next}(C_L, D_1) \\
(C_L \subseteq \Diamond^+ D_1 \cap \neg \Diamond^+ \Diamond^+ D_1)
\]
Encoding the Halting Problem with $\mathcal{ALC}_F$

$C_0 \equiv C_\mathcal{L} \cap \Diamond^+ C_{\langle s_0, b \rangle}$

$\text{next}(C_\mathcal{L}, D_1)$

$(C_\mathcal{L} \equiv \Diamond^+ D_1 \cap \neg \Diamond^+ \Diamond^+ D_1)$

$\text{next}(D_1, D_2)$

$C_{\langle s_0, b \rangle} \equiv D_1$
Encoding the Halting Problem with $\mathcal{ALC}_F$:

$C_0 \sqsubseteq C_{\mathcal{L}} \cap \Diamond^+ C_{\langle s_0, b \rangle}$

$\text{next}(C_{\mathcal{L}}, D_1)$

$(C_{\mathcal{L}} \sqsubseteq \Diamond^+ D_1 \cap \neg \Diamond^+ \Diamond^+ D_1)$

$\text{next}(D_1, D_2)$

$C_{\langle s_0, b \rangle} \sqsubseteq D_1$

$C_{\langle s_0, b \rangle} \sqsubseteq \square^+ C_b$

$\text{discover}(C, \{ C_x \mid x \in A \cup \{ \mathcal{L} \} \cup (S \times A) \})$
Encoding the Halting Problem with $\mathcal{ALCF}$

$C_0 \sqsubseteq C_\mathcal{L} \cap \Diamond^+ C_{\langle s_0, b \rangle}$

$\text{next}(C_\mathcal{L}, D_1)$

$(C_\mathcal{L} \sqsubseteq \Diamond^+ D_1 \cap \neg \Diamond^+ \Diamond^+ D_1)$

$\text{next}(D_1, D_2)$

$C_{\langle s_0, b \rangle} \sqsubseteq D_1$

$C_{\langle s_0, b \rangle} \sqsubseteq \Box^+ C_b$

$\text{discover}(C, \{C_x \mid x \in A \cup \{\mathcal{L}\} \cup (S \times A)\})$

$x_0 \rightarrow \langle \mathcal{L}, \langle s_0, b \rangle, b, b, \ldots \rangle$
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

\[
discover(C_s, \{C_{\langle s, a \rangle} \mid \langle s, a \rangle \in S \times A\})
\]
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

\[
\text{discover}(C_s, \{C_{\langle s, a \rangle} \mid \langle s, a \rangle \in S \times A\})
\]
\[
\text{next}(C_s, C_r)
\]
\[
\text{next}(C_r, D_3)
\]
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

\[
\text{discover}(C_s, \{C_{\langle s, a \rangle} \mid \langle s, a \rangle \in S \times A\}) \\
\text{next}(C_s, C_r) \\
\text{next}(C_r, D_3) \\
C_L \sqsubseteq C_l \sqcup \Diamond^+ C_l \\
\text{next}(C_l, C_s)
\]
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$\top \sqsubseteq \exists R. \top$ (with $R$ global)
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$\top \subseteq \exists R. \top$ (with $R$ global)

$\delta(\mathcal{L}, \langle s_0, b \rangle, b) = \langle \mathcal{L}, b, \langle s', b \rangle \rangle$

$C_I \subseteq C_\mathcal{L} \rightarrow \forall R. C_\mathcal{L}$
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$\top \subseteq \exists R. \top$ (with $R$ global)

$\delta(L, \langle s_0, b \rangle, b) = \langle L, b, \langle s', b \rangle \rangle$

$C_l \sqsubseteq C_{\langle s_0, b \rangle} \rightarrow \forall R. C_{\langle s_0, b \rangle}$

$C_b \sqsubseteq C_b \rightarrow \forall R. C_b$
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$$\top \sqsubseteq \exists R. \top \quad \text{(with R global)}$$

$$\delta(\mathcal{L}, \langle s_0, b \rangle, b) = \langle \mathcal{L}, b, \langle s', b \rangle \rangle$$

$$C_l \sqsubseteq C_{\mathcal{L}} \rightarrow \forall R. C_{\mathcal{L}}$$

$$C_s \sqsubseteq C_{\langle s_0, b \rangle} \rightarrow \forall R. C_b$$

$$C_r \sqsubseteq C_b \rightarrow \forall R. C_{\langle s', b \rangle}$$
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$$\top \sqsubseteq \exists R. \top \text{ (with } R \text{ global)}$$

$$\delta(\mathcal{L}, \langle s_0, b \rangle, b) = \langle \mathcal{L}, b, \langle s', b \rangle \rangle$$

$$C_l \sqsubseteq C_\mathcal{L} \rightarrow \forall R. C_\mathcal{L}$$

$$C_s \sqsubseteq C_{\langle s_0, b \rangle} \rightarrow \forall R. C_b$$

$$C_r \sqsubseteq C_b \rightarrow \forall R. C_{\langle s', b \rangle}$$

$$C_a \sqsubseteq (\neg C_l \sqcap \neg C_s \sqcap \neg C_r) \rightarrow \forall R. C_a$$
Encoding the Halting Problem with $\mathcal{ALC}_F$ (Cont.)

$$\top \sqsubseteq \exists R. \top \text{ (with } R \text{ global)}$$

$$\delta(\mathcal{L}, \langle s_0, b \rangle, b) = \langle \mathcal{L}, b, \langle s', b \rangle \rangle$$

$$C_i \sqsubseteq C_\mathcal{L} \rightarrow \forall R. C_\mathcal{L}$$

$$C_s \sqsubseteq C_{\langle s_0, b \rangle} \rightarrow \forall R. C_b$$

$$C_r \sqsubseteq C_b \rightarrow \forall R. C_{\langle s', b \rangle}$$

$$C_a \sqsubseteq (\neg C_i \sqcap \neg C_s \sqcap \neg C_r) \rightarrow \forall R. C_a$$

$x_1 \rightarrow \langle \mathcal{L}, b, \langle s', b \rangle, b, b, \ldots \rangle$
The chain of \( R \)-successor, \( \langle x_0, x_1, x_2, \ldots \rangle \), represents a computation of \( M \);

- The following axioms:
  
  \[
  \text{discover}(C_s, \{ C_{\langle s, a \rangle} \mid \langle s, a \rangle \in S \times A \})
  \]

  \[
  \text{discover}(S1, \{ C_{\langle s_1, a \rangle} \mid a \in A \cup \{ \$ \} \})
  \]

  \[
  C_s \sqsubseteq \neg S1
  \]

  Guarantee that \( M \) does not halt.
Reducing $\mathcal{ALC}_F$ Axioms to $\mathcal{ER}_{VT}$ Schema

- To capture standard $\mathcal{ALC}$ axioms we use the translation presented in [Berrarri:Calì:Calvanese:DeGiacomo:03] apart from axioms of the form $\mathcal{C} \sqsubseteq \forall R . \mathcal{C}$ and $\top \sqsubseteq \exists R . \mathcal{C}$:
Reducing $\mathcal{ALC}_F$ Axioms to $\mathcal{ER}_{VT}$ Schema (Cont.)

- Axioms of the form $C \sqsubseteq \Diamond^+ D$ are captured using total dynamic extension:
  
  $C \xrightarrow{\text{T-DEX}} D$

- Axioms of the form $C \sqsubseteq \Box^+ D$ are captured using dynamic evolution and status classes (in particular, disabled status):
  
  $C \xrightarrow{\text{DEV}} \text{Disabled-CD}$
  
  $D \xrightarrow{\text{DEV}} \text{Disabled-CD}$
Reducing $\mathcal{ALC}_F$ Axioms to $\mathcal{ER}_{VT}$ Schema (Cont.)

- Axioms of the form $\text{next}(C, D) \equiv \lozenge^+ D \land \square^+ \square^+ \neg D$ are mapped by using the dynamic constraints:
Conclusions

- We presented the temporal data model $\mathcal{ER}_{VT}$ which combines a linear and visual syntax with a rigorous set-theoretic semantics.

- $\mathcal{ER}_{VT}$ captures both timestamping and evolution constraints.

- The formalization of each construct gives rise to a set of constraints as a logical consequence of its semantics.

- Quality criteria as schema consistency and logical implication of implicit constraints have been semantically defined.

- Using a description logic translation reasoning over $\mathcal{ER}_{VT}$ has been showed decidable (if we give up temporal relationships).
Conclusions

• We presented the temporal data model $\mathcal{E}\mathcal{R}_{VT}$ which combines a linear and visual syntax with a rigorous set-theoretic semantics.

• $\mathcal{E}\mathcal{R}_{VT}$ captures both timestamping and evolution constraints.

• The formalization of each construct gives rise to a set of constraints as a logical consequence of its semantics.

• Quality criteria as schema consistency and logical implication of implicit constraints have been semantically defined.

• Using a description logic translation reasoning over $\mathcal{E}\mathcal{R}_{VT}$ has been showed decidable (if we give up temporal relationships).

• Open Problem. Does reasoning on $\mathcal{E}\mathcal{R}_{VT}$ with full timestamping but without evolution constraints become decidable?
  
  – Hint. Check the decidability of the epistemic description logic $S5 \times \mathcal{DL}$. 