Decidable Reasoning over Timestamped Conceptual Models

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Abstract. We show that reasoning in the temporal conceptual model $\mathcal{ER}^{-}_{VT}$, a fragment of $\mathcal{ER}_{VT}$ that only allows *timestamping* is complete for 2-ExpTime. The membership result is based on an embedding of the conceptual model into the description logic $\mathsf{S5}_{\mathcal{ALCQI}}$. Hardness is obtained by reducing a fragment of $\mathsf{S5}_{\mathcal{ALCQI}}$, namely $\mathsf{S5}_{\mathcal{ALC}}$ with global roles only, to $\mathcal{ER}_{VT}$.

1 Introduction

This paper studies the problem of reasoning over temporal conceptual data models, in particular the $\mathcal{ER}_{VT}$ [3] model, a model equipped with both a linear and a graphical syntax, and accompanied by a model-theoretic semantics. The $\mathcal{ER}_{VT}$ model is a temporal extension of the EER model (the Extended Entity-Relationship data model [11]). In addition to the classical constructors, such as inheritance (isa) between entities and between relationships, cardinality constraints restricting the participation of entities in relationships, and disjointness and covering constraints, it is also able to express the following *temporal constraints*:

Timestamping that allows to classify constructs as *snapshot*—whose instances must have a global lifespan—and as *temporary*—whose instances must have a limited lifespan.

Dynamic Constructs that describe how an object can (or must) change its class membership over time. Such constraints are often called *transition* constraints [13] and govern *object migration*.

Constraints expressed in the $\mathcal{ER}_{VT}$ models can be captured using Temporal Description Logics (TDLs), in particular the logic $\mathcal{ALCQI}_{\mathcal{US}}$—an undecidable temporal extension of $\mathcal{ALCQI}$ [6] with the LTL (linear-time temporal logic) temporal modalities *until* and *since* [3, 4]. In addition, it has been shown that reasoning in temporal class diagrams alone (and hence in $\mathcal{ER}_{VT}$ schemas) is also undecidable [1] (hence the undecidability is not caused by choosing a loose embedding).

The contribution of this paper is showing that $\mathsf{S5}_{\mathcal{ALCQI}}$ [5]—a temporal description logic that combines a simpler modal logic, $\mathsf{S5}$, with the description logic $\mathcal{ALCQI}$—is sufficient to capture $\mathcal{ER}^-_{VT}$, a fragment of $\mathcal{ER}_{VT}$ that uses timestamping as its sole temporal construct. The embedding then provides a 2-ExpTime upper bound for reasoning in $\mathcal{ER}^-_{VT}$, as reasoning in $\mathsf{S5}_{\mathcal{ALCQI}}$ is complete for 2-ExpTime. In addition, the paper provides a matching 2-ExpTime lower bound for reasoning in $\mathcal{ER}^-_{VT}$, hence showing that the embedding is complexity-wise optimal.

The rest of the paper is organized as follows: Sections 2 and 3 provide the necessary background and definitions for $\mathsf{S5}_{\mathcal{ALCQI}}$ and $\mathcal{ER}_{VT}$, respectively. Section 4 shows how $\mathcal{ER}^-_{VT}$ diagrams can be captured in $\mathsf{S5}_{\mathcal{ALCQI}}$. Section 5 shows $\mathcal{ER}^-_{VT}$ patterns that capture a sufficiently large fragment of $\mathsf{S5}_{\mathcal{ALCQI}}$ needed to show hardness of reasoning in $\mathcal{ER}_{VT}$.

2 The Logic $\mathsf{S5}_{\mathcal{ALCQI}}$

The logic $\mathsf{S5}_{\mathcal{ALCQI}}$ is a combination of the modal logic $\mathsf{S5}$ and the description logic $\mathcal{ALCQI}$. It is similar in spirit to the multi-dimensional description logics proposed, e.g., in [12, 14]. The syntax of formulae in $\mathsf{S5}_{\mathcal{ALCQI}}$ is built from disjoint countably infinite sets $\mathsf{N_C}$ and $\mathsf{N_R}$ of primitive *concept names* and *role names*. We assume that $\mathsf{N_R}$ is partitioned into two countably infinite sets $\mathsf{N_{glo}}$ and $\mathsf{N_{loc}}$ of *global role names* and *local role names*. The set ROL of *roles* is defined as $\{r, r^-, \Diamond r, \Diamond r^-, \Box r, \Box r^-\}$, with $r \in \mathsf{N_R}$. The set of concepts CON is defined inductively: $\mathsf{N_C} \subseteq \mathsf{CON}$; if $C, D \in \mathsf{CON}$, $r \in \mathsf{ROL}$, and $n \in \mathbb{N}$, then the following are also in CON: $\neg C$, $C \sqcap D$, $(\geqslant n\ r\ C)$, and $\Diamond C$. A *TBox* is a finite set of *general concept inclusions (GCIs)* $C \sqsubseteq D$ with $C, D \in \mathsf{CON}$.

The concept constructors $C \sqcup D$, $\exists r.C$, $\forall r.C$, $(\leqslant n\ r\ C)$, $(= n\ r\ C)$, $\Box C$, $\top$, and $\bot$ are defined as abbreviations in the usual way. Concerning roles, note that we allow only single applications of boxes and diamonds, while inverse is applicable only to role names. It is easily seen that any role obtained by nesting modal operators and inverse in an arbitrary way can be converted into an equivalent role in this restricted form: multiple temporal operators are absorbed and inverse commutes over temporal operators.

An $\mathsf{S5}_{\mathcal{ALCQI}}$-*interpretation* $\mathfrak{I}$ is a pair $(W, \mathcal{I})$ with W a non-empty set of *worlds* and $\mathcal{I}$ a function assigning to each $w \in W$ an $\mathcal{ALCQI}$-*interpretation* $\mathcal{I}(w) = (\Delta, \cdot^{\mathcal{I},w})$, where the *domain* Δ is a non-empty set and $\cdot^{\mathcal{I},w}$ is a function mapping each $A \in \mathsf{N_C}$ to a subset $A^{\mathcal{I},w} \subseteq \Delta$ and each $r \in \mathsf{N_R}$ to a relation $r^{\mathcal{I},w} \subseteq \Delta \times \Delta$, such that if $r \in \mathsf{N_{glo}}$, then $r^{\mathcal{I},w} = r^{\mathcal{I},v}$ for all $w, v \in W$. We extend the mapping $\cdot^{\mathcal{I},w}$ to complex roles and concepts as shown below:

$$
\begin{aligned}
(r^-)^{\mathcal{I},w} &:= \{(y,x) \in \Delta \times \Delta \mid (x,y) \in r^{\mathcal{I},w}\} \\
(\Diamond r)^{\mathcal{I},w} &:= \{(x,y) \in \Delta \times \Delta \mid \exists v \in W : (x,y) \in r^{\mathcal{I},v}\} \\
(\Box r)^{\mathcal{I},w} &:= \{(x,y) \in \Delta \times \Delta \mid \forall v \in W : (x,y) \in r^{\mathcal{I},v}\} \\
(\neg C)^{\mathcal{I},w} &:= \Delta \setminus C^{\mathcal{I},w} \\
(C \sqcap D)^{\mathcal{I},w} &:= C^{\mathcal{I},w} \cap D^{\mathcal{I},w} \\
(\geqslant n\ r\ C)^{\mathcal{I},w} &:= \{x \in \Delta \mid \sharp\{y \in \Delta \mid (x,y) \in r^{\mathcal{I},w} \text{ and } y \in C^{\mathcal{I},w}\} \geq n\} \\
(\Diamond C)^{\mathcal{I},w} &:= \{x \in \Delta \mid \exists v \in W : x \in C^{\mathcal{I},v}\}
\end{aligned}
$$

An $\mathsf{S5}_{\mathcal{ALCQI}}$-interpretation $\mathfrak{I} = (W, \mathcal{I})$ is a *model* of a TBox $\mathcal{T}$ iff it satisfies $C^{\mathcal{I},w} \subseteq D^{\mathcal{I},w}$ for all $C \sqsubseteq D \in \mathcal{T}$ and $w \in W$. It is a *model* of a concept C if $C^{\mathcal{I},w} \neq \emptyset$ for some $w \in W$.

3 The $\mathcal{ER}_{VT}$ Conceptual Language

In this section, the temporal model $\mathcal{ER}_{VT}$ [3] is briefly introduced. We concentrate on a fragment $\mathcal{ER}_{VT}^{-}$ that only allows for timestamping as that is the main focus of this paper.

An $\mathcal{ER}_{VT}^{-}$ schema is a tuple

$$\Sigma = (\mathcal{L}, \textsc{rel}, \textsc{att}, \textsc{card}, \textsc{isa}, \textsc{disj}, \textsc{cover}, \textsc{key}, \textsc{s}, \textsc{t}),$$

in which $\mathcal{L}$ stands for a finite alphabet partitioned into the sets $\mathcal{E}$ (*entity* symbols), $\mathcal{A}$ (*attribute* symbols), $\mathcal{R}$ (*relationship* symbols), $\mathcal{U}$ (*role* symbols), and $\mathcal{D}$ (*domain* symbols). $\mathcal{E}$ is furthermore partitioned into a set $\mathcal{E}^S$ of *snapshot entities* (the S-*marked* entities in Figure 1)[1]; a set $\mathcal{E}^M$ of M*ixed entities* (the *unmarked* entities in Figure 1); and a set $\mathcal{E}^T$ of *temporary entities* (the T-*marked* entities in Figure 1). A similar partition applies to the set $\mathcal{R}$, too. $\textsc{att}$ is a function that maps an entity symbol in $\mathcal{E}$ to an $\mathcal{A}$-labeled tuple over $\mathcal{D}$, $\textsc{att}(E) = \langle A_1 : D_1, \ldots, A_h : D_h \rangle$ (e.g., $\textsc{att}(\texttt{Project}) = \langle \texttt{ProjectCode} : \texttt{String} \rangle$). $\textsc{rel}$ is a function that maps a relationship symbol in $\mathcal{R}$ to an $\mathcal{U}$-labeled tuple over $\mathcal{E}$, $\textsc{rel}(R) = \langle U_1 : E_1, \ldots, U_k : E_k \rangle$, and k is the *arity* of R (e.g., $\textsc{rel}(\texttt{Manages}) = \langle \texttt{man} : \texttt{TopManager}, \texttt{prj} : \texttt{Project} \rangle$). $\textsc{card}$ is a function $\mathcal{E} \times \mathcal{R} \times \mathcal{U} \mapsto \mathbb{N} \times (\mathbb{N} \cup \{\infty\})$ denoting cardinality constraints. We denote $\textsc{cmin}(E, R, U)$ and $\textsc{cmax}(E, R, U)$ the first and second component of $\textsc{card}$ (e.g., $\textsc{card}(\texttt{TopManager}, \texttt{Manages}, \texttt{man}) = (1, 1)$). $\textsc{isa}$ is a binary relationship $\textsc{isa} \subseteq (\mathcal{E} \times \mathcal{E}) \cup (\mathcal{R} \times \mathcal{R})$. $\textsc{isa}$ between relationships is restricted to relationships with the same arity ($\texttt{Manager}\ \textsc{isa}\ \texttt{Employee}$ in Figure 1). $\textsc{disj}, \textsc{cover}$ are binary relations over $2^{\mathcal{E}} \times \mathcal{E}$, describing disjointness and covering partitions, respectively (e.g., $\texttt{Department}$ and $\texttt{InterestGroup}$ are both disjoint and they cover $\texttt{OrganizationalUnit}$). $\textsc{key}$ is a function that maps a class symbol in $\mathcal{C}$ to its key attribute, $\textsc{key}(E) = A$. Keys are visualized as underlined attributes. $\textsc{s}, \textsc{t}$ are binary relations over $\mathcal{E} \times \mathcal{A}$ containing, respectively, the snapshot and temporary attributes of an entity (see S,T marked attributes in Figure 1).

The model-theoretic semantics associated with the $\mathcal{ER}_{VT}^{-}$ modeling language adopts the *snapshot* representation of abstract temporal databases and temporal conceptual models [10]. Following this paradigm, given a set $\mathcal{T}$ of time points (or chronons), a temporal database can be regarded as a mapping from time points in $\mathcal{T}$ to standard relational databases, with the same interpretation of constants and the same domain in time.

[1] We adopt an EER graphical style where entities are in boxes and relationships inside diamonds, $\textsc{isa}$ are directed lines, generalized hierarchies could be disjoint (circle with a 'd' inside) or covering (double directed lines).

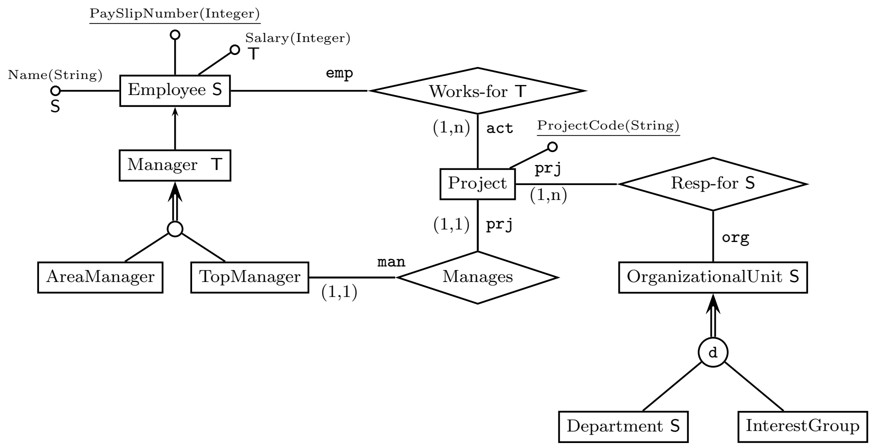

Fig. 1. An $\mathcal{ER}_{VT}^{-}$ diagram

Definition 1 **($\mathcal{ER}_{VT}^{-}$ Semantics).** Let Σ be an $\mathcal{ER}_{VT}^{-}$ schema. $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}} \cup \Delta_D^{\mathcal{B}}, \cdot^{\mathcal{B},t})$ is a *temporal database state* for the schema Σ, where

- $\Delta^{\mathcal{B}}$ is a nonempty set disjoint from $\Delta_D^{\mathcal{B}}$.
- $\Delta_D^{\mathcal{B}} = \bigcup_{D_i \in \mathcal{D}} \Delta_{D_i}^{\mathcal{B}}$ is the set of basic domain values used in the schema Σ such that $\Delta_{D_i}^{\mathcal{B}} \cap \Delta_{D_j}^{\mathcal{B}} = \emptyset$ for $i \neq j$—we call $\Delta_{D_i}^{\mathcal{B}}$ *active domain.*
- $\cdot^{\mathcal{B},t}$ is a function that for each $t \in \mathcal{T}$ maps
  - Every domain symbol $D_i \in \mathcal{D}$ to the corresponding active domain $D_i^{\mathcal{B},t} = \Delta_{D_i}^{\mathcal{B}}$—then, $D_i^{\mathcal{B},t}$ does not depend on the time t of evaluation.
  - Every entity $E \in \mathcal{E}$ to a set $E^{\mathcal{B},t} \subseteq \Delta^{\mathcal{B}}$.
  - Every relationship $R \in \mathcal{R}$ to a set $R^{\mathcal{B},t}$ of $\mathcal{U}$-labeled tuples over $\Delta^{\mathcal{B}}$.
  - Every attribute $A \in \mathcal{A}$ to a set $A^{\mathcal{B},t} \subseteq \Delta^{\mathcal{B}} \times \Delta_D^{\mathcal{B}}$.

$\mathcal{B}$ is a *legal temporal database state* if it satisfies all of the integrity constraints expressed in the schema (we mention here just the timestamped constructs, see [3] for more details):

- For each snapshot entity, $E \in \mathcal{E}^S$, then, $e \in E^{\mathcal{B},t} \to \forall t' \in \mathcal{T}. e \in E^{\mathcal{B},t'}$
- For each temporary entity, $E \in \mathcal{E}^T$, then, $e \in E^{\mathcal{B},t} \to \exists t' \neq t. e \notin E^{\mathcal{B},t'}$
- For each snapshot relationship, $R \in \mathcal{R}^S$, then, $r \in R^{\mathcal{B},t} \to \forall t' \in \mathcal{T}. r \in R^{\mathcal{B},t'}$
- For each temporary relationship, $R \in \mathcal{R}^T$, then, $r \in R^{\mathcal{B},t} \to \exists t' \neq t. r \notin R^{\mathcal{B},t'}$
- For each entity, $E \in \mathcal{E}$, if $\text{ATT}(E) = \langle A_1 : D_1, \ldots, A_h : D_h \rangle$, and $\langle E, A_i \rangle \in \text{S}$, then, $(e \in E^{\mathcal{B},t} \wedge \langle e, a_i \rangle \in A_i^{\mathcal{B},t}) \to \forall t' \in \mathcal{T}. \langle e, a_i \rangle \in A_i^{\mathcal{B},t'}$
- For each entity, $E \in \mathcal{E}$, if $\text{ATT}(E) = \langle A_1 : D_1, \ldots, A_h : D_h \rangle$, and $\langle E, A_i \rangle \in \text{T}$, then, $(e \in E^{\mathcal{B},t} \wedge \langle e, a_i \rangle \in A_i^{\mathcal{B},t}) \to \exists t' \neq t. \langle e, a_i \rangle \notin A_i^{\mathcal{B},t'}$

Reasoning tasks over conceptual schemas include verifying whether an entity, a relationship, and/or a schema are *consistent*, and checking whether an entity (relationship) *subsumes* another entity (relationship, respectively). The model-theoretic semantics associated with a conceptual schema, Σ, allows us to define formally the following reasoning tasks:

Schema consistency: Σ is *consistent* if there exists a legal database state $\mathcal{B}$ for Σ such that $E^{\mathcal{B},t} \neq \emptyset$, for some entity $E \in \mathcal{E}$, and for some $t \in \mathcal{T}$.

Entity consistency: An entity $E \in \mathcal{E}$ is *consistent w.r.t. a schema* Σ if there exists a legal database state $\mathcal{B}$ for Σ such that $E^{\mathcal{B},t} \neq \emptyset$, for some $t \in \mathcal{T}$.

Relationship consistency: A relationship $R \in \mathcal{R}$ is *consistent w.r.t. a schema* Σ if there exists a legal database state $\mathcal{B}$ for Σ such that $R^{\mathcal{B},t} \neq \emptyset$, for some $t \in \mathcal{T}$.

Entity subsumption: An entity $E_1 \in \mathcal{E}$ *subsumes* an entity $E_2 \in \mathcal{E}$ w.r.t. a schema Σ if $E_2^{\mathcal{B},t} \subseteq E_1^{\mathcal{B},t}$, for every legal database state $\mathcal{B}$ for Σ and for every $t \in \mathcal{T}$.

The reasoning tasks of Schema/Entity/Relationship consistency and Entity subsumption are reducible to each other (see [2]).

4 Encoding in $\mathsf{S5}_{\mathcal{ALCQI}}$

We now show how the temporal description logic $\mathsf{S5}_{\mathcal{ALCQI}}$ can capture temporal conceptual schemas expressed in $\mathcal{ER}^-_{VT}$. This characterization allows us to support reasoning on temporal conceptual models by using the reasoning services of $\mathsf{S5}_{\mathcal{ALCQI}}$. The correspondence is based on a mapping function Φ—extending the one introduced in [9] for non-temporal EER models—from $\mathcal{ER}^-_{VT}$ schemas to $\mathsf{S5}_{\mathcal{ALCQI}}$ knowledge bases.

Informally, the encoding works as follows. Both entity and relationship symbols in the $\mathcal{ER}^-_{VT}$ diagram are mapped into $\mathsf{S5}_{\mathcal{ALCQI}}$ concept names (i.e., relationships are reified). Domain symbols are mapped into additional concept names, pairwise disjoint. Both attributes of entities and roles of relationships are mapped to role names in $\mathsf{S5}_{\mathcal{ALCQI}}$. ISA links between entities or between relationships are mapped using GCI's. Generalized hierarchies with disjointness and covering constraints can be captured using Boolean connectives. Cardinality constraints are mapped using number restrictions in $\mathsf{S5}_{\mathcal{ALCQI}}$. Timestamping constraints are mapped using the $\mathsf{S5}$ modality. Thus, the worlds of the $\mathsf{S5}_{\mathcal{ALCQI}}$ interpretation are used as time points.

Definition 2 (Mapping $\mathcal{ER}^-_{VT}$ into $\mathsf{S5}_{\mathcal{ALCQI}}$). *Let Σ be an $\mathcal{ER}^-_{VT}$ schema. The $\mathsf{S5}_{\mathcal{ALCQI}}$ knowledge base $\Phi(\Sigma) = (\mathsf{N_C}, \mathsf{N_R}, \Gamma)$ is defined as follows. The set $\mathsf{N_C}$ of concept names is such that: for each symbol $X \in \mathcal{D} \cup \mathcal{E} \cup \mathcal{R}$, then, $\Phi(X) \in \mathsf{N_C}$. The set $\mathsf{N_R} = \mathsf{N_{loc}} \cup \mathsf{N_{glo}}$ of atomic roles is such that: for each attribute $A \in \mathcal{A}$, then, $\Phi(A) \in \mathsf{N_{loc}}$; for each role symbol $U \in \mathcal{U}$, then, $\Phi(U) \in \mathsf{N_{glo}}$. Φ is functional over $\mathcal{L}$. The set Γ contains the following $\mathsf{S5}_{\mathcal{ALCQI}}$ GCIs.*

– $\top \sqsubseteq (\leq 1\, \Phi(A)\, \top)$, *for each $A \in \mathcal{A}$ (attributes are single-valued)*
– *For each relationship $R \in \mathcal{R}$ such that* $\text{REL}(R) = \langle U_1 : E_1, \ldots, U_k : E_k \rangle$,
  $\Phi(R) \sqsubseteq \exists \Phi(U_1).\Phi(E_1) \sqcap \ldots \sqcap \exists \Phi(U_k).\Phi(E_k)$*—reification axiom*
  $\top \sqsubseteq (\leq 1\, \Phi(U_i)\, \top)$, *with $i = 1, \ldots, k$ and $\Phi(U_i)$ global roles*

– $\Phi(E) \sqsubseteq \Box\Phi(E)$, *for each snapshot entity* $E \in \mathcal{E}^S$
– $\Phi(R) \sqsubseteq \Box\Phi(R)$, *for each snapshot relationship* $R \in \mathcal{R}^S$
– $\Phi(E) \sqsubseteq \exists\Box\Phi(A_i).\top$, *for each snapshot attribute* A_i *with* $\langle E, A_i\rangle \in \textsc{s}$
– $\Phi(E) \sqsubseteq \Diamond\neg\Phi(E)$, *for each temporary entity* $E \in \mathcal{E}^T$
– $\Phi(R) \sqsubseteq \Diamond\neg\Phi(R)$, *for each temporary relationship* $R \in \mathcal{R}^T$
– $\Phi(E) \sqsubseteq \forall\Box\Phi(A_i).\bot$, *for each temporary attribute* A_i *with* $\langle E, A_i\rangle \in \textsc{t}$

For the mapping for the remaining atemporal constructors see [3].

The correctness of the mapping can be shown by establishing a precise correspondence between legal database states of $\mathcal{ER}^-_{VT}$ schemas and models of the corresponding $\mathsf{S5}_{\mathcal{ALCQI}}$ TBox.

Proposition 1 (Correctness of the encoding). *Let Σ be an $\mathcal{ER}^-_{VT}$ schema. Then, Σ admits a legal database state if and only if the corresponding $\mathsf{S5}_{\mathcal{ALCQI}}$ knowledge base $\Phi(\Sigma)$ has a model.*

Proof.(Sketch)
$'' \Rightarrow''$ Let $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_D, \cdot^{\mathcal{B},t})$ be a legal temporal database state for Σ. To define an interpretation $(\mathcal{T},\mathcal{I})$ of $\Phi(\Sigma)$ we introduce a new set, $\Delta_{\mathcal{R}}$, disjoint from $\Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_D$ and a bijective mapping, $\phi_{\mathcal{R}} : \bigcup_{R\in\mathcal{R},t\in\mathcal{T}} R^{\mathcal{B},t} \mapsto \Delta_{\mathcal{R}}$. Then, for each $t \in \mathcal{T}$ the interpretation $\mathcal{I}(t) = (\Delta, \cdot^{\mathcal{I},t})$ is such that:

1. $\Delta = \Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_D \cup \Delta_{\mathcal{R}}$;
2. For for each symbol $X \in \mathcal{E} \cup \mathcal{A} \cup \mathcal{D}$, then $\Phi(X)^{\mathcal{I},t} = X^{\mathcal{B},t}$;
3. For each relationship $R \in \mathcal{R}$, then $\Phi(R)^{\mathcal{I},t} = \phi_{\mathcal{R}}(R^{\mathcal{B},t})$;
4. For each $U_i \in \mathcal{U}$, if $\textsc{rel}(R) = \langle U_1 : E_1, \ldots, U_i : E_i, \ldots, U_k : E_k\rangle$ for some $R \in \mathcal{R}$, then: $\Phi(U_i)^{\mathcal{I},t} = \{(r, e_i) \in \Delta_{\mathcal{R}} \times \Delta^{\mathcal{B}} \mid r = \phi_{\mathcal{R}}(e_1, \ldots, e_i, \ldots, e_k)$ and $\exists t' \in \mathcal{T} : (e_1, \ldots, e_i, \ldots, e_k) \in R^{\mathcal{B},t'}\}$.

It is now sufficient to show that $(\mathcal{T},\mathcal{I})$ satisfies all the GCI's of Definition 2.

$'' \Leftarrow''$ In [5] (theorem 7) we proved that if an $\mathsf{S5}_{\mathcal{ALCQI}}$ KB has a model than it has a tree shaped model. We now prove that for each tree model of $\Phi(\Sigma)$, $(\mathcal{T},\mathcal{I})$, with $\mathcal{I}(t) = (\Delta, \cdot^{\mathcal{I},t})$, there is a legal temporal database state $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_D, \cdot^{\mathcal{B},t})$ for Σ.
We first define the set of active domains in Σ, $\Delta^{\mathcal{B}}_D$, starting from $\Phi(D_i)^{\mathcal{I},t}$. Let $\mathcal{B}_\Delta$ be a one-to-one partial mapping, then, $\Delta^{\mathcal{B}}_{D_i} \equiv \mathcal{B}_\Delta(\Phi(D_i)^{\mathcal{I},t})$, for some $t \in \mathcal{T}$, and $\Delta^{\mathcal{B}}_D = \bigcup_{D_i\in\mathcal{D}} \Delta^{\mathcal{B}}_{D_i}$. We can now define the temporal database state $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}} \cup \Delta^{\mathcal{B}}_D, \cdot^{\mathcal{B},t})$ for each $t \in \mathcal{T}$:

1. $\Delta^{\mathcal{B}} = \Delta \setminus \Delta^{\mathcal{B}}_D$
2. For each $E \in \mathcal{E}$: $E^{\mathcal{B},t} = \Phi(E)^{\mathcal{I},t}$;
3. For each n-ary relationship $R \in \mathcal{R}$, if $\textsc{rel}(R) = \langle U_1 : E_1, \ldots, U_k : E_k\rangle$, then: $R^{\mathcal{B},t} = \{\langle U_1 : e_1, \ldots, U_k : e_k\rangle \mid \exists r \in \Phi(R)^{\mathcal{I},t} : (r, e_1) \in \Phi(U_1)^{\mathcal{I},t} \wedge \ldots \wedge (r, e_k) \in \Phi(U_k)^{\mathcal{I},t}\}$;
4. For each $D_i \in \mathcal{D}$: $D_i^{\mathcal{B},t} = \Delta^{\mathcal{B}}_{D_i}$;

5. For each $A \in \mathcal{A}$: $\langle d_1, d_2 \rangle \in A^{\mathcal{I},t}$ iff $\langle d_1, \mathcal{B}_\Delta(d_2) \rangle \in A^{\mathcal{B},t}$.

Note that, since $(\mathcal{T}, \mathcal{I})$ is a tree model, and for each $U \in \mathcal{U}$ its mapping is a global and functional role, then there is bijective mapping between tuples $\langle U_1 : e_1, \ldots, U_k : e_k \rangle \in R^{\mathcal{B},t}$ and objects $r \in \Phi(R)^{\mathcal{I},t} : (r, e_1) \in \Phi(U_1)^{\mathcal{I},t} \wedge \ldots \wedge (r, e_k) \in \Phi(U_k)^{\mathcal{I},t}$. We can now prove that $\mathcal{B}$ is a legal temporal database state by showing that $\mathcal{B}$ satisfies all the integrity constraints of Definition 1. ❑

Since checking KB satisfiability is 2-ExpTime-complete [5], the complexity result follows from the above Proposition immediately.

Proposition 2. *Checking satisfiability of $\mathcal{ER}^-_{VT}$ schemas is in 2-ExpTime.*

5 Reasoning with $\mathcal{ER}^-_{VT}$ is 2-ExpTime-hard

This section shows that reasoning in $\mathcal{ER}^-_{VT}$ is hard for 2-ExpTime. Since [5] has shown that already $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$, a logic denoting the modal product $\mathsf{S5} \times \mathcal{ALC}$ where roles are always global, is 2-ExpTime-hard in the following we show a reduction from $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ GCI's into $\mathcal{ER}^-_{VT}$ schemas.

First we show that we can restrict GCI's to *primitive inclusions* of the form $A \sqsubseteq C$ where A is an atomic concept and C is a concept that conforms to the following grammar,

$$C \rightarrow A \mid \neg A \mid A_1 \sqcup A_2 \mid \forall R.A \mid \exists R.A \mid \Box A \mid \Diamond A,$$

allowing on the right-hand side of subsumption constraints only concept expressions built with at most one constructor, with the exception of conjunction that is disallowed altogether.

Lemma 1. *Concept satisfiability w.r.t. an $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ KB can be linearly reduced to atomic concept satisfiability w.r.t. a primitive $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ KB.*

Proof. *(Sketch)* The proof extends to the $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ case a well known result proved in [8]. Let Γ be an $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ KB and A_Γ be an atomic concept such that:

$$\Gamma_1 = \{A_\Gamma \sqsubseteq \bigsqcap_{C_1 \sqsubseteq C_2 \in \Gamma} (\neg C_1 \sqcup C_2) \sqcap \bigsqcap_{P \in \mathsf{N_R}} (\forall P.A_\Gamma \sqcap \forall P^-.A_\Gamma), A_\Gamma \sqsubseteq \Box A_\Gamma\}$$

Then, a concept C is satisfiable w.r.t. Γ iff the atomic concept A_C is satisfiable w.r.t. $\Gamma_1 \cup \{A_C \sqsubseteq A_\Gamma \sqcap C\}$.

The proof is concluded by converting the right-hand sides of constraints in $\Gamma_1 \cup \{A_C \sqsubseteq A_\Gamma \sqcap C\}$ to NNF and then exhaustively applying the following rules:

1. $A \sqsubseteq C_1 \sqcap C_2$ into $A \sqsubseteq C_1$ and $A \sqsubseteq C_2$;
2. $A \sqsubseteq C_1 \sqcup C_2$ into $A \sqsubseteq A_1 \sqcup A_2$ and $A_1 \sqsubseteq C_1$ and $A_2 \sqsubseteq C_2$;
3. $A \sqsubseteq \exists R.C$ into $A \sqsubseteq \exists R.A_1$ and $A_1 \sqsubseteq C$;
4. $A \sqsubseteq \forall R.C$ into $A \sqsubseteq \forall R.A_1$ and $A_1 \sqsubseteq C$;
5. $A \sqsubseteq \Box C$ into $A \sqsubseteq \Box A_1$ and $A_1 \sqsubseteq C$;
6. $A \sqsubseteq \Diamond C$ into $A \sqsubseteq \Diamond A_1$ and $A_1 \sqsubseteq C$.

❑

Fig. 2. Encoding of axioms: (a) $A \sqsubseteq \neg B$; (b) $A \sqsubseteq B_1 \sqcup B_2$.

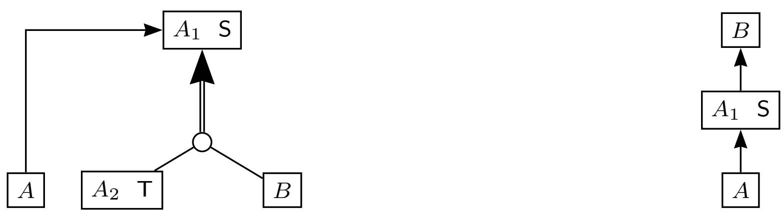

Fig. 3. Encoding of axioms: (a) $A \sqsubseteq \Diamond B$; (b) $A \sqsubseteq \Box B$.

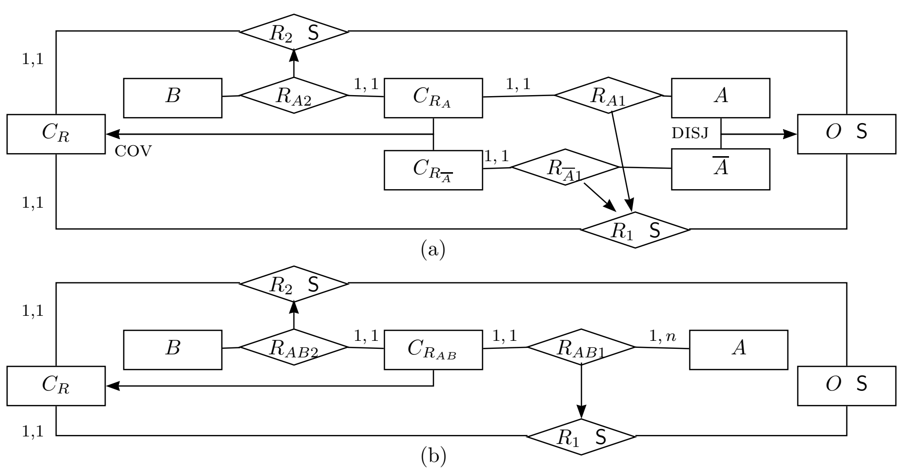

Fig. 4. Encoding of axioms: (a) $A \sqsubseteq \forall R.B$; (b) $A \sqsubseteq \exists R.B$.

Now we can reduce satisfiability of atomic concepts w.r.t. a primitive $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ theory $\mathbf{KB}$ to entity consistency in an $\mathcal{ER}^{-}_{VT}$ diagram $\Sigma(\mathbf{KB})$. The mapping is based on a temporal extension of a reduction designed for capturing $\mathcal{ALC}$ axioms in EER [7].

For each atomic concept in $\mathbf{KB}$ we introduce an entity in $\Sigma(\mathbf{KB})$. To simulate the universal concept, $\top$, we introduce a snapshot class, O, that generalizes all the entities in $\Sigma(\mathbf{KB})$. Since the hardness proof in [5] uses solely global roles we map GCI's of the form $A \sqsubseteq \exists R.B$ and $A \sqsubseteq \forall R.B$ assuming that $R \in \mathsf{N}_{\mathsf{glo}}$. Furthermore, roles are mapped with the help of reification [2]. The primitive inclusions in $\mathbf{KB}$ that were obtained with the help of Lemma 1 are mapped into $\mathcal{ER}^{-}_{VT}$ diagrams as follows:

1. $A \sqsubseteq B$ by A ISA B;
2. $A \sqsubseteq \neg B$ by diagram in Figure 2(a);
3. $A \sqsubseteq B_1 \sqcup B_2$ by diagram in Figure 2(b);
4. $A \sqsubseteq \Diamond B$ by diagram in Figure 3(a);
5. $A \sqsubseteq \Box B$ by diagram in Figure 3(b);
6. $A \sqsubseteq \forall R.B$ by diagram in Figure 4(a);
7. $A \sqsubseteq \exists R.B$ by diagram in Figure 4(b).

Proposition 3. *An atomic concepts A is satisfiable w.r.t. a primitive* $\mathbf{KB}$ *in* $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ *iff the entity A is consistent w.r.t. the $\mathcal{ER}^{-}_{VT}$ schema $\Sigma(\mathbf{KB})$.*

Proof. ($\Leftarrow$) Let $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}}, \cdot^{\mathcal{B},t})$ be a legal database for $\Sigma(\mathbf{KB})$ such that $E^{\mathcal{B},t} \neq \emptyset$ for some $t \in \mathcal{T}$. We construct a model $\mathcal{M} = (\mathcal{T}, \mathcal{I})$ of $\mathbf{KB}$, where $\mathcal{I}(t) = (\Delta, \cdot^{\mathcal{I},t})$, by taking $\Delta = \Delta^{\mathcal{B}}$, $A^{\mathcal{I},t} = A^{\mathcal{B},t}$, for all concept names A in $\mathcal{K}$, and $R^{\mathcal{I},t} = (R_1^{-} \circ R_2)^{\mathcal{B},t}$, for all role names R in $\mathbf{KB}$, where $\circ$ denotes the binary relation composition. Clearly, $E^{\mathcal{I},t} \neq \emptyset$. Let us show that $\mathcal{M}$ is indeed a model of $\mathbf{KB}$. The cases of axioms of the form $A \sqsubseteq B$, $A \sqsubseteq \neg B$ and $A \sqsubseteq B_1 \sqcup B_2$ are treated as in [7]. Let us consider the remaining cases.

Case $A \sqsubseteq \forall R.B$—with $R \in \mathsf{N}_{\mathsf{glo}}$. Let $o \in A^{\mathcal{I},t}$ and $o' \in \Delta$ with $(o, o') \in R^{\mathcal{I},t}$. Since $R^{\mathcal{I},t} = (R_1^{-} \circ R_2)^{\mathcal{B},t}$ and R_1, R_2 are snapshot relationships, then, R is indeed interpreted as a global role. We show now that $o \in (\forall R.B)^{\mathcal{I},t}$. Since $R^{\mathcal{I},t} = (R_1^{-} \circ R_2)^{\mathcal{B},t}$, there is $o'' \in \Delta^{\mathcal{B}}$ with $(o, o'') \in (R_1^{-})^{\mathcal{B},t}$ and $(o'', o') \in R_2^{\mathcal{B},t}$. Then $o'' \in C_R^{\mathcal{B},t}$ and, by the covering constraint, $o'' \in C_{R_A}^{\mathcal{B},t} \cup C_{R_{\overline{A}}}^{\mathcal{B},t}$. We claim that $o'' \in C_{R_A}^{\mathcal{B},t}$. Indeed, suppose otherwise; then $o'' \in C_{R_{\overline{A}}}^{\mathcal{B},t}$, and so there is a unique $a \in \Delta^{\mathcal{B}}$ such that $(o'', a) \in R_{\overline{A}1}^{\mathcal{B},t}$ and $a \in \overline{A}^{\mathcal{B},t}$; it follows from $R_{\overline{A}1}^{\mathcal{B},t} \subseteq R_1^{\mathcal{B},t}$ and the cardinality constraint on C_R that $a = o$, contrary to $o \in A^{\mathcal{B},t} = A^{\mathcal{I},t}$ and the disjointness of A and $\overline{A}$. Since $o'' \in C_{R_A}^{\mathcal{B},t}$, there is a unique $b \in \Delta^{\mathcal{B}}$ with $(o'', b) \in R_{A2}^{\mathcal{B},t}$ and $b \in B^{\mathcal{B},t}$. From $R_{A2}^{\mathcal{B},t} \subseteq R_2^{\mathcal{B},t}$ and the cardinality constraint on C_R, we conclude that $b = o'$. Thus, $o' \in B^{\mathcal{B},t} = B^{\mathcal{I},t}$ and $o \in (\forall R.B)^{\mathcal{I},t}$.

Case $A \sqsubseteq \exists R.B$—with $R \in \mathsf{N}_{\mathsf{glo}}$. Let $o \in A^{\mathcal{I},t}$. Since $o \in A^{\mathcal{I},t} = A^{\mathcal{B},t}$, there is $o' \in \Delta^{\mathcal{B}}$ with $(o, o') \in (R_{AB1}^{-})^{\mathcal{B},t}$ and $o' \in C_{R_{AB}}^{\mathcal{B},t}$. As $R_{AB1}^{\mathcal{B},t} \subseteq R_1^{\mathcal{B},t}$, we have $(o, o') \in (R_1^{-})^{\mathcal{B},t}$, and, as $o' \in C_{R_{AB}}^{\mathcal{B},t}$, there is $o'' \in \Delta^{\mathcal{B}}$ such that

$(o', o'') \in R_{AB2}^{\mathcal{B},t} \subseteq R_2^{\mathcal{B},t}$ and $o'' \in B^{\mathcal{B},t} = B^{\mathcal{I},t}$. Thus, as $R^{\mathcal{I},t} = (R_1^- \circ R_2)^{\mathcal{B},t}$, we obtain $(o, o'') \in R^{\mathcal{I},t}$ and $o'' \in B^{\mathcal{I},t}$, i.e., $o \in (\exists R.B)^{\mathcal{I},t}$.

Case $A \sqsubseteq \Diamond B$. Let $o \in A^{\mathcal{I},t} = A^{\mathcal{B},t}$. Then, for all $t \in \mathcal{T}$, $o \in A_1^{\mathcal{B},t}$. Due to the covering constraint either $o \in B^{\mathcal{B},t}$—thus the thesis is proved—or $o \in A_2^{\mathcal{B},t}$. In this second case, since A_2 is a temporary entity there is a time t' such that $o \notin A_2^{\mathcal{B},t'}$ and thus $o \in B^{\mathcal{B},t'} = B^{\mathcal{I},t'}$.

Case $A \sqsubseteq \Box B$. Let $o \in A^{\mathcal{I},t} = A^{\mathcal{B},t}$. Then, $o \in A_1^{\mathcal{B},t}$ and since A_1 is a snapshot entity, then, $o \in A_1^{\mathcal{B},t}$ for all $t \in \mathcal{T}$. Then, $o \in B^{\mathcal{B},t} = B^{\mathcal{I},t}$ for all $t \in \mathcal{T}$.

($\Rightarrow$) Let $\mathcal{M} = (\mathcal{T}, \mathcal{I})$, where $\mathcal{I}(t) = (\Delta, \cdot^{\mathcal{I},t})$, be an $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ model of $\mathbf{KB}$ such that $E^{\mathcal{I},t} \neq \emptyset$ for some $t \in \mathcal{T}$. We construct a legal database state $\mathcal{B} = (\mathcal{T}, \Delta^{\mathcal{B}}, \cdot^{\mathcal{B},t})$ for $\Sigma(\mathbf{KB})$ such that $E^{\mathcal{B},t} \neq \emptyset$. Let $\Delta^{\mathcal{B}} = \Delta \cup \Gamma$, where Γ is the disjoint union of the $\Delta_R = \{(o, o') \in \Delta \times \Delta \mid (o, o') \in R^{\mathcal{I},t}, t \in \mathcal{T}\}$, for all $R \in \mathsf{N}_{\mathsf{glo}}$. We set $A^{\mathcal{B},t} = A^{\mathcal{I},t}$ and $\overline{A}^{\mathcal{B},t} = (\neg A)^{\mathcal{I},t}$, for all concept names A, $O^{\mathcal{B},t} = \Delta$ for every $t \in \mathcal{T}$, for the entity O, and $C_R^{\mathcal{B},t} = \Delta_R$ for every $t \in \mathcal{T}$, for all $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ role names R. Next, for every primitive $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ GCI of the form $A \sqsubseteq \forall R.B$, we set

- $C_{R_A}^{\mathcal{B},t} = \{(o, o') \in \Delta_R \mid o \in A^{\mathcal{I},t}\}$, $C_{R_{\overline{A}}}^{\mathcal{B},t} = \{(o, o') \in \Delta_R \mid o \in (\neg A)^{\mathcal{I},t}\}$,
- $R_1^{\mathcal{B},t} = \{((o, o'), o) \in \Delta_R \times \Delta \mid (o, o') \in R^{\mathcal{I},t}\}$,
- $R_2^{\mathcal{B}} = \{((o, o'), o') \in \Delta_R \times \Delta \mid (o, o') \in R^{\mathcal{I},t}\}$,
- $R_{A1}^{\mathcal{B},t} = \{((o, o'), o) \in R_1^{\mathcal{B},t} \mid o \in A^{\mathcal{I},t}\}$, $R_{\overline{A}1}^{\mathcal{B},t} = \{((o, o'), o) \in R_1^{\mathcal{B}} \mid o \in (\neg A)^{\mathcal{I},t}\}$,
- $R_{A2}^{\mathcal{B},t} = \{((o, o'), o') \in R_2^{\mathcal{B}} \mid o \in A^{\mathcal{I},t}\}$,

and, for every primitive $\mathsf{S5}^{\mathsf{glo}}_{\mathcal{ALC}}$ axiom of the form $A \sqsubseteq \exists R.B$, we set

- $C_{R_{AB}}^{\mathcal{B},t} = \{(o, o') \in \Delta_R \mid o \in A^{\mathcal{I},t}$ and $o' \in B^{\mathcal{I},t}\}$,
- $R_1^{\mathcal{B},t} = \{((o, o'), o) \in \Delta_R \times \Delta \mid (o, o') \in R^{\mathcal{I},t}\}$,
- $R_2^{\mathcal{B},t} = \{((o, o'), o') \in \Delta_R \times \Delta \mid (o, o') \in R^{\mathcal{I},t}\}$,
- $R_{AB1}^{\mathcal{B},t} = \{((o, o'), o) \in R_1^{\mathcal{B},t} \mid (o, o') \in C_{R_{AB}}^{\mathcal{B},t}\}$.
- $R_{AB2}^{\mathcal{B},t} = \{((o, o'), o') \in R_2^{\mathcal{B},t} \mid (o, o') \in C_{R_{AB}}^{\mathcal{B},t}\}$.

It is now easy to show that $\mathcal{B}$ is a legal database state for $\Sigma(\mathbf{KB})$ and $E^{\mathcal{B},t} \neq \emptyset$. ❑

A tight complexity bound for reasoning in $\mathcal{ER}_{VT}^-$ is a direct consequence of the above Proposition and of Proposition 2.

Theorem 1. *Reasoning in $\mathcal{ER}_{VT}^-$ is 2-*EXPTIME*-complete.*

6 Conclusion

The paper shows that even very simple temporal extensions of the EER model, such as timestamping, lead to a considerable increase in computational complexity for the underlying reasoning tasks. Future research is needed to design fragments of the EER model for which temporal extensions are computationally more amenable while they still retain sufficient expressiveness.

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