Formal Methods
Lecture V – Part II
CTL Model Checking with Fairness Constraints

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M. Benerecetti, A. Cimatti, M. Fisher, F. Giunchiglia, M. Pistore, M. Roveri, R.Sebastiani.

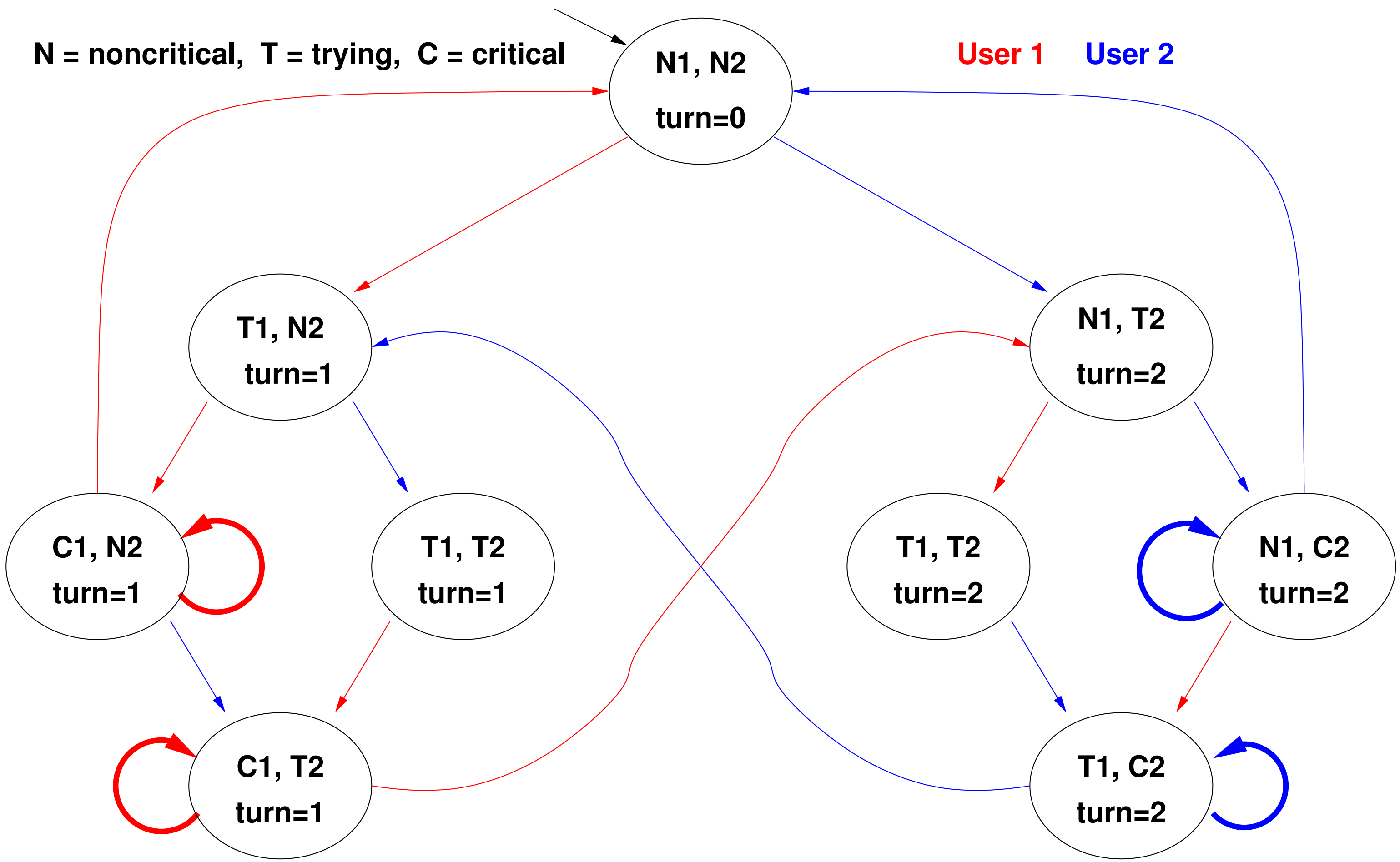
Fair Kripke Models: An Example

- Consider a variant of the mutual exclusion protocol in which one process can stay in the critical section as long as it likes.
- Do the Liveness conditions still hold?

$$\mathcal{M} \models \boxed{\mathrm{P}}\, \Box (T_1 \Rightarrow \boxed{\mathrm{P}}\, \Diamond C_1);$$

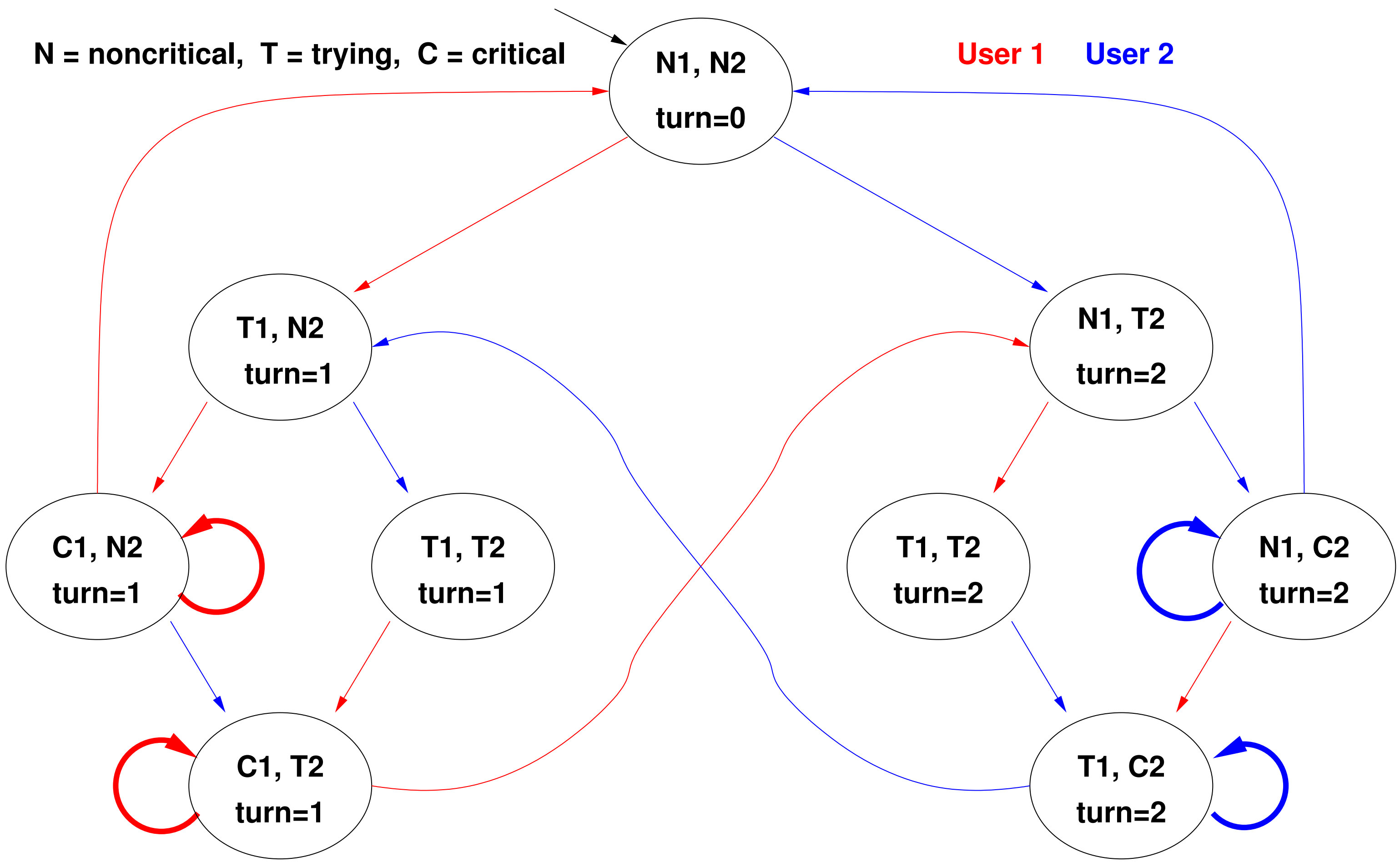
$$\mathcal{M} \models \boxed{\mathrm{P}}\, \Box (T_2 \Rightarrow \boxed{\mathrm{P}}\, \Diamond C_2).$$

Fair Kripke Models: An Example (Cont.)



$\mathcal{M} \models \boxed{\text{P}} \, \Box (T_2 \Rightarrow \boxed{\text{P}} \, \Diamond C_2)$? $\mathcal{M} \models \boxed{\text{P}} \, \Box (T_1 \Rightarrow \boxed{\text{P}} \, \Diamond C_1)$?

Fair Kripke Models: An Example (Cont.)



$\mathcal{M} \models \boxed{\mathrm{P}} \Box (T_2 \Rightarrow \boxed{\mathrm{P}} \Diamond C_2)$? $\mathcal{M} \models \boxed{\mathrm{P}} \Box (T_1 \Rightarrow \boxed{\mathrm{P}} \Diamond C_1)$?

NO: E.g., it can cycle forever in $\{C_1, T_2, turn = 1\}$

Unfair Protocol: one process might never be served!

Fairness Conditions in LTL

Fairness Conditions in LTL. $\Box\Diamond\varphi \Rightarrow \psi$, where ψ is the formula to be verified.

- Using LTL the fairness conditions of the example can be expressed as:

$$\mathcal{M} \models \Box\Diamond\neg C_2 \Rightarrow \Box(T_1 \Rightarrow \Diamond C_1)$$

$$\mathcal{M} \models \Box\Diamond\neg C_1 \Rightarrow \Box(T_2 \Rightarrow \Diamond C_2)$$

Fairness Conditions in CTL

Fairness Conditions in CTL. In CTL fairness constraints cannot be expressed!

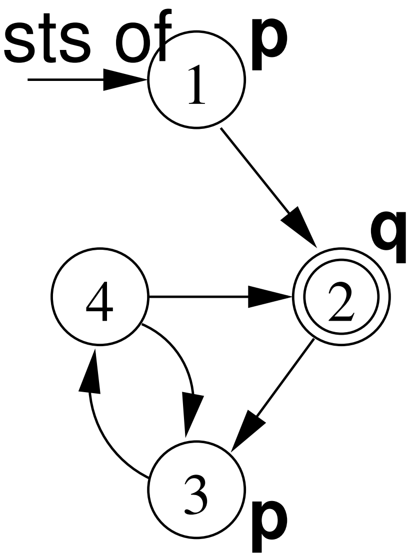
Solution. Impose Fairness Constraints on top of the Kripke Model.

- We call Fair Computation Paths those paths verifying a fairness constraint infinitely often;
- We call Fair Kripke Models those models restricted to fair paths.

Fair Kripke Models

- A Fair Kripke model $\mathcal{M}_F := \langle S, R, I, AP, L, F \rangle$ consists of
  - a set of states S;
  - a set of initial states $I \subseteq S$;
  - a set of transitions $R \subseteq S \times S$;
  - a set of atomic propositions AP;
  - a labeling $L : S \mapsto 2^{AP}$;
  - a set of fairness conditions $F = \{f_1, \ldots, f_n\}$, with $f_i \subseteq S$.

  E.g., $\{\{2\}\} := \{\{s \mid \mathcal{M}, s \models q\}\}$ can be a set of fair conditions of the Kripke model above.



- Fair path π: At least one state for each f_i must occur *infinitely often* in π.

  E.g., every path visiting *infinitely often* state 2 is a fair path.

M.C. with Fair Kripke Models

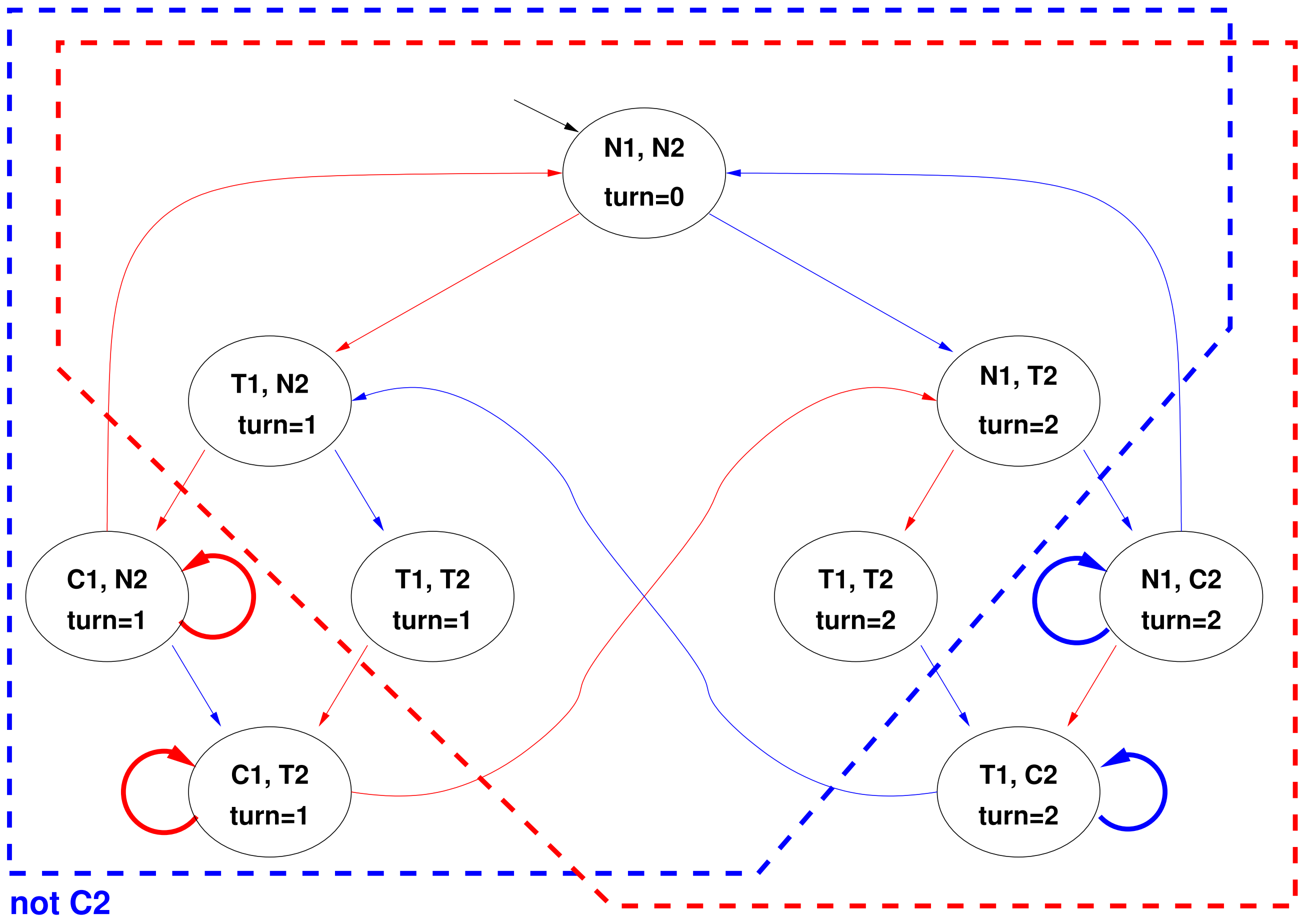
Fair Kripke Models restrict the CTL Model Checking process to fair paths:

> Path quantifiers apply only to fair paths:

- $\mathcal{M}_F, s_i \models \boxed{\mathrm{P}} \ \Box\varphi$ iff for every fair path $\pi = (s_i, s_{i+1}, \ldots), \forall j \geq i. M, s_j \models \varphi$.
- $\mathcal{M}_F, s_i \models \Diamond\!\mathrm{P} \ \Box\varphi$ iff for some fair path $\pi = (s_i, s_{i+1}, \ldots), \forall j \geq i. \mathcal{M}, s_j \models \varphi$.

Fairness Constraints: An Example

$$F := \{\{s \mid \mathcal{M}, s \models \neg C1\}, \{s \mid \mathcal{M}, s \models \neg C2\}\}$$

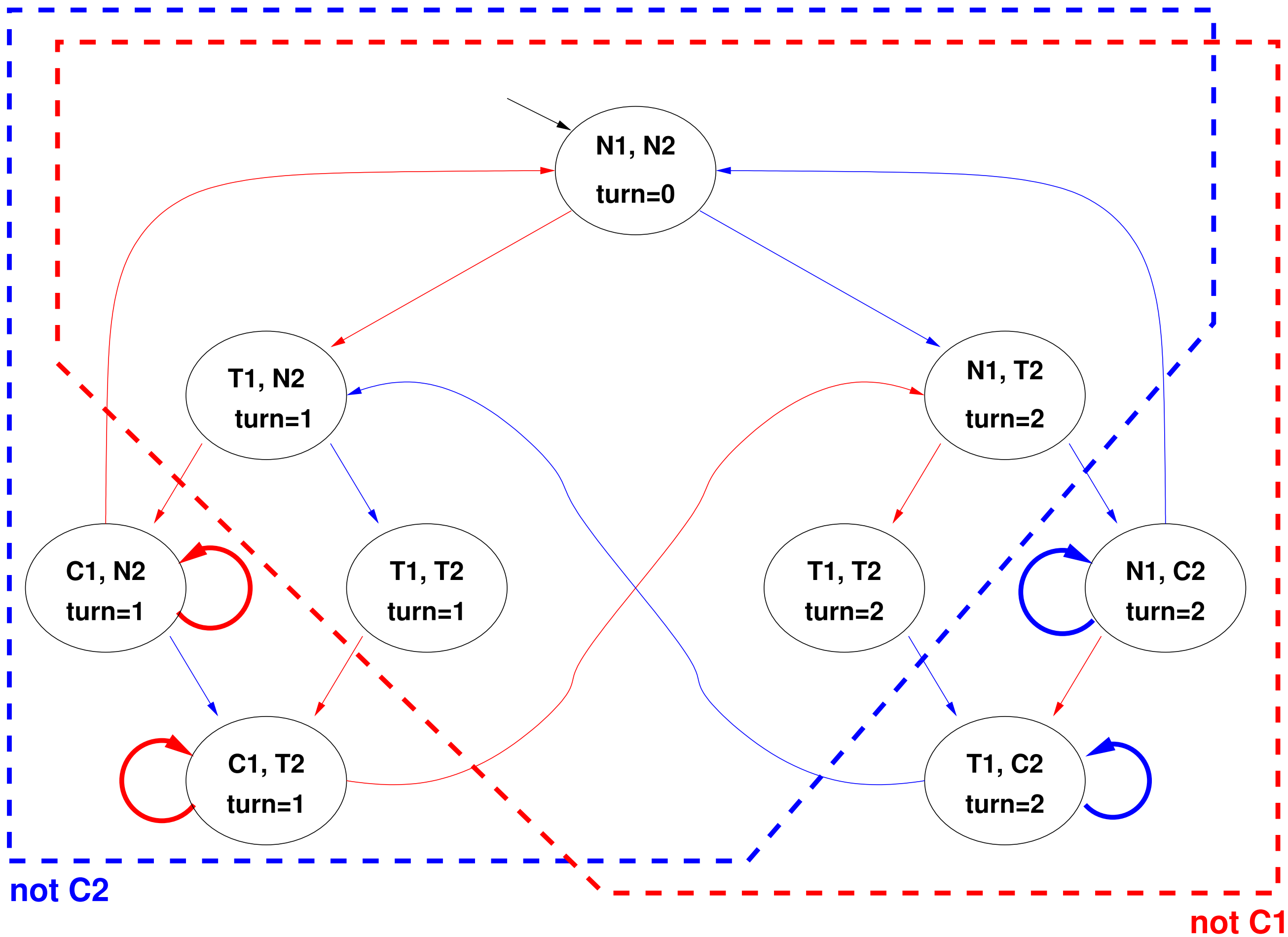


$\mathcal{M}_F \models \boxed{\mathrm{P}} \Box (T_1 \Rightarrow \boxed{\mathrm{P}} \Diamond C_1)$?

$\mathcal{M}_F \models \boxed{\mathrm{P}} \Box (T_2 \Rightarrow \boxed{\mathrm{P}} \Diamond C_2)$?

Fairness Constraints: An Example

$$F := \{\{s \mid \mathcal{M}, s \models \neg C1\}, \{s \mid \mathcal{M}, s \models \neg C2\}\}$$



$\mathcal{M}_F \models \boxed{\mathrm{P}}\,\Box(T_1 \Rightarrow \boxed{\mathrm{P}}\,\Diamond C_1)$? $\qquad$ $\mathcal{M}_F \models \boxed{\mathrm{P}}\,\Box(T_2 \Rightarrow \boxed{\mathrm{P}}\,\Diamond C_2)$?

YES: every **fair path** satisfies the conditions.