Faculty of Computer Science
Free University of Bozen-Bolzano
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Formal Methods Exam – 25.September.2007

STUDENT NAME:

STUDENT NUMBER:

STUDENT SIGNATURE:

This exam will constitute the 80% of the overall course assessment.

1 Exercise: Proving Equivalences in LTL and CTL

Formally prove the following equivalences between LTL and CTL formulas:

- **LTL equivalence.** $\Diamond\varphi \equiv \top\,\mathcal{U}\,\varphi$.
- **CTL equivalence.** $\Diamond_{\mathrm{P}}\,\Box\varphi \equiv \neg\,\boxed{\scriptstyle\mathrm{P}}\,\Diamond\neg\varphi$.

Prove that the following pairs of formulas are not equivalent by exhibiting a model of the first formula which is not a model of the other:

- **LTL.** $\Box(\varphi \vee \psi)$ is not equivalent to $\Box\varphi \vee \Box\psi$.
- **CTL.** $\boxed{\scriptstyle\mathrm{P}}\,\Diamond(\varphi \vee \psi)$ is not equivalent to $\boxed{\scriptstyle\mathrm{P}}\,\Diamond\varphi \vee \boxed{\scriptstyle\mathrm{P}}\,\Diamond\psi$.

Finally, answer the following question:

- Show the Syntax and the Semantics of LTL and define the notion of formula satisfiability.

2 Exercise: Expressing Properties in LTL

Let's consider LTL interpreted only over linear temporal structures. Consider a language whose alphabet contains three atomic propositions: $\{j1, j2, u\}$ with the following meaning:

$$\begin{aligned} j1 &= \mathsf{Job1} \\ j2 &= \mathsf{Job2} \\ u &= \mathsf{Unemployed} \end{aligned}$$

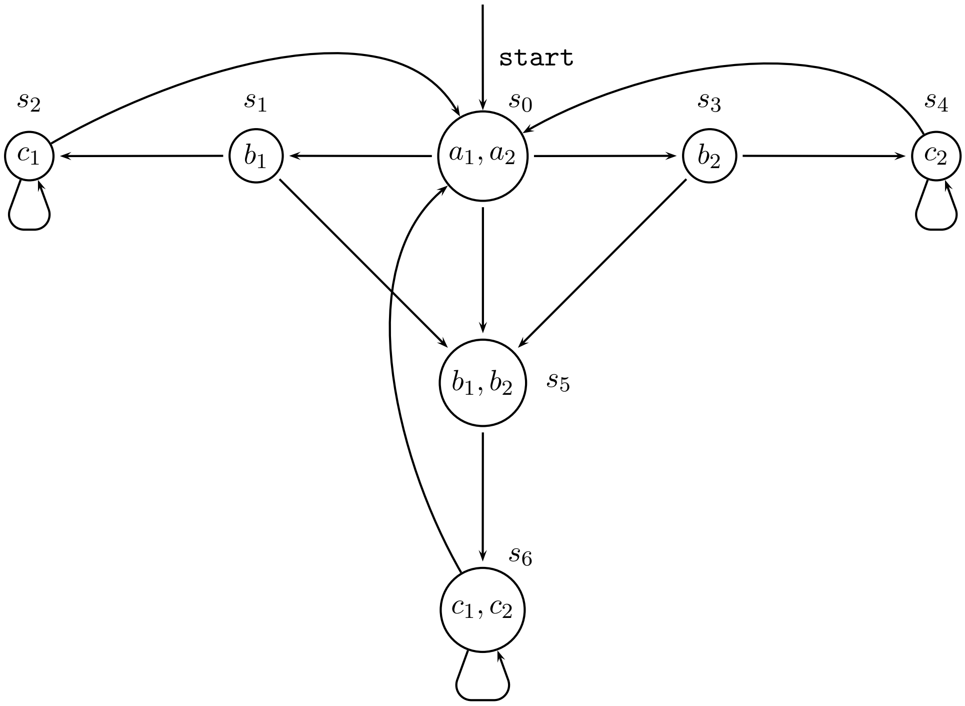
Let $\mathcal{T}$ be the following set of LTL formulas:

1. $\Box(j1 \rightarrow \neg j2)$
2. $\Box(j1 \rightarrow \neg u)$
3. $\Box(j2 \rightarrow \neg u)$
4. $\Box(j1 \;\rightarrow\; (j1\ \mathcal{U}\ \Box\neg j1))$
5. $\Box(j2 \;\rightarrow\; (j2\ \mathcal{U}\ \Box\neg j2))$

- Write an LTL formula, ψ, such that $\mathcal{T} \cup \psi$ expresses the following property:
  "It is always the case that Job1 precedes Job2"
- Write an LTL formula, φ, such that $\mathcal{T} \cup \psi \cup \varphi$ expresses the following property:
  "Unemployed is true exactly in all instants between Job1 and Job2."

3 Exercise: Model Checking in LTL

You are given the following Kripke model $\mathcal{M}$:



Extract from the above graphical representation of $\mathcal{M}$ its formal definition.
Furthermore, for each of the following **LTL** formulas φ:

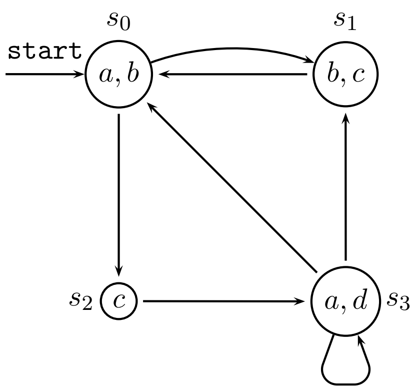
1. $(a_1 \wedge a_2) \wedge (a_2 \vee b_1) \wedge \neg b_2$
2. $\Box(a_1 \vee \neg c_2)$
3. $\Box(b_2 \rightarrow \Diamond c_2)$
4. $\Box(b_1 \rightarrow \Diamond(c_1 \wedge \neg c_2))$
5. $\Box\Diamond b_2 \rightarrow \Box\Diamond c_2$
6. $\bigcirc((b_1 \vee b_2)\,\mathcal{U}\,(c_1 \vee a_2))$

reply to the following questions:

1. Find a path from the initial state which satisfies φ.
2. Check whether $\mathcal{M} \models \varphi$, and in case the answer is negative exhibit a path that does not satisfy the formula.

4 Exercise: Model Checking in CTL

You are given the following Kripke model $\mathcal{M}$:



For each of the following **CTL** formulas φ:

1. $\boxed{\scriptstyle P}\,\Diamond(b \wedge d)$

2. $\langle\!\scriptstyle P\!\rangle\,\Box(c \vee \langle\!\scriptstyle P\!\rangle\,\bigcirc(c \wedge d))$

3. $\langle\!\scriptstyle P\!\rangle\,((a \vee c)\,\mathcal{U}\,(\langle\!\scriptstyle P\!\rangle\,\Box b))$

4. $\boxed{\scriptstyle P}\,\Diamond c \rightarrow \langle\!\scriptstyle P\!\rangle\,\Box b$

check whether $\mathcal{M} \models \varphi$ holds by using the labeling algorithm.

5 Exercise: OBDD

Construct the OBDD in canonical form for the following formula:

$$(a \rightarrow b) \rightarrow (c \vee \neg d)$$

by showing all the partial OBDD's needed to reach the final OBDD.

Furthermore, discuss the rules to reduce an OBDD to its canonical form.