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Formal Methods Exam – 4.July.2008

STUDENT NAME:

STUDENT NUMBER:

STUDENT SIGNATURE:

This exam will constitute the 80% of the overall course assessment.

1 Proving Equivalences in LTL and CTL

Formally prove the following equivalences between LTL and CTL formulas:

- **LTL equivalence.** $\Diamond\varphi \equiv \top\,\mathcal{U}\,\varphi$.
- **CTL equivalence.** $\boxed{\mathrm{P}}\Diamond\varphi \equiv \neg\diamondsuit\Box\neg\varphi$.

Prove that the following pairs of formulas are not equivalent by exhibiting a model of the first formula which is not a model of the other:

- **LTL.** $\Box(\varphi \vee \psi)$ is not equivalent to $\Box\varphi \vee \Box\psi$.
- **CTL.** $\boxed{\mathrm{P}}\Diamond(\varphi \vee \psi)$ is not equivalent to $\boxed{\mathrm{P}}\Diamond\varphi \vee \boxed{\mathrm{P}}\Diamond\psi$.

Finally, answer the following question:

- Show the Syntax and the Semantics over **Kripke structures** (i.e., the so called *path-semantics*) of LTL and define the notion of formula satisfiability.

2 Expressing Properties in LTL

Express the following properties in LTL assumed to be true at all points in time:

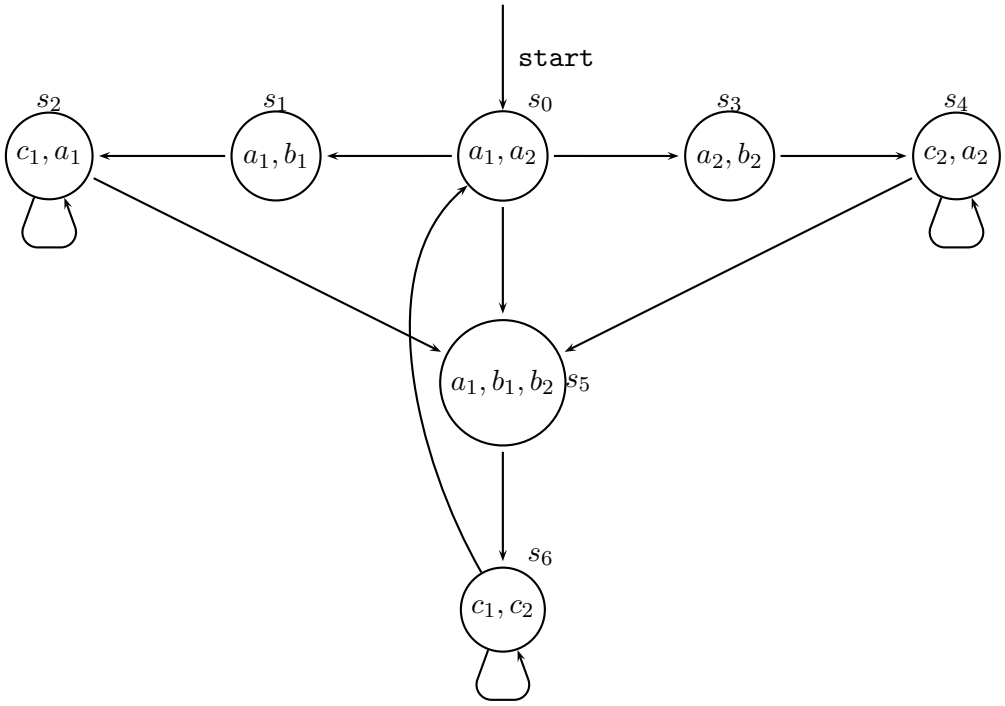
1. Between the events S and T the event W is never true.
2. It is never the case that events E_1 and E_2 happen at the same time.
3. A person is alive till he dies. After he dies a person cannot be alive again.
4. If event P is true then there exists a future time (not including the current time) where the event Q is true.

Finally, answer the following question:

- Discuss on the expressive power of LTL Vs. CTL.

3 Model Checking in LTL

You are given the following Kripke model $\mathcal{M}$:



Extract from the above graphical representation of $\mathcal{M}$ its formal definition.
Furthermore, for each of the following **LTL** formulas φ:

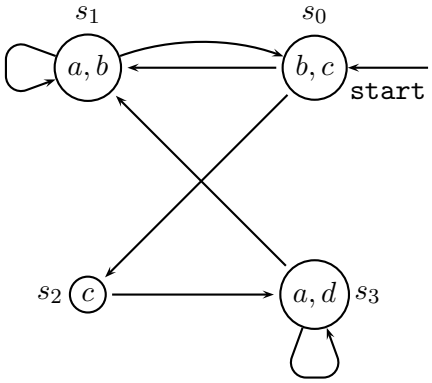
1. $\big((a_1 \wedge \neg b_2) \vee b_1\big) \rightarrow \bigcirc b_2 \vee \bigcirc \bigcirc (c_1 \vee c_2)$
2. $\Box(\neg c_2 \vee \bigcirc a_1)$
3. $\Box\big(\bigcirc b_1 \rightarrow \Diamond(c_1 \wedge \neg b_1)\big)$
4. $\Box\Diamond b_2 \rightarrow \Box\Diamond(a_1 \wedge a_2)$
5. $(a_1 \vee a_2)\,\mathcal{U}\,(c_1 \vee c_2)$

reply to the following questions:

1. Find a path from the initial state which satisfies φ.
2. Check whether $\mathcal{M} \models \varphi$, and in case the answer is negative exhibit a path that does not satisfy the formula.

4 Model Checking in CTL

You are given the following Kripke model $\mathcal{M}$:



For each of the following **CTL** formulas φ:

1. $\boxdot_{\mathrm{P}} \Diamond (b \wedge c)$
2. $\boxdot_{\mathrm{P}} \Box (b \vee (c \,\mathcal{U}\, a))$
3. $\diamondsuit_{\mathrm{P}} \Box (a \vee \diamondsuit_{\mathrm{P}} \bigcirc (b \wedge d))$
4. $\diamondsuit_{\mathrm{P}} \Diamond (c \wedge \boxdot_{\mathrm{P}} \Box \neg c)$

check whether $\mathcal{M} \models \varphi$ holds by using the labeling algorithm.

5 Symbolic Model Checking

Given the Kripke model of the Exercise 4 do the following:

1. Write the characteristic function of the initial state, $\xi(s_0)$.
2. Construct the OBDD in canonical form for $\xi(s_0)$ by showing all the partial OBDD's needed to reach the final OBDD.
3. Check whether $\mathcal{M} \models \neg a \rightarrow (b \wedge c)$ holds by using the symbolic model checking algorithm.

Furthermore, explain how to build an OBDD for *PreImages* CTL formulas, i.e. $B_{\diamondsuit_{\mathrm{P}} \bigcirc \varphi}$.