This exam will constitute the 80% of the overall course assessment.
1 Proving Equivalences in LTL and CTL

Formally prove the following equivalences between LTL and CTL formulas:

- **LTL equivalence.** \( \Diamond \varphi \equiv \top \cup \varphi \).
- **CTL equivalence.** \( \Box \Diamond \varphi \equiv \neg \Diamond \neg \varphi \).

Prove that the following pairs of formulas are not equivalent by exhibiting a model of the first formula which is not a model of the other:

- **LTL.** \( \Box (\varphi \lor \psi) \) is not equivalent to \( \Box \varphi \lor \Box \psi \).
- **CTL.** \( \Box \Diamond (\varphi \lor \psi) \) is not equivalent to \( \Box \Diamond \varphi \lor \Box \Diamond \psi \).

Finally, answer the following question:

- Show the Syntax and the Semantics over Kripke structures (i.e., the so called path-semantics) of LTL and define the notion of formula satisfiability.

2 Expressing Properties in LTL

Express the following properties in LTL assumed to be true at all points in time:

1. Between the events \( S \) and \( T \) the event \( W \) is never true.
2. It is never the case that events \( E_1 \) and \( E_2 \) happen at the same time.
3. A person is alive till he dies. After he dies a person cannot be alive again.
4. If event \( P \) is true then there exists a future time (not including the current time) where the event \( Q \) is true.

Finally, answer the following question:

- Discuss on the expressive power of LTL Vs. CTL.
3 Model Checking in LTL

You are given the following Kripke model $M$:

![Diagram of Kripke model](image)

Extract from the above graphical representation of $M$ its formal definition.
Furthermore, for each of the following LTL formulas $\varphi$:

1. $(a_1 \land \neg b_2) \lor b_1 \rightarrow \bigcirc b_2 \lor \bigcirc (c_1 \lor c_2)$
2. $\Box (\neg c_2 \lor \bigcirc a_1)$
3. $\Box (\bigcirc b_1 \rightarrow \Diamond (c_1 \land \neg b_1))$
4. $\Box \Diamond b_2 \rightarrow \Box \Diamond (a_1 \land a_2)$
5. $(a_1 \lor a_2) \mathcal{U} (c_1 \lor c_2)$

reply to the following questions:

1. Find a path from the initial state which satisfies $\varphi$.

2. Check whether $M \models \varphi$, and in case the answer is negative exhibit a path that does not satisfy the formula.
4 Model Checking in CTL

You are given the following Kripke model $M$:

For each of the following CTL formulas $\varphi$:

1. $\square \diamond (b \land c)$
2. $\square \Box (b \lor (c \mathcal{U} a))$
3. $\phi \Box (a \lor \diamond \circ (b \land d))$
4. $\diamond \diamond (c \land \square \neg c)$

check whether $M \models \varphi$ holds by using the labeling algorithm.

5 Symbolic Model Checking

Given the Kripke model of the Exercise 4 do the following:

1. Write the characteristic function of the initial state, $\xi(s_0)$.

2. Construct the OBDD in canonical form for $\xi(s_0)$ by showing all the partial OBDD’s needed to reach the final OBDD.

3. Check whether $M \models \neg a \rightarrow (b \land c)$ holds by using the symbolic model checking algorithm.

Furthermore, explain how to build an OBDD for PreImages CTL formulas, i.e. $B \diamond \circ \varphi$. 

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