Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Discrete Mathematics and Logic – A.Y. 2016/2017
Final Exam – Logic Part – 01/February/2017
Prof. Alessandro Artale – *Time: 60 minutes*

This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID in the solution sheet.

Problem 1 [10 points] **Completeness Proof.**

- Show that the Tableaux calculus in First Order Logic is *Complete*. Formulate the Theorem and present its proof considering **only the case for the existential quantifier rule**.

Problem 2 [6 points] **Satisfiability.**
Determine whether each of the following formulas is *satisfiable* by means of the Tableaux calculus and, in case it is satisfiable, exibit a model.

1. $(\neg A \wedge C) \wedge \neg(C \wedge \neg B) \wedge (D \to C) \wedge \neg(\neg A \vee B)$; [3 Points]

2. $\exists x.\, [A(x) \wedge \forall y.\, (R(x,y) \to \neg A(x))]$; [3 Points]

Problem 3 [8 points] **Entailment.**
Check whether each of the following *entailment* holds, using the Tableaux calculus:

1. $(A \to C) \wedge (B \to C) \models (A \vee B) \to C$; [3 Points]

2. $\exists x.\, P(x) \wedge \forall y.\, (P(y) \to Q(y)) \models \exists z.\, Q(z)$; [3 Points]

3. Finally, give the formal definition of "formula φ entails formula ψ". [2 Points]

Problem 4 [4 points] **Tableaux Termination.**
Discuss why the Tableaux calculus for FOL does not always terminate and show an example of a formula such that the Tableaux does not terminate.

Problem 5 [6 points] **Formalization in FOL.**
Consider a first-order language with the following predicates:
Student(X) – X is a student.
Loves(X,Y) – X loves Y.
Sister(X,Y) – X is a sister of Y.
Takes(X,Y) – X takes the course Y.
Express the following sentences in FOL using the above predicates and the constants `Bill, Analysis` and `Geaometry`:

1. No students love Bill.

2. Bill has at least one sister.

3. Bill has at most one sister.

4. Every student who takes Analysis also takes Geometry.