Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Discrete Mathematics and Logic – A.Y. 2016/2017
Final Exam – Discrete Mathematics Part – 01/February/2017
Prof. Alessandro Artale – *Time: 60 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID in the solution sheet.`

Problem 1 [10 points] **Induction.**

- Show that for any $n \geq 0$, $5^n - 1$ is divisible by 4. [4 POINTS]
- **Loop Invariant.** The following while loop is annotated with a pre- and a post-condition and also a loop invariant. Furthermore, assume the integer $m \geq 1$ in the guard of the while loop. Use the *loop invariant theorem* to prove the correctness of the loop with respect to the pre- and post-conditions. [6 POINTS]

  [*Pre-condition:* greatest $= A[1]$ and $i = 1$]
  while $i \neq m$ **do**
  i := i + 1
  if($A[i] >$ greatest) **then** greatest := $A[i]$
  end while

  [*Post-condition:* greatest = maximum value of $A[1], \ldots, A[m]$]

  Loop Invariant I(n): greatest is the maximum value of $A[1], A[2], \ldots, A[n+1]$ and $i = n+1$.

Problem 2 [12 points] **Sets.**

- Given the following sets:
  - $A = \{Alice, Paul, Mary\}$,
  - $B = \{CS110, CS111, MAT222, MAT221\}$ and
  - $C = \{CS110, CS112, MAT222, FIS333, MAT221\}$.

  Show the following set: $(B \cap C) \times A$. [2 POINTS]
- Powerset property: Show that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. [5 POINTS]
- **Halting Problem.** Discuss the Halting Problem. Formulate the Halting problem Theorem and give an idea on how it can be proved. [5 POINTS]

Problem 3 [4 points] **Cardinality.**

- Give the definition of **2 sets have the same cardinality** and also the definition of a set being **countably infinite**. [2 POINTS]
- Determine whether the following set is **finite**, **countably infinite** or **uncountable**. In case the set is countably infinite, show a one-to-one correspondence from the set of positive integers. [2 POINTS]
  - The set of positive integers multiple of 7.

Problem 4 [8 points] **Relations and Trees.**

- Let $A = \{0, 1, 2, 3, 4\}$ and R the following equivalence relation over A:

$$R = \{(0,0), (3,1), (1,1), (1,3), (2,2), (0,4), (4,4), (3,3), (4,0)\}$$

  Show the equivalence class of each element in A with respect to R. [3 POINTS]

- Say whether the following relation is a partial order relation. In case it is not, say what is the missing property. [3 POINTS]
  - $R = \{(0,0), (1,1), (0,3), (1,2), (0,2), (1,3), (2,2), (3,3), (3,2)\}$;

- Find all non-isomorphic trees with 5 vertices. Provide an explanation with your answer. [2 POINTS]