Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Discrete Mathematics and Logic – A.Y. 2015/2016
Final Exam Exam – Logic – 04/July/2016
Prof. Alessandro Artale – *Time: 60 minutes*

This is a closed book exam: the only resources allowed are blank paper, pens, and
your head. Explain your reasoning. Write clearly, in the sense of logic, language
and legibility. The clarity of your explanations affects your grade.

Problem 1 [12 points] **Soundness and Completeness.**

1. Show that the Tableaux calculus in Propositional Logic is *Sound*. Formulate the Theorem and
   present its proof. [10 POINTS]

2. Suppose we have an algorithm for cecking the satisfiability of a formula that for every input formula
   the output is always yes. Explain whether such an algorithm is sound or complete. [2 POINTS]

Problem 2 [10 points] **Satisfiability.**
Determine whether each of the following formulas is *satisfiable* by means of the Tableaux method and,
in case it is satisfiable, exibit a model.

1. $p \wedge (q \vee r) \wedge \neg(p \wedge q) \wedge \neg(p \wedge r)$. [4 POINTS]

2. $\forall x. (\neg A(x) \vee B(x)) \wedge \forall x. C(x) \wedge \exists y. (\neg A(y) \wedge B(y) \wedge C(y))$. [6 POINTS]

Problem 3 [8 points] **Entailment.**
Check whether each of the following *entailment* holds, using the Tableaux method:

1. $A \rightarrow (B \wedge C) \models (\neg B \vee \neg C) \rightarrow \neg A$

2. $\forall x. (A(x) \vee B(x)) \models \exists x. (A(x) \wedge B(x))$

Problem 4 [4 points] **Formalization in Propositional Logic.**
The island of Tufa has two tribes, the Tu's who always tell the truth, and the Fa's who always lie. A
traveller encountered three residents A, B, and C of Tufa, and each made a statement to the traveller:

- A said: "A or B tells the truth if C lies."

- B said: "If A or C tell the truth, then it is not the case that exactly one of us is telling the truth."

- C said: "A or B is lying if A or C is telling the truth."

Formalize the above scenario in Propositional Logic, using as propositions A, B and C.