Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Discrete Mathematics and Logic – A.Y. 2015/2016
Final Exam Exam – Logic – 05/February/2016
Prof. Alessandro Artale – *Time: 60 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID in the solution sheet.`

Problem 1 [12 points] **Completeness Proof.**

- Show that the Tableaux calculus in Propositional Logic is *Complete*. Formulate the Theorem and present its proof.

Problem 2 [8 points] **Satisfiability.**
Determine whether each of the following formulas is *satisfiable* by means of the Tableaux method and, in case it is satisfiable, exibit a model.

1. $(\neg A \wedge B) \wedge \neg(B \wedge \neg C) \wedge (C \rightarrow D) \wedge \neg(\neg A \vee D)$
2. $\forall x.\, B(x) \wedge \forall x.\, C(x) \rightarrow \exists y.\, (B(y) \wedge C(y))$

Problem 3 [10 points] **Entailment.**
Check whether each of the following *entailment* holds, using the Tableaux method:

1. $(A \rightarrow B) \wedge (\neg B \rightarrow C) \models \neg C \rightarrow A$
2. $\forall x.\, (A(x) \wedge B(x)) \models \exists x.\, A(x) \rightarrow \exists x.\, B(x)$

Problem 4 [4 points] **Formalization in FOL.**
Consider a first-order language with the following predicates:
c(X,Y) – X is a child of Y.
d(X,Y) – X is a daughter of Y.
s(X,Y) – X is a son of Y.
m(X) – X is male.
Assuming that the domain contains only people, express the following sentences:

- Every son is a male child.
- Every daughter is a child who is not male.
- Everyone is a child of someone.
- Everyone is either a son or a daughter of someone.