Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Discrete Mathematics and Logic – A.Y. 2015/2016
Final Exam – Discrete Mathematics – 05/February/2016
Prof. Alessandro Artale – *Time: 60 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade. Write your name and ID in the solution sheet.`

Problem 1 [12 points] **Induction.**

- Show that for any $n \geq 2$, $5^n + 9 < 6^n$. [4 POINTS]
- **Loop Invariant.** The following while loop is annotated with a pre- and a post-condition and also a loop invariant. Use the *loop invariant theorem* to prove the correctness of the loop with respect to the pre- and post-conditions. [8 POINTS]

[*Pre-condition:* greatest $= A[1]$ and $i = 1$]
while $i \neq m$ **do**
i := i + 1
if($A[i] >$ greatest) **then** greatest := $A[i]$
end while

[*Post-condition:* greatest = maximum value of $A[1], \ldots, A[m]$]

Loop Invariant I(n): greatest is the maximum value of $A[1], A[2], \ldots, A[n+1]$ and $i = n + 1$.

Problem 2 [6 points] **Sets.**

- Given the following sets: $A = \{a, b, c, d\}$, $B = \{x, y\}$. Show the Cartesian Product $A \times B$. [2 POINTS]
- Powerset property: Show that $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$. [4 POINTS]

Problem 3 [8 points] **Cardinality.**

- Give the definition of **2 sets have the same cardinality** and also the definition of a set being **countably infinite**. [2 POINTS]
- Let A and B be two countably infinite sets. Show that $A \cup B$ is a countably infinite set. [6 POINTS]

Problem 4 [8 points] **Relations and Trees.**

- Say which of the following relations is an equivalence relation. In case it is not, say what is the missing property. [2 POINTS]
  1. $R1 = \{(a, a), (a, c), (c, d), (c, c), (c, a), (d, c), (d, d)\}$;
  2. $R2 = \{(0, 0), (0, 1), (0, 2), (2, 1), (2, 2), (1, 0), (1, 2), (1, 1), (3, 3)\}$.
- Let $A = \{0, 1, 2, 3, 4\}$ and R the following equivalence relation over A:

$$R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}$$

  Show the equivalence class of each element in A with respect to R. [4 POINTS]
- Find all non-isomorphic trees with 5 vertices. Provide an explanation with your answer. [2 POINTS]