Formal Languages and Compilers
Lab II: CFGs Clean-Up Form

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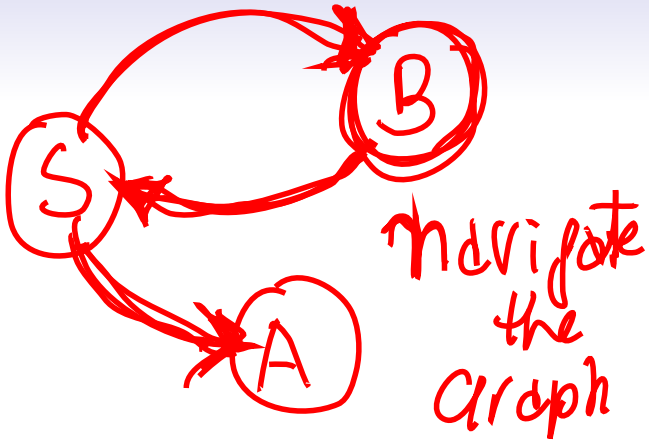
Formal Languages and Compilers — BSc course

2020/21 – Second Semester

Board



Example 1. Eliminate Unit Productions

$$
\begin{aligned}
S &\longrightarrow A \mid B \\
A &\longrightarrow Sa \mid a \\
B &\longrightarrow b \mid S
\end{aligned}
$$

Show all the steps, starting from the construction of the *Graph of Unit Productions*.

$S \rightarrow Sa \mid a \mid b$

$A \rightarrow Sa \mid a$

$B \rightarrow b \mid Sa \mid a$

Board

N = nullable symb.

Board

Graph

Example 2. Eliminate ϵ-Productions, and Unit Productions, Non-generating symbols, and Non-reachable symbols

H — Generating symbols

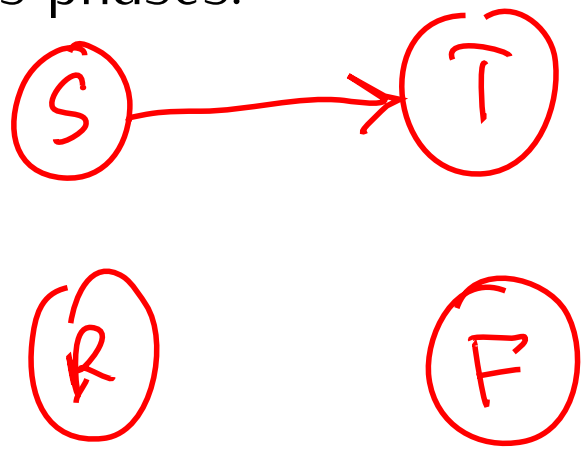
R — set of reachable symb.

$$
\begin{array}{lcl}
S & \longrightarrow & T + S \mid T \\
T & \longrightarrow & aR \mid a \\
R & \longrightarrow & aR \mid \epsilon \mid a \\
F & \longrightarrow & RTa \mid Ta
\end{array}
$$

Show all the steps in all eliminations phases.

$N_0 = \{R\}$

$N_1 = N_0 = N$



Board

$S \to T+S \mid aR \mid a$

$T \to aR \mid a$

$R \to aR \mid a$

$F \to RTa \mid Ta$

Generating Symb.

$G_0 = \{a, S, T, R\}$

$G_1 = G_0 \cup \{F\} = G$

Reachable Symb.

$R_0 = \{S\}$

$R_1 = \{S, T, R\}$

$R_2 = R_1 = R$

$NR = \{F\}$

$S \to T+S \mid aR \mid a$

$T \to aR \mid a$

$R \to aR \mid a$

Board

Board

Example 3. Eliminate ϵ-Productions, and Unit Productions, Non-generating symbols, and Non-reachable symbols

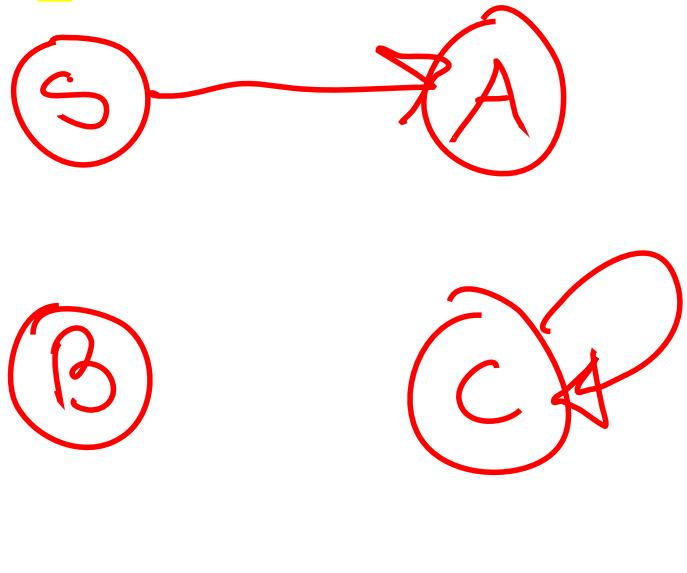
$$
\begin{aligned}
S &\longrightarrow A \mid ABa \mid AbA \mid \epsilon \mid Ba \mid b \mid bA \mid Ab \\
A &\longrightarrow Aa \mid \epsilon \mid a \qquad\qquad Aa \mid a \\
B &\longrightarrow Bb \mid BC \\
C &\longrightarrow CB \mid CA \mid bB \mid \epsilon
\end{aligned}
$$

Show all the steps in all eliminations phases.

$N_0 = \{A\}$

$N_1 = \{A, S\}$

$N_2 = N_1 = N$



Board

$G_0 = \{a, b, S, A\}$

$G_1 = G_0 = G$

$NG = \{B, C\}$

$$\begin{cases} S \to \varepsilon \mid Ab \mid bA \mid b \mid bA \mid Ab \mid Aa \mid a \\ A \to Aa \mid a \end{cases}$$

$R_0 = \{S\}$

$R_1 = \{S, A\}$

Board

Board

Example 4. Eliminate ϵ-Productions, and Unit Productions, Non-generating symbols, and Non-reachable symbols

$$
\begin{aligned}
S &\longrightarrow B \mid ABa \mid BaDb \\
A &\longrightarrow Ab \mid AC \\
B &\longrightarrow BaD \mid \epsilon \\
C &\longrightarrow CB \mid CA \mid bA \\
D &\longrightarrow DA \mid B \mid aDD \mid ABC
\end{aligned}
$$

Show all the steps in all eliminations phases.

1. ϵ-Production

Set of Nullable symbols:

$N_0 = \{B\}$

$N_1 = N_0 \cup \{S, D\}$

$N_2 = N_1$

$N = \{S, B, D\}$

(cont→)

$N = \{S, B, D\}$

New Grammar after elimination of ϵ-Productions

$S \to \epsilon \mid B \mid ABa \mid BaDb \mid Aa \mid aDb \mid Bab \mid ab$

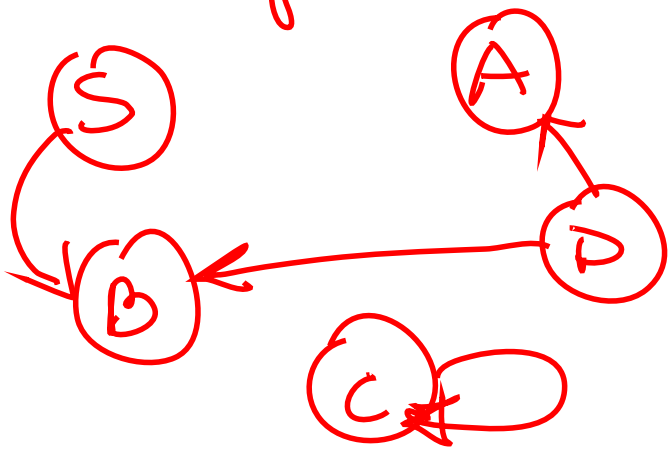
$A \to Ab \mid AC$

$B \to BaD \mid aD \mid Ba \mid a$

$C \to CB \mid CA \mid bA \mid C$

$D \to DA \mid B \mid aDD \mid ABC \mid A \mid aD \mid a \mid AC$

2. Removing Unit Productions:



Graph of unit productions (cont→)

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New Grammar:

$S \to \varepsilon \mid ABa \mid BaDb \mid Aa \mid aDb \mid Bab \mid ab \mid BaD \mid aD \mid Ba$

$A \to Ab \mid AC$

$B \to BaD \mid aD \mid Bab$

$C \to CB \mid CA \mid b$

$D \to DA \mid aDD \mid ABC \mid aD \mid a \mid AC \mid Ab \mid BaD \mid Ba$

3. Removing Non-Generating Symbols:

$G_0 = \{a, b\}$

$G_1 = G_0 \cup \{S, B, D\}$

$G_2 = G_1$

$G = \{S, B, D\}$

Non-generating:

$NG = \{A, C\}$

New Grammer after removing {A, C} :

$S \to \varepsilon \mid BaDb \mid aDb \mid Bab \mid ab \mid BaD \mid aD \mid Ba \mid a$

$B \to BaD \mid aD \mid Ba \mid a$

$D \to aDD \mid aD \mid a \mid BaD \mid Ba$

4. Removing Non-Reachable symbols:

$R_0 = \{S\}$

$R_1 = \{S, B, D\}$

$R = \{S, B, D\}$ and all symbols are Reachable.

Board