Formal Languages and Compilers
Lab I: Languages and Grammars

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Formal Languages and Compilers — BSc course

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Board - CFLs

$L = \{0^n 1^n \mid n \geq 0\}$

$\boxed{000111} \in L$

Reason by induction

$V_T = \{0, 1\}$

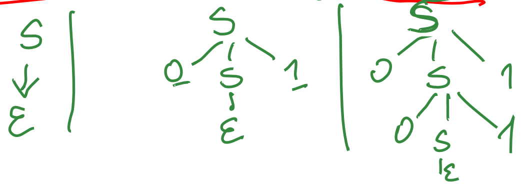
$\varepsilon \in L$

$L = \{\varepsilon, 01, 0011, 000111, \dots$

$S \to \varepsilon \mid 0S1$

Base: $w = \varepsilon$

Induction $w_n = \boxed{0\, w_{n-1}\, 1}$



Board

w

$\underbrace{010}_{w1} \mid \underbrace{101}_{w2}$

COMPLEMENT LANGUAGE

$L = \{ w \in \{0,1\}^* \mid$ W=w1w2 such that each bit in w1 is complemented in w2 $\}$

$\boxed{\varepsilon \in L}$

Reason by Induction.

Base: ε

Induction: $w_n = 0\, w_{n-1}\, 1$; $1\, w_{n-1}\, 0$

$S \to \varepsilon \mid 0S1 \mid 1S0$ S

"000111"

Grammar Productions

$S \to A \mid B$

$A \to 0B1 \mid \varepsilon$

$B \to 1A0 \mid \varepsilon$

$S \to A$

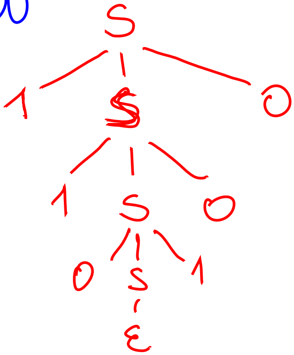
$A \to 0A1 \mid 1A0 \mid \varepsilon$

Board

$S \to \varepsilon \mid 0S1 \mid 1S0$

Show the Derivation Tree for:

$w = 110100$



Board CFL

Palindroms over $\{0,1\}$

Language

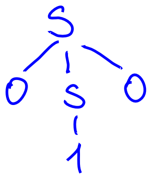
Example: 0110 $\boxed{\varepsilon \in L}$

Reason By Induction:

Base: $w_0 = \varepsilon, 0, 1$

Induction: $w_n = \begin{cases} 0 w_{n-1} 0 \\ 1 w_{n-1} 1 \end{cases}$

010



Grammar Productions

$S \to \varepsilon \mid 0 \mid 1 \mid 0S0 \mid 1S1$

$S \to \varepsilon \mid a \mid b \mid c \mid aSa \mid bSb \mid cSc$

if $V_T = \{a, b, c\}$

Board - CFL

$L = \{ 0^n 1^n \mid n \geq 1 \}$.

Note: the empty string does not belong to L.

Reason by Induction:

Board

$L = \{ w \in \{0,1\}^* \mid w \text{ has equal number of } 0\text{'s} \ \& \ 1\text{'s} \}$

$\varepsilon \in L$

w^1

$0 \ \boxed{1\,1\,1\,0\,1\,0\,0}$ 1

$1 \ \underbrace{\boxed{1\,0\,1\,0\,0}}_{w^0}$ 2

w

Base $w^1_0 = 1$ || $w^0_0 = 0$

$$w^1_n = \begin{cases} 1 \cdot w_{n-1} \\ 0 \cdot w^1_{n-1} \cdot w^1_{n-1} \end{cases} \qquad w^0_n = \begin{cases} 0\, w_{n-1} \\ 1 \cdot w^0_{n-1} \cdot w^0_{n-1} \end{cases}$$

Board

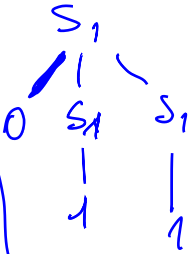
$$S \to \varepsilon \mid 0S_1 \mid 1S_0$$

$$S_1 \to 1 \mid 1S \mid 0S_1S_1$$

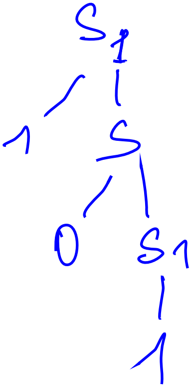
$$S_0 \to 0 \mid 0S \mid 1S_0S_0$$

Board

$w^1 = 0\ 1\ 1$



$w^1 = 1\ 0\ 1$



Board

$$L = \{0^n 1^n 2^m \mid n \geq 0, m \geq 0\}$$

Examples: 00112222, 01, 222, 000111

Board

$L = \{a^n b^m \mid n \geq 0, m \geq 0\}$

$L_1 = \{a^n \mid n \geq 0\}$

$L_2 = \{b^m \mid m \geq 0\}$

$L = L_1 \cdot L_2$

Board

$$L = \{ 0^i 1^j 2^k \mid i, j, k \geq 0,\ k = i + j \}$$

$$L = \{ 0^n 1^m 2^m \mid n \geq 0,\ m \geq 0 \}$$

$V = \{0, 1, 2\}$

Board

$L = \{ 0^i 1^j 2^k \mid i, j, k \geq 0 \text{ and } k = i + j \}$

Examples: $*\boxed{111222}$ $\quad k = j$

$-\ \underbrace{00}\ \boxed{111222}\ \underbrace{22}$

$\underline{L1} = \{ 1^j 2^k \mid j, k \geq 0 ,\ k = j \}$

$\underline{L2} = \underline{\underline{0^i}\ (L1)\ \underline{2^i}}$ $\qquad \underline{k} = i + \underline{j}$

$w_1 = \begin{cases} \varepsilon & \text{base} \\ 1\, w_1^{n-1}\, 2 \end{cases}$ $\qquad \underline{w} = \underline{w_1} \mid 0\, w_{n-1}\, 2$

Board

$$w = w_1 \mid 0w2$$

$$S \rightarrow S_1 \mid 0S2$$
$$S_1 \rightarrow \varepsilon \mid 1S_12$$