Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science
Formal Languages and Compilers– A.Y. 2018/19 Mid-Term Exam – 04/08/2019
Prof. Alessandro Artale – *Time: 120 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade.`

Problem 1 [9 points] Decide which of the following statements is TRUE and which is FALSE. You must give a **clear explanation** of your answer to receive full credit.

(a) Let L_1, L_2 be any two regular languages over the same alphabet Σ, then the language $L = \{w \in \Sigma^* \mid w \in L_1 \text{ or } w \notin L_2\}$ is regular.

(b) If a language L is constituted by a **finite** set of strings, then L is a regular language.

(c) The language $L = \{a^n b^n \mid n \geq 1\}$ is regular (use the Pumping Lemma).

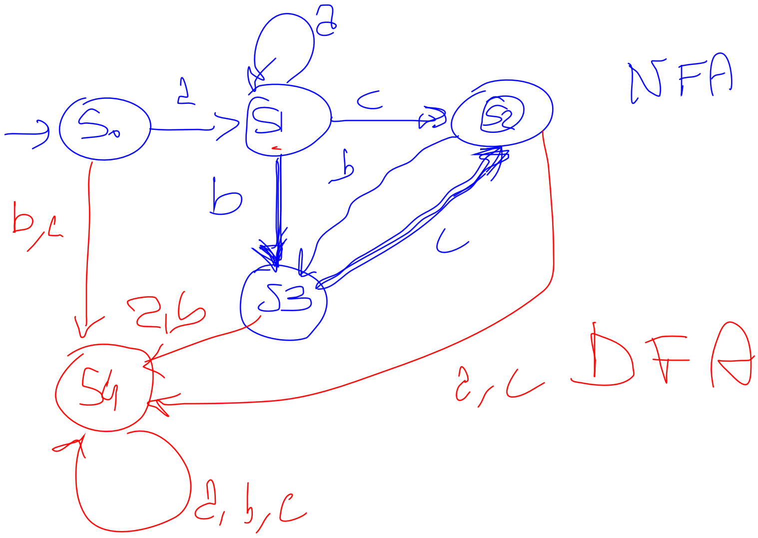
a) $L = L_1 \cup \overline{L_2}$ is Reg, because R L are closed under union & complement

b) Assume $L = \{w_1, \ldots, w_n\}$
RE$_L$ $w_1 | w_2 | \ldots | w_n$

c) $|w| \geq n$ $w = xyz$ s t
1 $|xy| \leq n$
2 $|y| \geq 1$
3 xy^kz, $k \geq 0$ $\in L$
$w = a^n b^n$ $|w| = 2n$
xy is made only of 'a'
3 $\Rightarrow$ we can pump as many 'a' as we want $\notin L$

Problem 2 [12 points]

(a) Construct the automaton (either NFA or DFA) that recognises the language over the alphabet $\{a, b, c\}$ constituted by all strings starting with the letter a, ending with the letter c, and never containing any of the following as a substring: ba, bb, ca, cc. E.g., $abcbc \in \mathcal{L}(A)$ and $ac \in \mathcal{L}(A)$, while $ababc \notin \mathcal{L}(A)$ since it contains ba, and $abbc \notin \mathcal{L}(A)$ since it contains bb. [3 POINTS]



$\delta_{NFA}(s, a) = \emptyset$

(b) Construct the RE for the language over the alphabet $V = \{a, b, c\}$ where if a string contains two or more letters a, they are not adjacent. E.g., $bcbbbcc \in L(RE)$ since there are no as, $cbcacbb \in L(RE)$ since it contains a single a, $bcbabcaba \in L(RE)$ since the as are not adjacent, while $baac \notin L(RE)$ since the as are adjacent. [3 POINTS]

$(b \mid c \mid ab \mid ac)^* (a \mid \varepsilon)$

$(b|c)^* a \mid (a(b|c)^+)^* \mid ((b|c)^+ a)^* (b|c)^*$

aba

(c) Given two DFA's over the same alphabet V, say $A = (Q_A, V, q_0^A, \delta_A, F_A)$ and $B = (Q_B, V, q_0^B, \delta_B, F_B)$, formally describe the *Product Automaton*, say $A \times B = (Q_{A\times B}, V, q_0^{A\times B}, \delta_{A\times B}, F_{A\times B})$. In particular, say how the set of states $Q_{A\times B}$, the initial state $q_0^{A\times B}$, the transition function $\delta_{A\times B}$, and the set of final states $F_{A\times B}$ can be constructed to recognise the intersection $L(A) \cap L(B)$. [3 POINTS]

(d) Give the CFG for the language over the alphabet $V = \{a, b\}$, $L = \{a^n b^k \mid k > n\}$. [3 POINTS]

$n \geq 0$

$w = \begin{cases} B \\ w_A\, bb \dots b \end{cases}$ $\qquad w_A \to \begin{cases} \varepsilon \\ a\, w_A\, b \end{cases}$

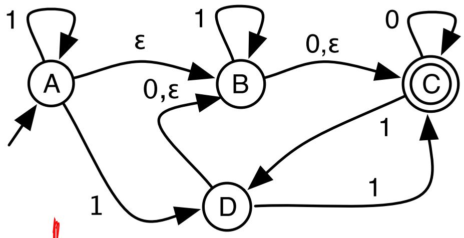
A —— B

$S \to AB$
$B \to b \mid bB$
$A \to \varepsilon \mid aAb$

$\leadsto$

$S \to AB \mid B$
$B \to b \mid bB$
$A \to ab \mid aAb$

Problem 3 [5 points] Consider the following ε-NFA A_ϵ over $\{0, 1\}$:



(a) Show the ϵ-closure of each state. [2 Points]

(b) Construct an NFA A_N such that $\mathcal{L}(A_N) = \mathcal{L}(A_\epsilon)$. Just show in a table format the transition function of the resulting NFA indicating the initial and the final states. [3 Points]

δ_E	0	1	ε
$\rightarrow$ A	$\emptyset$	$\{A, D\}$	$\{B\}$
B	$\{C\}$	$\{B\}$	$\{C\}$
* C	$\{C\}$	$\{D\}$	$\emptyset$
D	$\{B\}$	$\{C\}$	$\{B\}$

1

$*\ \varepsilon\text{-}Cl(A) = \{A, B, C\}$

$*\ \varepsilon\text{-}Cl(B) = \{B, C\}$

$*\ \varepsilon\text{-}Cl(C) = \{C\}$

$*\ \varepsilon\text{-}Cl(D) = \{D, B, C\}$

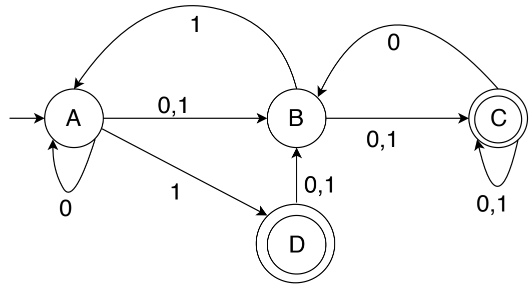
2

$\delta_N(A, 0)$

δ_N	0	1
$* \rightarrow$ A	$\{C\}$	$\{A, B, D\}$
* B	$\{C\}$	$\{B, D\}$
* C	$\{C\}$	$\{D\}$
* D	$\{B, C\}$	$\{B, C, D\}$

3

Problem 4 [4 points] Consider the following NFA, A_N, over $\{0, 1\}$:



(a) Construct a DFA, A_D, such that $\mathcal{L}(A_D) = \mathcal{L}(A_N)$. Just show in a table format the transition function of the resulting DFA indicating the initial and the final states. [3 Points]

(b) Show **all** possible sequences of states of A_N that are traversed for the string 1001. [1 Point]

δ_N	0	1
→ A	{A, B}	{B, D}
B	{C}	{A, C}
* C	{B, C}	{C}
* D	{B}	{B}

δ_D	0	1
→ [A]	[AB]	[BD]
[AB]	[ABC]	[A,B,C,D]
* [BD]	[BC]	[ABC]
* [ABC]	[A,B,C]	[A,B,C,D
* [ABCD]	[A,B,C]	[A,B,C,
* [BC]	[B,C]	[AC]
* [AC]	[A,B,C]	[BCD]
* [BCD]	[BC]	[ABC

Problem 5 [5 points]

(a) Apply the sequence of steps that are necessary to simplify a context free grammar and convert it into a Clean-up Form to the context free grammar $G = (\{S, A, B, C, D, E\}, \{a, b, c\}, P, S)$, where P consists of the following productions:

$$
\begin{array}{lllllll}
S & \longrightarrow & aABb \mid AE & \qquad & C & \longrightarrow & ABC \mid c \\
A & \longrightarrow & aAbE \mid \epsilon & & D & \longrightarrow & AAB \\
B & \longrightarrow & abB \mid \epsilon & & E & \longrightarrow & CA
\end{array}
$$

The normalizations steps must be carried on in the **correct order** and the **algorithms** used for each step must become evident to get full mark.

1. ε-PROD

$N_0 = \{A, B\}$, $N_1 = \{A, B, D\}$

$N_2 = \{A, B, D\} = N_1 = N$

$S \to aABb \mid AE \mid aBb \mid aAb \mid ab \mid E$

$A \to aAbE \mid abE$

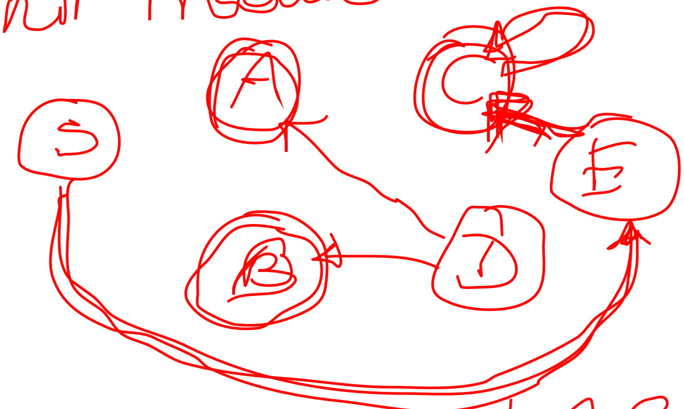
$B \to abB \mid ab$

$C \to ABC \mid c \mid BC \mid AC \mid \cancel{\epsilon}$

$D \to AAB \mid AB \mid AA \mid B \mid A$

$E \to CA \mid C$

2- Unit Production



$S \to \_\_ \mid CA \mid ABC \mid c \mid BC \mid AC$

$A \to aAbE \mid abE$

$B \to$

$C \to ABC \mid c \mid BC \mid AC$

$D \to \_\_\_\_ \mid abB \mid ab \mid aAbE \mid abE$

$E \to CA \mid ABC \mid c \mid BC \mid AC$

(Cont.)

3 - Non-Generating Symbols

- $S \to aABb \mid AE \mid aBb \mid aAb \mid ab \mid CA \mid ABC \mid c \mid BC \mid AC$

✓ $A \to aAbE \mid ab$ E

✓ $B \to abB \mid a$

✓ $C \to ABC \mid c \mid BC \mid AC$

- ~~$D \to ADB \mid AB \mid AA \mid aAbE \mid abE \mid abB \mid ab$~~

✓ $E \to CA \mid ABC \mid c \mid BC \mid AC$

$H_0 = \{a, b, c\}$ $\quad H_1 = H_0 \cup \{S, B, C, D, E\}$

$H_2 = H_1 \cup \{A\} = V$

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4 Non-Reachable

$R_0 = \{S\}$ $\quad R_1 = \{S, A, B, E, C\}$

$R_2 = R_1$ $\quad NR = \{D\}$