Free University of Bozen-Bolzano – Faculty of Computer Science
Bachelor in Computer Science and Engineering
Formal Languages and Compilers – A.Y. 2016/2017
MidTerm Exam – Formal Languages part – 12.December.2016
Prof. Alessandro Artale – *Time: 1^{h}50 minutes*

`This is a closed book exam: the only resources allowed are blank paper, pens, and your head. Explain your reasoning. Write clearly, in the sense of logic, language and legibility. The clarity of your explanations affects your grade.`

Problem 1 [9 points] Decide which of the following statements is TRUE and which is FALSE. You must give an explanation of your answer to receive marks.

(a) For all languages L_1 and L_2, if $L_1^* = L_2^*$, then $L_1 = L_2$.

(b) Let L_1, L_2 be two *Regular Languages* over the same alphabet Σ, then the language $L = \{w \in \Sigma^* \mid w \notin L_1 \text{ and } w \in L_2\}$ is regular.

(c) The language $L = \{0^n 1^n \mid n \geq 1\}$ is not regular (use the Pumping Lemma).

1a) NO $L_1 = L - \{\varepsilon\}$ $L_2 = L \cup \{\varepsilon\}$ $L_1 \neq L_2$
$L_1^* = L_2^*$

1b) $L = \overline{L_1} \cap L_2$ Reg Lang are closed under complement e Intersection
So, L is regular

Problem 2 [9 points]

(a) Construct a Context Free Grammar for the language L over the alphabet $\{a, b, c\}$ such that $L = \{a^n b^m c^k \mid k = n + m \text{ with } n, m, k \geq 0\}$. E.g., $abbccc \in L$, $aaabcccc \in L$, $abbbcccc \in L$, $bc \in L$, $bbcc \in L$, $aacc \in L$ while $b \notin L$, $ab \notin L$, $abc \notin L$.

w_A

aab | bbcc | ccc

ε A

$w = w_A$

$b^m c^m, m \geq 0$

$aa\_ \; w_A \; cc\_ \; c_B \leftarrow B$

ac

$S \to A \mid B$

$A \to \varepsilon \mid bAc$

$B \to A \mid ac \mid aBc$

$S \to A \mid aSc$

$A \to bc \mid bAc \mid \varepsilon$

(b) Construct a regular expression RE and the corresponding automaton (either DFA or NFA) that generates the language over the alphabet $\{x, y, z\}$ constituted by all strings in which each x is (not necessarily immediately) preceded by some z, and this z comes after any other x.
E.g., $\varepsilon \in \mathcal{L}(RE)$, $zyyz \in \mathcal{L}(RE)$, $zxyyzzyxyz \in \mathcal{L}(RE)$, $zxyyxz \notin \mathcal{L}(RE)$, $xyyzxyy \notin \mathcal{L}(RE)$.

$V = \{x, y, z\}$

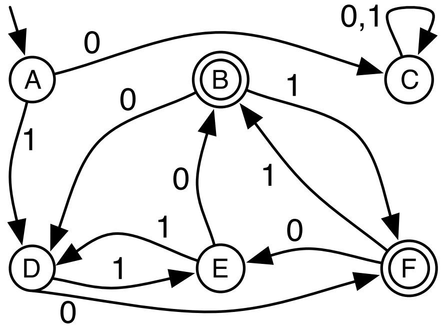
$yz\,yy\,zz \in L$

$zyy\,x\,zyyx\,zx \in L$

RE $(y \mid z \mid zy^* x)^*$

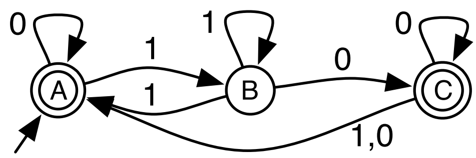
(c) Given two DFA's, said A and B, describe the notion of *Product Automaton*. Furthermore, show how this notion can be used to prove that regular languages are closed under *Intersection* (i.e., if L_1 and L_2 are regular languages, then so is $L_1 \cap L_2$).

Problem 3 [4 points] Consider the following DFA A over $\{0, 1\}$:



(a) Construct a DFA A_m with minimal number of states such that $\mathcal{L}(A_m) = \mathcal{L}(A)$. The algorithm you have followed to construct A_m should become evident in your construction.

Problem 4 [6 points] Consider the following NFA A_n over $\{0, 1\}$:



(a) Construct a DFA A_d such that $\mathcal{L}(A_d) = \mathcal{L}(A_n)$. The algorithm you have followed to construct the DFA A_d should become evident in your construction.

(b) Show all possible sequences of states of the NFA A_n that are traversed for the string 0100.

Problem 5 [6 points] Apply the sequence of steps that are necessary to simplify a context free grammar and convert it into a Clean-up Form to the context free grammar $G = (\{S, A, B, C, D\}, \{a, b\}, P, S)$, where P consists of the following productions:

$$
\begin{array}{lcl@{\qquad\qquad}lcl}
S & \longrightarrow & B \mid ABa \mid BaDb & C & \longrightarrow & CB \mid CA \mid bA \\
A & \longrightarrow & Ab \mid AC & D & \longrightarrow & DA \mid B \mid aDD \mid ABC \\
B & \longrightarrow & BaD \mid \varepsilon & & &
\end{array}
$$

Note1: The sequece of steps must be in the right order!
Note2: The algorithm applied in each step of the semplification procedure must be `Clearly Presented` showing the various intermediate steps.

(cont.)