Algorithms for Data Processing
Lecture II: Graphs and Trees

Alessandro Artale

Free University of Bozen-Bolzano
Faculty of Computer Science
http://www.inf.unibz.it/˜artale
artale@inf.unibz.it

2019/20 – First Semester
MSc in Computational Data Science — UNIBZ

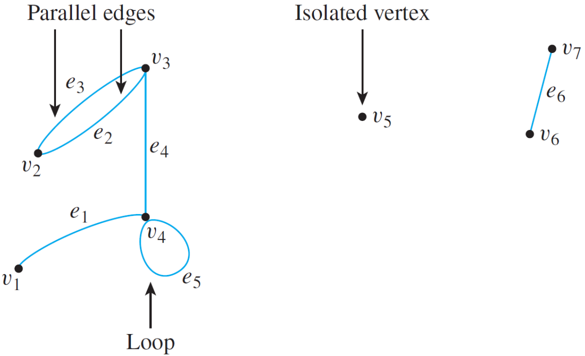
Graphs — Definitions

A graph $G = (V, E)$ is composed of:

- a set of vertices V;
- a set of edges $E \subseteq V \times V$ connecting the vertices;
- An edge $e \in E$ between a pair of vertices is denoted as $e = (u, v)$;
- Loop: An edge which connects a vertex to itself, i.e., $e = (u, u)$;
- Given an edge $e = (u, v)$, the vertices u, v are its endpoints;
- An edge is said to be incident on each of its endpoints;
- Vertex v is adjacent to vertex u iff $(u, v) \in E$;
- If a graph is undirected, we represent an edge between u and v by having both $(u, v) \in E$ and $(v, u) \in E$.

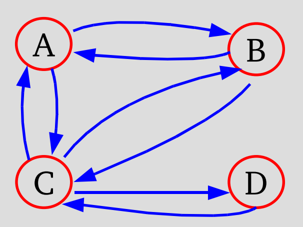
Graphs: Definitions and Basic Properties

The edges may be straight or curved and should either connect one vertex to another or a vertex to itself, as shown below.



Graphs — Definitions/2

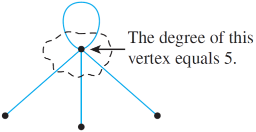
- The general definition of directed graph is similar to the definition of graph, except that one associates an ordered pair of vertices with each edge instead of a set of vertices.
- Thus each edge of a directed graph can be drawn as an arrow going from the first vertex to the second vertex of the ordered pair.



Graphs — Degree

- Degree of a vertex v: denoted as $\mathbf{deg}(v)$, is the number of edges incident on v, with a loop counted twice.
- Total degree of a graph: Sum of the degree of all vertices.
- The total degree of G is twice the number of edges of G:
$$\mathbf{deg}(G) = 2e.$$

The degree of a vertex can be obtained from the drawing of a graph by counting how many end segments of edges are incident on the vertex.

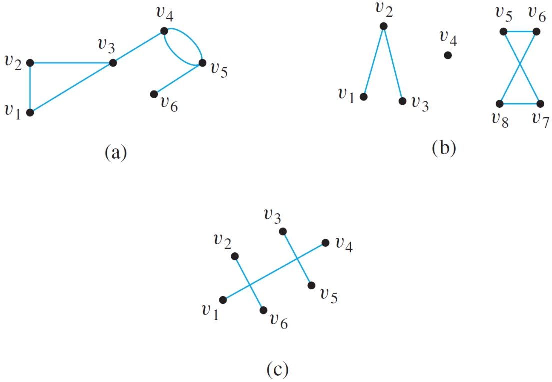


Paths and Connectivity

- Path: Denoted as P, is a sequence of vertices, $v_1, v_2, \ldots v_k$ such that v_{i+1} is adjacent to v_i for $i = 1, \ldots, k-1$;
  - P is called a path from v_1 to v_k.
- Simple Path: A path with no repeated vertices.
- Cycle: As a simple path, but $k > 2$ and $v_1 = v_k$.
- Connected Graph: Any two vertices are connected by some path.
- Distance between vertices u, v: The number of edges in the shortest path from u to v.

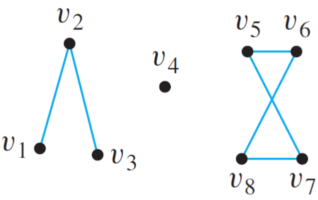
Example 3 – *Connected and Disconnected Graphs*

Which of the following graphs are connected?

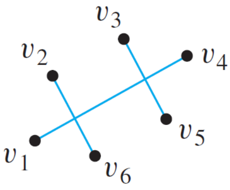


Connected Components

- A Connected Component of a graph is a connected subgraph of the largest possible size.
- The graphs in (b) and (c) are both made up of three pieces, each of which is itself a connected graph.



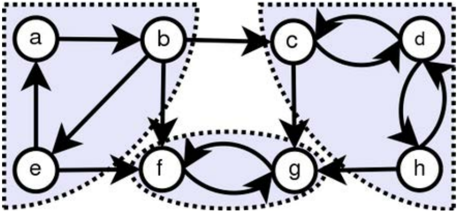
(b)



(c)

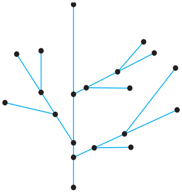
Connectivity in Directed Graphs

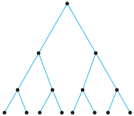
- Strong Connectivity: Defined for Directed Graphs: For any two vertices u, v, there is a path form u to v (and thus, also from v to u).
- The Strongly Connected Components (or the diconnected components) of a Directed Graph form a partition into subgraphs that are themselves strongly connected.



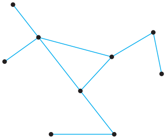
Trees

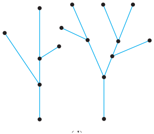
Tree: A connected graph that does not contain any cycle.





The following are not trees (the last is a forest):





Characterising Trees

- If a Tree has only one or two vertices then they are both terminal vertices;
- If a Tree has at least three vertices then it has at least one vertex of degree 1, called terminal vertex or leaf, and a vertex with degree greater than 1, called internal vertex.

The following are notable properties of Trees:

1. A Tree with N vertices has $N - 1$ edges;
2. If G is a connected graph with N vertices and $N - 1$ edges then G is a tree.

Thank You!