Parthood relations, between semantics and ontology

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Aims and methods

- Formal analysis of parthood concepts
  the engine is part of the car, the door has a handle, the front of the car, a cow from the herd, this syrup is made of water and sugar…
- Formal (lexical) semantics
  represent meaning in logical formulas
- Formal ontology
  analyze the nature and the structure of 'reality' in logical theories
- Logic
  - to fully characterize the semantics (model theory)
  - to support inference

Plan

- Mereology
- The transitivity puzzle
- Several parthoods
- Functional parthood
- Functional dependence
- Transitivities solved

Mereology

- Formal relation of Parthood
- Generic relation: search for invariants across domains
- One of the pillars of formal ontology
  – Basic structure for concepts (unary predicates): taxonomic “is-a” relation (→)
  – Basic structure for entities: parthood (P) *(my hand,my body), (today,this week), (this room,the university), (one student,the class), (mereology,formal ontology)…*

A Bit of History

- Lesniewski 1927-1931, *On the Foundations of Mathematics*
- Tarski 1935, link with Boolean algebra
- Leonard & Goodman 1940, the "calculus of individuals", a first-order theory
- Contemporary studies: Peter Simons (1986), Achille Varzi (1996)
- All 'ontologies' use a parthood relation, in the best cases fully specified with respect to Simons's work

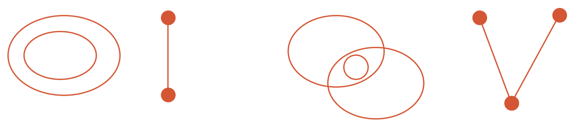
Basic Mereology

- P, partial order:

(M1)	$\forall x\ P(x,x)$	Reflexivity
(M2)	$\forall xyz\ ((P(x,y) \wedge P(y,z)) \rightarrow P(x,z))$	Transitivity
(M3)	$\forall xy\ ((P(x,y) \wedge P(y,x)) \rightarrow x=y)$	Antisymmetry

- Definitions

$PP(x,y) \equiv_{df} P(x,y) \wedge \neg P(y,x)$	Proper part
$O(x,y) \equiv_{df} \exists z\ (P(z,x) \wedge P(z,y))$	Overlap



Merologies

- Partial order can be many things!
  - weights, numbers, instants, preferences…
- Further specifications and *varieties* of mereologies
  - Supplementation, Extensionality
  - Sums, products, complements
  - General fusion
  - Atomicity / infinite divisibility
  - Mereotopology, mereogeometry…

Transitivity?

- *This horn is part of Margot the cow*, *Margot is part of my herd*, but *This horn is not part of my herd*
- Winston, Chaffin, Hermann 1987
  - Not one, but several parthood relations:
    - **component-integral object** (*handle-door, engine-car*)
    - **member-collection** (*tree-forest, cow-herd*)
    - **portion-mass** (*slice-pie*)
    - **stuff-object** (*steel-bike, sugar-bottle of syrup*)
    - **feature-activity** (*paying-shopping*)
    - **place-area** (*oasis-desert*)
  - Each relation is transitive, intransitivities occur with mixed parthoods

Transitivity?

- Lyons 1977, Cruse 1986
  - *The jacket has a sleeve,*
    The sleeve has a cuff, and
    The jacket has a cuff
  - *The house has a door*,
    The door has a handle, but
    The house does not have a handle
- Also Simons, Johansson, Varzi…
- $\varphi\text{-}Part(x,y) \equiv_{df} P(x,y) \wedge \varphi(x,y)$
- What is φ?

Ontological grounds for Parthood variety

- Number
  - Singular entities / collections
  - Also linguistically relevant (plurals, count/mass distinction)
  - ➢ **Member-collection** (tree-forest, cow-herd)
  - ➢ **Subcollection-collection** (Benelux-EU)
- Categories
  - Essence, Identity, Constitution…
  - Fundamental categories ('top-level'):
    physical object, amount of matter, substance, perdurant
  - Again, linguistic counterpart with the count/mass distinction
  - ➢ **Portion-whole** (slice-pie)
  - ➢ **Substance-whole** (steel-bike, sugar-bottle of syrup)
  - ➢ Event parthood? Location parthood??

Missing relations

- **Piece-whole** (*front-car, left part-table*…)
  Internal localization nouns
  - ➢ Simplest relation, mereotopology suffices

- **Component-integral whole** / functional part (*handle-door, cuff-sleeve, engine-car, cell-heart, electron-atom*…)
  The main parthood relation!!

Functional parthood

- Function is crucial
  - The part "plays a role" within the whole
  - The whole (and the part) is / are "integral entities"
- Which notion of function?
  - Functional parthood equally applies to artefacts, organisms, inanimate natural entities
  - Functional parthood equally applies to flawed or broken artefacts and organisms
- Function is a very complex matter. No off-the-shelf solution.

Function and types

- Normativity
  - need to refer to some “ideal entity”, either a prototype or a universal
- Parthood expressions are sensitive to descriptions
  - *La testa del letto / ??la testa del mobile*
  - *The house has a door, the door has a door handle, and the house has a door handle.*
- ➢ Functional parthood refers to lexical types

The idea: functional dependence

- The cuff is part of the sleeve

  All cuffs “functioning-as-a-cuff” require some related sleeve “functioning-as-a-sleeve”
- The engine is part of the car

  All engines “functioning-as-a-engine” require some related machine “functioning-as-a-machine”, for instance, a car “functioning-as-a-car”
- The wall is part of the house

  All entities “functioning-as-a-house” require some related entity “functioning-as-a-wall”

Type reification

- Sorted modal first-order logic
  - Particulars: *x, y, z*
  - Lexical types: *X, Y, Z*
    concepts associated to a public definition

- Classification
  CF(x, X, t)
  x exists at time t and satisfies at time *t* all the conditions required by the definition of *X* for it to be a *X*.

Dependence

- Specific dependence
  $D(x,y) \equiv \Box \forall t (E(x,t) \rightarrow E(y,t)) \wedge$
  $\Diamond \exists t\, E(x,t) \wedge \neg \Box \forall t\, E(y,t)$
  the existence of *x* implies the existence of *y*

- Generic dependence
  $GD(X,Y) \equiv \Box \forall x,t\, (CF(x,X,t) \rightarrow \exists y\, CF(y,Y,t))$
  $\wedge \Diamond \exists x,t\, CF(x,X,t) \wedge \neg \Box \forall t\, \exists y\, CF(y,Y,t)$
  all *X*s imply the existence of some *Y*

Functional dependence

$GFD(X,Y) \equiv$
$\Box \forall x,t\ ((CF(x,X,t) \wedge Functioning(x,X,t)) \rightarrow$
$\exists y\ (CF(y,Y,t) \wedge Functioning(y,Y,t))) \wedge$
$\Diamond \exists x,t\ (CF(x,X,t) \wedge Functioning(x,X,t)) \wedge$
$\neg \Box \forall t\ \exists y\ (CF(y,Y,t) \wedge Functioning(y,Y,t))$

all *X*s, when functioning as a X, imply the existence of some *Y* that is functioning as a *Y*

Individual functional dependence

- $IFD(x,X,y,Y,t) \equiv$
  $GFD(X,Y) \wedge CF(x,X,t) \wedge CF(y,Y,t) \wedge$
  $\forall t'\ ((P(t',t) \wedge Functioning(x,X,t')) \rightarrow$
  $Functioning(y,Y,t'))$
- Indirect individual functional dependence
  $IIFD(x,X,y,Y,t) \equiv$
  $CF(y,Y,t) \wedge \exists Z\ (Subclass(Y,Z) \wedge IFD(x,X,y,Z,t))$

Axioms and theorems

- □ is S5
- (*Functioning*(*x,X,t*) ∧ *P*(*t'*,*t*)) → *Functioning*(*x,X,t'*)
- (*Functioning*(*x,X,t*)∧ *Subclass*(*X,Y*)) → *Functioning*(*x,Y,t*)

(T1) (*GFD*(*X,Y*) ∧ *GFD*(*Y,Z*)) → *GFD*(*X,Z*)

(T2) *GFD*(*X,Y*) ∧ *Subclass*(*Y,Z*) ∧ ¬□∀*t*∃*x* (*CF*(*x,Z,t*)) → *GFD*(*X,Z*)

(T3) *IFD*(*x,X,y,Y,t*) → *IIFD*(*x,X,y,Y,t*)

Defining Functional Parthood

FP-D1(*x,X,y,Y,t*) ≡ *PP*(*x,y,t*) ∧ *Obj*(*x*) ∧ *Obj*(*y*) ∧ *IFD*(*x,X,y,Y,t*)

FP-D2(*x,X,y,Y,t*) ≡ *PP*(*x,y,t*) ∧ *Obj*(*x*) ∧ *Obj*(*y*) ∧ *IFD*(*y,Y,x,X,t*)

FP-I1(*x,X,y,Y,t*) ≡ *PP*(*x,y,t*) ∧ *Obj*(*x*) ∧ *Obj*(*y*) ∧ *IIFD*(*x,X,y,Y,t*)

FP-I2(*x,X,y,Y,t*) ≡ *PP*(*x,y,t*) ∧ *Obj*(*x*) ∧ *Obj*(*y*) ∧ *IIFD*(*y,Y,x,X,t*)

Transitivity

Theorems:

(T4) $(FP\text{-}D1(x,X,y,Y,t) \wedge FP\text{-}D1(y,Y,z,Z,t)) \rightarrow FP\text{-}D1(x,X,z,Z,t)$

(T5) $(FP\text{-}D2(x,X,y,Y,t) \wedge FP\text{-}D2(y,Y,z,Z,t)) \rightarrow FP\text{-}D2(x,X,z,Z,t)$

(T6) $(FP\text{-}D1(x,X,y,Y,t) \wedge FP\text{-}I1(y,Y,z,Z,t)) \rightarrow FP\text{-}I1(x,X,z,Z,t)$

(T7) $(FP\text{-}I2(x,X,y,Y,t) \wedge FP\text{-}D2(y,Y,z,Z,t)) \rightarrow FP\text{-}I2(x,X,z,Z,t)$

Intransitivities

Non-theorems:

(i) $(FP\text{-}I1(x,X,y,Y,t) \wedge FP\text{-}D1(y,Y,z,Z,t)) \rightarrow (FP\text{-}I1(x,X,z,Z,t) \vee FP\text{-}I2(x,X,z,Z,t))$

(ii) $(FP\text{-}D2(x,X,y,Y,t) \wedge FP\text{-}I2(y,Y,z,Z,t)) \rightarrow (FP\text{-}I1(x,X,z,Z,t) \vee FP\text{-}I2(x,X,z,Z,t))$

(iii) $(FP\text{-}D1(x,X,y,Y,t) \wedge FP\text{-}D2(y,Y,z,Z,t)) \rightarrow (FP\text{-}I1(x,X,z,Z,t) \vee FP\text{-}I2(x,X,z,Z,t))$

(iv) $(FP\text{-}D2(x,X,y,Y,t) \wedge FP\text{-}D1(y,Y,z,Z,t)) \rightarrow (FP\text{-}I1(x,X,z,Z,t) \vee FP\text{-}I2(x,X,z,Z,t))$

Puzzles solved

My hand is part of my arm,
My arm is part of my body, and
My hand is part of my body

(T4) (*FP-D1*(*x,Hand,y,Arm,t*) ∧
FP-D1(*y,Arm,z,Body,t*)) →
FP-D1(*x,Hand,z,Body,t*)

Puzzles solved

The jacket has a sleeve,
The sleeve has a cuff, and
The jacket has a cuff

(T6) (*FP-D1*(*x,Cuff,y,Sleeve,t*) ∧
FP-I1(*y,Sleeve,z,Jacket,t*)) →
FP-I1(*x,Cuff,z,Jacket,t*)

Subclass(*Jacket, Garment*)

Puzzles solved

This electron is part of this atom,
This atom is part of this molecule, and
This electron is part of this molecule

(T5) (*FP-D2*(*x*,*Electron*,*y*,*Atom*,*t*) ∧
FP-D2(*y*,*Atom*,*z*,*Molecule*,*t*)) →
FP-D2(*x*,*Electron*,*z*,*Molecule*,*t*)

Puzzles solved

This brick is part of this wall,
This wall is part of the house, and
This brick is part of the house

(T7) (*FP-I2*(*x*,*Brick*,*y*,*Wall*,*t*) ∧
FP-D2(*y*,*Wall*,*z*,*House*,*t*)) →
FP-I2(*x*,*Brick*,*z*,*House*,*t*)

Subclass(*Brick*, *Building-material*)

Puzzles solved

The house has a door,
The door has a handle, but
The house does not have a handle

(i') (*FP-I1*(*x*,*Handle*,*y*,*Door*,*t*) ∧
FP-I1(*y*,*Door*,*z*,*House*,*t*)) →
(*FP-I1*(*x*,*Handle*,*z*,*House*,*t*) ∨
FP-I2(*x*,*Handle*,*z*,*House*,*t*))

- *Subclass*(*Door, Objects-that-can-be-moved-or-used-by-hand*)
- *Subclass*(*House, Building-room-cupboard-vehicle*)

(T3) *IFD*(*x*,*X*,*y*,*Y*,*t*) → *IIFD*(*x*,*X*,*y*,*Y*,*t*)

Puzzles solved

The nucleus is part of this cell,
This cell is part of the heart, but
The nucleus is not part of the heart

(iii) (*FP-D1*(*x*,*Nucleus*,*y*,*Cell*,*t*) ∧
FP-D2(*y*,*Cell*,*z*,*Heart*,*t*)) →
(*FP-I1*(*x*,*Nucleus*,*z*,*Heart*,*t*) ∨
FP-I2(*x*,*Nucleus*,*z*,*Heart*,*t*))

Puzzles solved

This cell is part of the heart,
The heart is part of the circulatory system, but
This cell is not part of the circulatory system

(iv) (*FP-D2*(*x,Cell,y,Heart,t*) ∧
FP-D1(*y,Heart,z, CSyst,t*)) →
(*FP-I1*(*x, Cell,z,CSyst,t*) ∨ *FP-I2*(*x, Cell,z,CSyst,t*))

(ii) (*FP-D2*(*x,Cell,y,Heart,t*) ∧
FP-I2(*y,Heart,z,CSyst,t*)) →
(*FP-I1*(*x,Cell,z,CSyst,t*)∨*FP-I2*(*x,Cell,z,CSyst,t*))

- *Subclass*(*Heart, Pump*)

Linguistic evidence

- Determinative compound nouns
  - Parts determined by wholes
    - *a car engine*, *a door handle, a bag handle*: FP-I1
  - Wholes determined by parts
    - *a brick wall, a motor boat, a sail boat*: FP-I2
  - FP-D1: **an arm hand*, **a sleeve cuff*, ??*a machine engine*
  - FP-D2: **a hand arm*, **a wall house*, **a motor automobile*
- Direct functional dependence implies non-informativeness (Grice's maxim of quantity)

Conclusion

- Some ontology is required to do semantics

but in addition,

- Ontological tools help doing Semantics

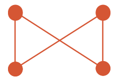
- Ontology in isolation from language and cognition is an utopy

Additional slides

Extensional Mereology

- Supplementation

(M4) $\forall xy\ (PP(x,y) \rightarrow \exists z\ (P(z,y) \wedge \neg O(z,x)))$
Weak supplementation



(M5) $\forall xy\ (\neg P(y,x) \rightarrow \exists z\ (P(z,y) \wedge \neg O(z,x)))$
Strong supplementation

- Extensionality
  (E1) $\forall xy\ ((\exists z\ PP(z,x) \wedge \forall z\ (PP(z,x) \leftrightarrow PP(z,y))) \rightarrow x=y)$
  (E2) $\forall xy\ (\forall z\ (O(z,x) \leftrightarrow O(z,y)) \rightarrow x=y)$
- Theorems
  - M+(M5) $\vdash$ (M4)
  - M+(M5) $\vdash$ (E1); M+(M5) $\vdash$ (E2)

General Extensional Mereology

- General fusion
  (M6) $\exists x\ \phi(x) \rightarrow \exists z\ \forall y\ (O(y,z) \leftrightarrow \exists x\ (\phi(x) \wedge O(y,x)))$
  - Axiom schema, useful for infinite domains
  - unicity guaranteed by (E2), z is noted $\sigma x\ \phi(x)$
- Russell's description operator ι often used
  - $\sigma x\ \phi(x) = \iota z\ \forall y\ (O(y,z) \leftrightarrow \exists x\ (\phi(x) \wedge O(y,x)))$
- Sum, product and complement as fusions
  - $x+y = \sigma z\ (P(z,x) \vee P(z,y))$
  - $x \bullet y = \sigma z\ (P(z,x) \wedge P(z,y))$
  - $\sim x = \sigma z\ (\neg O(z,x))$
- Universe
  - $U = \sigma x\ P(x,x)$
- NB: no null element!

Concepts, Descriptions,Classification

(A1) $DS(x) \rightarrow NASO(x)$
(A2) $CN(x) \rightarrow NASO(x)$
(A3) $DS(x) \rightarrow \neg CN(x)$
(A4) $US(x, y) \rightarrow (CN(x) \wedge DS(y))$
(A5) $DF(x, y) \rightarrow US(x, y)$
(A6) $CN(x) \rightarrow \exists y(DF(x, y))$
(A7) $DS(x) \rightarrow \exists y(US(y, x))$
(A8) $(DF(x, y) \wedge DF(x, z)) \rightarrow y = z$

Concepts, Descriptions,Classification

(A9) $US(x, y) \rightarrow (PRE(y, t) \rightarrow PRE(x, t))$
(A10) $DF(x, y) \rightarrow (PRE(x, t) \rightarrow PRE(y, t))$
(A11) $CF(x, y, t) \rightarrow (ED(x) \wedge CN(y) \wedge TL(t))$
(A12) $CF(x, y, t) \rightarrow PRE(x, t)$
(A13) $(CF(x, y, t) \wedge DS(x)) \rightarrow \neg US(y, x)$
(A14) $CF(x, y, t) \rightarrow \neg CF(y, x, t)$
(A15) $(CF(x, y, t) \wedge CF(y, z, t)) \rightarrow \neg CF(x, z, t)$
(D-sub) $Subclass(x,y) \equiv CF(z, x, t) \rightarrow CF(z, y, t)$