Single agent or multiple agents

- Many domains are characterized by multiple agents rather than a single agent.
- Game theory studies what agents should do in a multi-agent setting.
- Agents can be cooperative, competitive or somewhere in between.
- Agents that are strategic can't be modeled as nature.

Multi-agent framework

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- Each agent's value depends on the outcome.

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- Two person, competitive (zero sum) $\Longrightarrow$ minimax.

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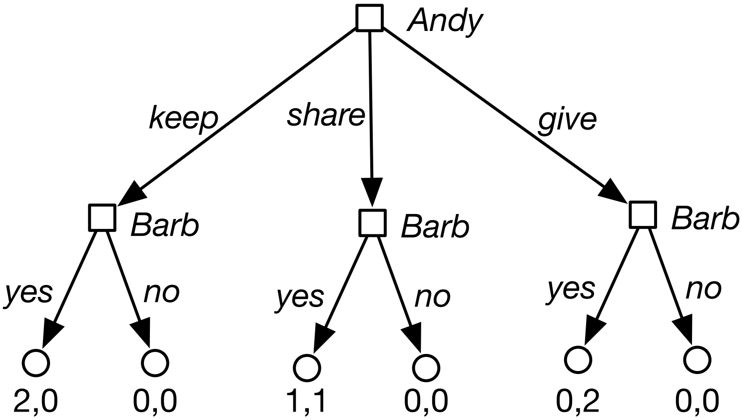
- a utility function $utility(\sigma, i)$ for action profile σ and agent $i \in I$, gives the expected utility for agent i when all agents follow action profile σ.

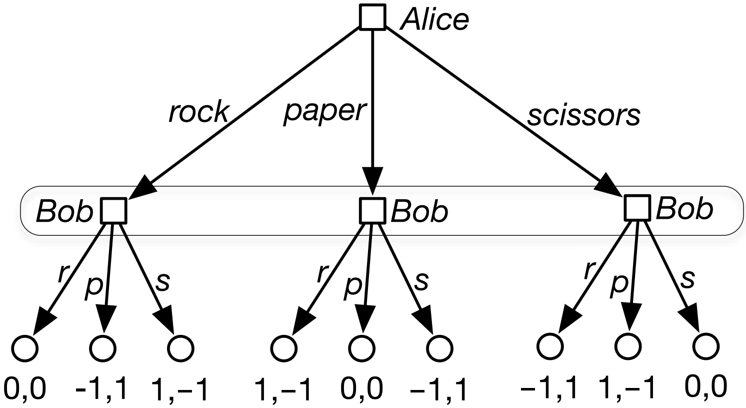
Rock-Paper-Scissors

		Bob		
		rock	*paper*	*scissors*
Alice	*rock*	$0, 0$	$-1, 1$	$1, -1$
	paper	$1, -1$	$0, 0$	$-1, 1$
	scissors	$-1, 1$	$1, -1$	0,0

Extensive Form of a Game

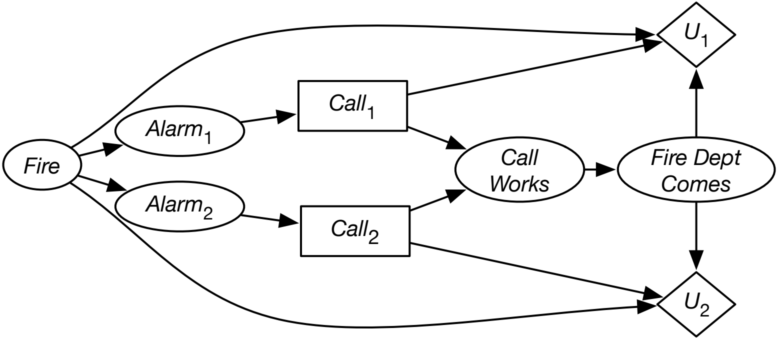


Extensive Form of an imperfect-information Game



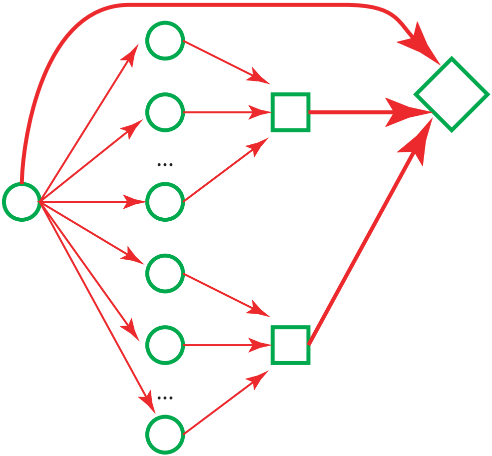
Bob cannot distinguish the nodes in an information set.

Multiagent Decision Networks



Value node for each agent.
Each decision node is owned by an agent.
Utility for each agent.

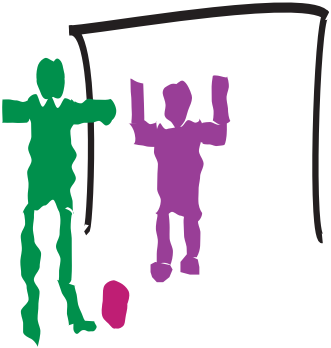
Multiple Agents, shared value



Complexity of Multi-agent decision theory

- It can be exponentially harder to find optimal multi-agent policy even with a shared values.
- Why? Because dynamic programming doesn't work:
  - If a decision node has n binary parents, dynamic programming lets us solve 2^n decision problems.
  - This is much better than d^{2^n} policies (where d is the number of decision alternatives).
- Multiple agents with shared values is equivalent to having a single forgetful agent.

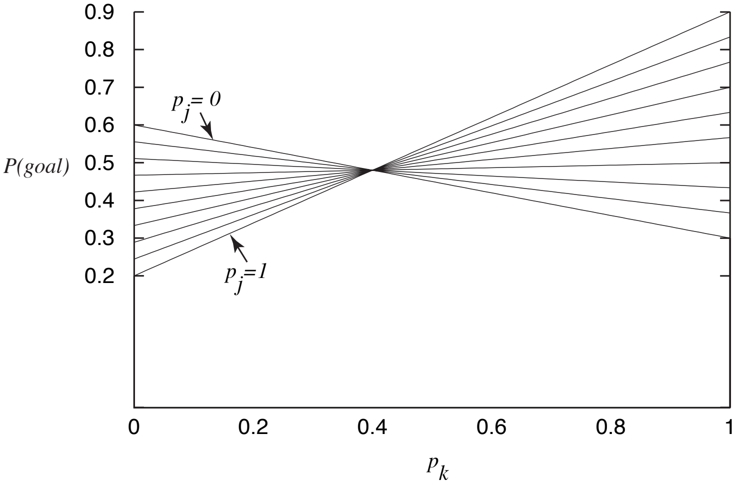
Partial Observability and Competition



		goalie	
		left	right
kicker	left	0.6	0.2
	right	0.3	0.9

Probability of a goal.

Stochastic Policies



Strategy Profiles

- Assume a general *n*-player game,
- A strategy for an agent is a probability distribution over the actions for this agent.
- A strategy profile is an assignment of a strategy to each agent.
- A strategy profile σ has a utility for each agent. Let $utility(\sigma, i)$ be the utility of strategy profile σ for agent i.
- If σ is a strategy profile:
  σ_i is the strategy of agent i in σ,
  σ_{-i} is the set of strategies of the other agents.
  Thus σ is $\sigma_i \sigma_{-i}$

Nash Equilibria

- σ_i is a best response to σ_{-i} if for all other strategies σ'_i for agent i,

$$utility(\sigma_i \sigma_{-i}, i) \geq utility(\sigma'_i \sigma_{-i}, i).$$

- A strategy profile σ is a Nash equilibrium if for each agent i, strategy σ_i is a best response to σ_{-i}. That is, a Nash equilibrium is a strategy profile such that no agent can be better by unilaterally deviating from that profile.
- Theorem [Nash, 1950] Every finite game has at least one Nash equilibrium.

Multiple Equilibria

Hawk-Dove Game:

		Agent 2	
		dove	hawk
Agent 1	dove	R/2,R/2	0,R
	hawk	R,0	-D,-D

D and R are both positive with $D >> R$.

Coordination

Just because you know the Nash equilibria doesn't mean you know what to do:

		Agent 2	
		shopping	football
Agent 1	shopping	2,1	0,0
	football	0,0	1,2

Prisoner's Dilemma

Two strangers are in a game show. They each have the choice:

- Take $100 for yourself
- Give $1000 to the other player

This can be depicted as the playoff matrix:

		Player 2	
		take	give
Player 1	take	100,100	1100,0
	give	0,1100	1000,1000

Tragedy of the Commons

Example:

- There are 100 agents.
- There is an common environment that is shared amongst all agents. Each agent has 1/100 of the shared environment.
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- If every agent does the action the total payoff is $1000 - 10000 = -9000$

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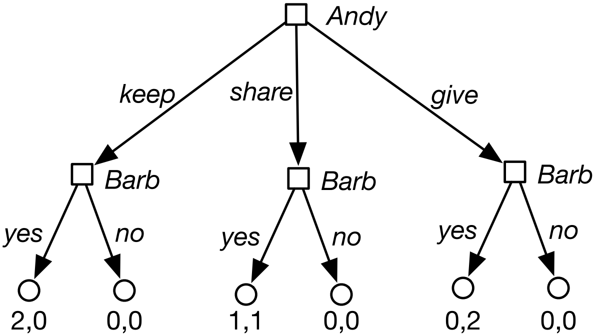
What are the Nash equilibria of:



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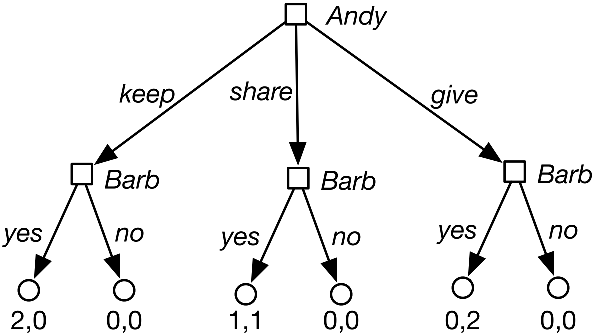

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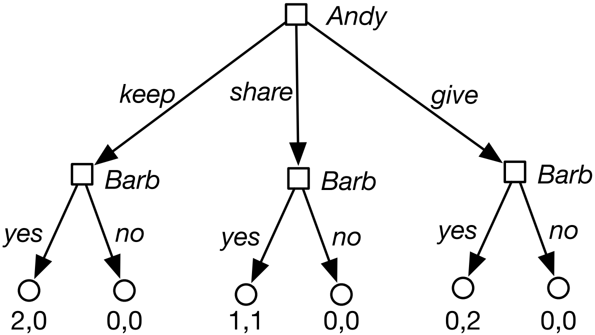

What if the 2,0 payoff was 1.9,0.1?
Should Barb be rational / predictable?

Computing Nash Equilibria

To compute a Nash equilibria for a game in strategic form:

- Eliminate dominated strategies
- Determine which actions will have non-zero probabilities. This is the support set.
- Determine the probability for the actions in the support set

Eliminating Dominated Strategies

Agent 2

		d_2	e_2	f_2
Agent 1	a_1	3,5	5,1	1,2
	b_1	1,1	2,9	6,4
	c_1	2,6	4,7	0,8

Computing probabilities in randomized strategies

Given a support set:

- Why would an agent will randomize between actions $a_1 \ldots a_k$?

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Search over support sets to find a Nash equilibrium

Learning to Coordinate

- Each agent maintains $P[A]$ a probability distribution over actions.
- Each agent maintains $Q[A]$ an estimate of value of doing A given policy of other agents.
- Repeat:
  - select action a using distribution P,
  - do a and observe payoff
  - update Q:

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  - select action a using distribution P,
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  - update Q: $Q[a] \leftarrow Q[a] + \alpha(payoff - Q[a])$
  - incremented probability of best action by δ.
  - decremented probability of other actions