Belief Revision in Description Logic

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(mostly joint work with Márcio Ribeiro)

Bolzano, FCCOD, 2014
Motivation

- Study the dynamics of ontologies, specially “OWL-like” DL ontologies.
- AGM Belief Revision deals with the problem of adding/removing information in a consistent way.
- AGM is most commonly applied to propositional classical logic and cannot be directly used with DLs.
- How can we adapt AGM so that it can deal with interesting DLs?
In this work

- Show reasons why AGM fails to apply to DLs.
- Adapt Contraction (easy).
- Adapt Revision (less easy).
Outline of the Talk

1. Motivation
2. The AGM paradigm
3. Contraction and DLs
4. Revision and DLs
5. Conclusions and Future Work
AGM Belief Revision

Three operations defined to deal with knowledge base dynamics:

- **Expansion** - adding knowledge (possibly inconsistent)
AGM Belief Revision

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- **Contraction** - removing knowledge
AGM Belief Revision

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Three operations defined to deal with knowledge base dynamics:

- **Expansion** - adding knowledge (possibly inconsistent)
- **Contraction** - removing knowledge
- **Revision** - adding knowledge consistently

Revision usually defined in terms of contraction:

\[ K \ast \alpha = (K - \neg \alpha) + \alpha \]
AGM Theory

For contraction and revision:

- **Rationality Postulates**
AGM Theory

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- **Construction**
AGM Theory

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**AGM Assumptions:** Tarskian, Compact, Deduction Theorem, Supra classical.
AGM contraction

(closure) \( K - \alpha = \text{Cn}(K - \alpha) \)

(success) If \( \alpha \notin \text{Cn}(\emptyset) \) then \( \alpha \notin K - \alpha \)

(inclusion) \( K - \alpha \subseteq K \)

(vacuity) If \( \alpha \notin K \) then \( K - \alpha = K \)

(recovery) \( K \subseteq K - \alpha + \alpha \)

(extensionality) If \( \text{Cn}(\alpha) = \text{Cn}(\beta) \) then \( K - \alpha = K - \beta \)
Applying to DL

- AGM cannot be applied to every logic. In particular it can not be applied to SHIF and SHOIN. [Flouris 2006]
- Solution: substitute recovery by relevance

  *(relevance)* If \( \beta \in K \setminus (K - \alpha) \), then there is \( K' \) s. t. \( K - \alpha \subseteq K' \subseteq K \) and \( \alpha \not\in \text{Cn}(K') \), but \( \alpha \in \text{Cn}(K' \cup \{\beta\}) \).

- Good property: AGM assumptions + 5 postulates \( \Rightarrow \) recovery and relevance are equivalent.
Results - contraction

Representation Theorem [RW06]

If the underlying logic is tarskian and compact, partial meet contraction is equivalent to the AGM postulates with relevance instead of recovery.
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Results - contraction

Representation Theorem [RW06]

If the underlying logic is tarskian and compact, partial meet contraction is equivalent to the AGM postulates with relevance instead of recovery.

Can we do the same for revision???
AGM Revision

(closure) \( K \star \alpha = \text{Cn}(K \star \alpha) \)

(success) \( \alpha \in K \star a \)

(inclusion) \( K \star \alpha \subseteq K + \alpha \)

(vacuity) If \( K + \alpha \) is consistent then \( K \star \alpha = K + \alpha \)

(consistency) If \( \alpha \) is consistent then \( K \star \alpha \) is consistent.

(extensionality) If \( \text{Cn}(\alpha) = \text{Cn}(\beta) \) then \( K \star \alpha = K \star \beta \)
Applying to DL

- Problem: no negation $\Rightarrow$ no Levi identity.
- Solution: Direct constructions.
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Definition

$X \in K \downarrow \alpha$ iff $X$ maximal subset of $K$ such that $X \cup \{\alpha\}$ is consistent.
Applying to DL

- Problem: no negation ⇒ no Levi identity.
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**Definition**

\[ X \in K \downarrow \alpha \text{ iff } X \text{ maximal subset of } K \text{ such that } X \cup \{\alpha\} \text{ is consistent.} \]

**Definition (Revision without negation)**

\[ K \ast_{\gamma} \alpha = \bigcap \gamma(K \downarrow \alpha) + \alpha \]

where \( \gamma \) selects at least one element of \( K \downarrow \alpha \).
Properties

1. Inconsistent explosion: Whenever $K$ is inconsistent, then for all formulas $\alpha$, $\alpha \in Cn(K)$

2. Distributivity: For all sets of formulas $X$, $Y$ and $W$, $Cn(X \cup (Cn(Y) \cap Cn(W))) = Cn(X \cup Y) \cap Cn(X \cup W)$
Motivation

The AGM paradigm

Contraction and DLs

Revision and DLs

Conclusions and Future Work

Properties

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Representation Theorem [RW09]

If the logic is monotonic and compact and satisfies Inconsistent explosion and Distributivity, then $*$ is a revision without negation iff it satisfies closure, success, inclusion, consistency, relevance and uniformity.

(uniformity) If for all $K' \subseteq K$, $K' \cup \{\alpha\}$ is inconsistent iff $K' \cup \{\beta\}$ is inconsistent then $K \cap K * \alpha = K \cap K * \beta$
Which Logics Satisfy Distributivity?

- Classical logic does.
- But what about DLs?
  - $\mathcal{ALC}$ does not.
  - $\mathcal{ALC}$ with empty $\mathcal{A}$Box does.
  - not many more...
New characterisation

Representation Theorem [RW14]

If the logic is monotonic and compact and satisfies Inconsistent explosion and Distributivity, then \( * \) is a revision without negation iff it satisfies closure, success, strong inclusion, consistency, relevance and uniformity.

\[(\text{strong inclusion}) \quad K \ast \alpha \subseteq (K \cap K \ast \alpha) + \alpha\]

In classical logics this postulate is equivalent to inclusion.
What was done

- Adapted AGM to DLs
  - Contraction - only 1 postulate changed
  - Revision - Contraction and postulates
- Provided representation results.
What we want to do

- Study other forms of revision for DLs avoiding negation.
- Apply the solutions to other fragments
  - Horn
  - ???