Temporal Dynamic Description Logic

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Temporal description logic (TDL)

• For capturing temporal aspects of concepts in ontologies.

\[
\neg \text{Doctor} \sqcap \Diamond \text{Doctor} \sqsubseteq \\
\Diamond (\text{PHDStudent} \sqcap \neg \text{Doctor} \sqcap (\text{PHDStudent} \mathbin{U} \text{Doctor}))
\]

\[
\text{PHDStudent} \sqsubseteq \exists \text{hasSup}.\text{Doctor}
\]
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\[ \neg \text{Doctor} \sqcap \Diamond \text{Doctor} \sqsubseteq \Diamond (\text{PHDStudent} \sqcap \neg \text{Doctor} \sqcap (\text{PHDStudent} \cup \text{Doctor})) \]

\[ \text{PHDStudent} \sqsubseteq \exists \text{hasSup} \cdot \text{Doctor} \]

- Two-dimensional logics [GKWZ03]
  - Temporal description logic
  - Dynamic description logic
  - ……
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- Two-dimensional logics [GKWZ03]
  - Temporal description logic
  - Dynamic description logic
  - ......
Different temporal extensions of DLs

- Explicit notion of time or implicit time
- Interval-based notion of time or point-based time
  - External representation of time or internal representation
- Linear time or branching time
Different temporal extensions

• Varying DL component: DL-Lite, EL, ALC, SHOIQ, …

• Different choice for applying temporal operators: concepts, TBox axioms, ABox assertions
  – \(\neg\)Doctor \(\sqcap\) Diamond Doctor \(\sqsubseteq\) Diamond (PHDStudent \(\cup\) Doctor)
  – Diamond (Citizen \(\sqsubseteq\) HASVote)
  – PHDStudent(Jack) \(\land\) Diamond (PHDStudent(Jack) \(\cup\) Doctor(Jack))

• Additional constraints on concepts and roles: rigid concepts, rigid roles

• interpretation domains: expanding, constant

• ……
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Dozens of combinations!

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Reasoning about actions

- Representation and Reasoning about Actions

- Situation Calculus [Mcc63]

- John McCarthy
  - father of AI, 1956
  - Winner of Turing Award, 1971
Action Formalisms

Based on propositional logics:
- Based on PDL [GL95]
- Based on LTL [CGV02]

Based on DL?
- Gap?

Based on first- or higher-order logics:
- Situation Calculus
- Fluent Calculus
- ........
**Action Formalisms**

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- based on PDL [GL95]
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- Situation Calculus
- Fluent Calculus

Action formalism based on DL [BLM05]

Gap ?
DL-Based Action Formalisms

- Background knowledge: RBox, TBox
- States: ABoxes
- Action: $\alpha = (\text{pre}, \text{occ}, \text{post})$
  - $\text{pre}$: ABox assertions
  - $\text{occ}$: primitive literals
  - $\text{post}$: set of conditional post-conditions, $\phi/\psi$

- Update ABox after the execution of actions.
Extension of the DL-based action formalism

Basic idea: construct more powerful formalism, action theory + description logic + dynamic logic

• Background knowledge: RBox, TBox
• Atomic actions: come from Baader et al.’s formalism
  \( \alpha \equiv (\text{pre, occ, post}) \)
• Complex actions:
  \( \pi, \pi' ::= \alpha \mid \phi ? \mid \pi \cup \pi' \mid \pi ; \pi' \mid \pi^* \)
• Formulas:
  \( \phi, \psi ::= C(p) \mid R(p,q) \mid <\pi>\phi \mid [\pi]\phi \mid \neg \phi \mid \phi \lor \psi \mid \phi \land \psi \)

• Dynamic description logic DDL(\( X@ \))
  \( X: \) DLs ranging from ALCO to ALCHOIQ, \( X@: \) extension of \( X \) with the @ constructor.
Features of DDL(X@) (1/3)

(1) Complex actions can be constructed

- **TBox:**
  
  \[
  \text{Customer} \equiv \text{Person} \sqcap \exists \text{holds.CreditCard} \\
  \text{VIPcustomer} \equiv \text{Customer} \sqcap \geq 10 \text{boughr.}(\text{Book} \sqcup \text{CD})
  \]

- **Atomic Actions:**
  
  \[
  \text{buybook}(a,b) \equiv (\{\text{Customer}(a), \text{Book}(b)\}, \{\}; \\
  \{\text{Instore}(b)/\neg \text{Instore}(b), \text{Instore}(b)/\text{bought}(a,b)\})
  \]

  \[
  \text{order}(b) \equiv (\{(\text{Book} \sqcup \text{CD})(b)\}, \{\}; \\
  \{\neg \text{Instore}(b)/\text{Instore}(b)\})
  \]

- **Complex Action:**
  
  \[
  \text{VIPbuybook}(a,b) \equiv \text{VIPcustomer}(a)? ; \\
  ( (\text{Instore}(b)? ; \text{buybook}(a,b) ) \cup \\
  (\neg \text{Instore}(b)? ; \text{order}(b); \text{buybook}(a,b)) )
  \]
Features of DDL(X@) (2/3)

(2) Properties on (complex) actions can be described directly

- necessary conditions for the execution of (complex) actions
  
  `<VIPbuybook(a,b)>true \rightarrow (VIPcustomer(a) \land Book(b))`
  
  `<VIPbuybook(a,b)>true \rightarrow Instore(b)`

- results on the execution of actions
  
  `[VIPbuybook(a,b)]bought(a,b)`
  
  `[buybook(a,b)]bought(a,b)`
(3) Reasoning problems on actions be reduced to the satisfiability problem of formulas

- Executability of actions
- Projection problem
- Consistency/realizability of actions
  - whether a given action makes sense w.r.t. the knowledge base
    buybook(a1,b); buybook(a2,b)

- Satisfiability problem
  - a Tableau decision algorithm is provided.
  - the complexity upper-bound is
    - EXPSpace if $X \in \{\text{ALCO}, \text{ALCHO}, \text{ALCOQ}, \text{ALCHOQ}\}$,
    - N2EXPTime if $X \in \{\text{ALCOI}, \text{ALCHOI}, \text{ALCOIQ}, \text{ALCHOIQ}\}$. 
Temporal extension of DDL(X@)

To investigate temporal properties of actions.

**Approach:**
- the ongoing of time is embodied as the execution of atomic actions (time units)
- two temporal assertions are introduced:

\[ \phi, \psi ::= C(p) \mid R(p,q) \mid <\pi>\phi \mid [\pi]\phi \mid \neg \phi \mid \phi \lor \psi \mid E(\phi U^\pi \psi) \mid A(\phi U^\pi \psi) \]

- \( E(\phi U^\pi \psi) \): there exists some path of \( \pi \) such that “\( \phi \) until \( \psi \)” holds.
- \( A(\phi U^\pi \psi) \): “\( \phi \) until \( \psi \)” holds in any path of \( \pi \).
Temporal extension of DDL(X@)

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  \]

**EX** $\phi = \text{def} \bigvee_{\alpha \in N_A} <\alpha>\phi$

$E(\phi U^\pi \psi) = \text{def} E(\phi U^{(a_1 \cup \ldots \cup a_n)^*} \psi)$

$A(\phi U^\pi \psi) = \text{def} A(\phi U^{(a_1 \cup \ldots \cup a_n)^*} \psi)$
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$$

**E(ϕU^πψ):** there exists some path of π such that “ϕ until ψ” holds.

**A(ϕU^πψ):** “ϕ until ψ” holds in any path of π.

$$
\begin{align*}
\text{EX } \phi & \stackrel{\text{def}}{=} \bigvee_{\alpha \in N_A} <\alpha>\phi \\
E(\phi U \psi) & \stackrel{\text{def}}{=} E(\phi U^{(a_1 \cup \ldots \cup a_n)^*} \psi) \\
A(\phi U \psi) & \stackrel{\text{def}}{=} A(\phi U^{(a_1 \cup \ldots \cup a_n)^*} \psi)
\end{align*}
$$

$$
\begin{align*}
\text{EF } \phi & = \text{def } E(\text{true}\text{U}\phi) \\
\text{AF } \phi & = \text{def } A(\text{true}\text{U}\phi) \\
\text{EG } \phi & = \text{def } \neg A F(\neg \phi) \\
\text{AG } \phi & = \text{def } \neg E F(\neg \phi) \\
\text{AX } \phi & = \text{def } \neg E X(\neg \phi)
\end{align*}
$$
Description example of TDDL(X@)

- **liveness property**: good things will eventually happen.
  \[
  \text{EF}(\exists \text{bought\neg.Customer}(b)) \\
  \text{E(Instore}(b) \text{ U}^{\text{VIPbuybook}(a,b)} \neg \text{Instore}(b))
  \]

- **safety property**: bad things will never happen.
  \[
  \text{AG} \neg (\geq 2 \text{ bought\neg.Customer}(b)) \\
  \text{AG ( Instore}(b) \lor (\exists \text{bought \neg.Customer}(b))
  \]

- Reduced to satisfiability problem of formulas.
- A **Tableau decision algorithm** is provided.
Limitation of DDL(X@)/TDDL(X@)

• TBox:
  – only concept definitions, no GCIs
  – acyclic

• RBox:
  – on transitive property

• Atomic action:
  – no defined concept name occurring in the effect set *post*.

Why?
  – difficulty of ABox updating.
Difficulty of ABox updating

Example.

• TBox:
  \( \text{Trans}(R), \ A \sqsubseteq \exists R.A, \ A \cap B \sqsubseteq \bot, \ B \sqsubseteq \forall R.B \)

• ABox:
  \( A(a) \)

• Update or new information:
  \( (\exists R.B)(a) \)
### Some results on ABox update

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<td>Acyclic TBox; no defined concept names occurring in U</td>
<td>ALC~ALC QIO</td>
<td>PMA semantics &amp; only primitive concept names are counted when measuring distance.</td>
<td>LLMW06, LLMW11</td>
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<td>DL-Lite$_F$</td>
<td></td>
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<td>Both revision and update. Based on $fcl_T(A)$.</td>
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Thank you!