On the Difference between Updating a Knowledge Base and Revising it:
Survey Talk on the KR ’1991 Paper by H. Katsuno and A. Mendelzon

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Outline

1. Introduction
2. Revision and Update
   - KB Revision
   - KB Update
3. Contraction and Erasure
   - Contraction
   - Erasure
4. Unifying Revision and Update Operations: Time Aspect
KB Evolution: Revision vs. Update

A Knowledge Base (KB) can eventually become inadequate and require change.

Notation

- $\psi$ is a KB. Models, $\text{Mod}(\psi)$, of $\psi$ describe possible worlds.
- $\mu$ specifies the change to be incorporated into $\phi$.

The authors argue that change caused by adding $\mu$ to $\psi$ are mainly of two different kinds.
Possible Causes for KB Evolution

- The world described by the KB $\psi$ changes.
  $\mu$ is called update in this case. Notation: $\psi \diamond \mu$.
- New knowledge about the world becomes available.
  $\psi$ requires revision $\mu$, denoted as $\psi \circ \mu$.

<table>
<thead>
<tr>
<th>Update</th>
<th>Revision</th>
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<tbody>
<tr>
<td>Make each possible world a model of $\mu$ by some minimal change.</td>
<td>Invalidate possible worlds which are far enough from $\mu$.</td>
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**Possible Causes for KB Evolution**

- The world described by the KB $\psi$ changes. $\mu$ is called **update** in this case. Notation: $\psi \triangleright \mu$.

- New knowledge about the world becomes available. $\psi$ requires **revision** $\mu$, denoted as $\psi \circ \mu$.

**Example**

$$\psi = \text{“Joe’s GF often cancels their dates lately”}$$
$$\quad \land \text{“She is 30 minutes late now”}$$
$$\quad \land (\heartsuit: \text{“She is serious about Joe”} \lor \spadesuit: \text{“... far less than about her cat”})$$
$$\mu = \text{“Came late because she was at a movie with another guy.”}$$
$$\quad \Rightarrow \neg \heartsuit :-( $$

**Update:** doesn’t allow you to infer $\spadesuit$  
**Revision:** allows you to also infer $\spadesuit$
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4. Unifying Revision and Update Operations: Time Aspect
KB Revision

Suppose old KB is given by $\psi$ and new knowledge $\mu$, the knowledge revision operator $\circ$ is defined as:

**Definition (KB Revision)**

$\psi \circ \mu$ is the propositional theory s.t. $\text{Mod}(\psi \circ \mu)$ are the set of models of $\mu$ that are closest to the set of models of $\psi$.

Closeness could be defined using Dalal’s [Dalal, 1988] notion of distance, hence,

$$\text{Mod}(\psi \circ \mu) = \{ I \in \text{Mod}(\mu) | \forall I' \in \text{Mod}(\mu) \text{ s.t. } \text{distance}(\text{Mod}(\psi), I') < \text{distance}(\text{Mod}(\psi), I)\}$$
Distance [Dalal, 1988]

\[ \text{diff}(l_1, l_2) = \{ p \in P \mid l_1(p) \neq l_2(p) \} , \]
\[ \text{distance}(l_1, l_2) = |\text{diff}(l_1, l_2)|. \]

For a set of models \( M \),
\[ \text{distance}(M, l_1) = \min\{ \text{distance}(l_2, l_1) \mid l_2 \in M \}. \]

Example

5 objects A, B, C, D, E and a table are in a room. The 5 Objects may be on or off the table. The sentence \( a \) intuitively means “Object A is on the table”. Similarly \( b, c, d, e \) are interpreted.

Suppose old KB \( \psi \) is the sentence
\[ \psi = (a \land \neg b \land \neg c \land \neg d \land \neg e) \lor (\neg a \land \neg b \land c \land d \land e) \]
Example (Contd.)

\[ \mu = (a \land b \land c \land d \land e) \lor (\neg a \land \neg b \land \neg c \land \neg d \land \neg e) \]

\[ \text{Mod}(\psi) = \{l_1, l_2\}, \text{ where } l_1 = \{a\}, \ l_2 = \{c, d, e\} \]

\[ \text{Mod}(\mu) = \{l_3, l_4\}, \text{ where } l_3 = \{a, b, c, d, e\}, \ l_4 = \{\}. \]

\[ \text{diff}(l_1, l_3) = \{b, c, d, e\}, \ \text{diff}(l_2, l_3) = \{a, b\}, \]

\[ \text{distance}(l_1, l_3) = 4, \quad \text{distance}(l_2, l_3) = 2, \]

hence, \[ \text{distance}(\text{Mod}(\psi), l_3) = \min\{4, 2\} = 2. \]

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Example (Contd.)

\[ \text{distance}(l_1, l_4) = 1, \text{distance}(l_2, l_4) = 3, \]

hence, \[ \text{distance}(\text{Mod}(\psi), l_4) = \min\{1, 3\} = 1. \]

Recall, \[ \text{distance}(\text{Mod}(\psi), l_3) = 2 \]

which means \[ \text{Mod}(\psi \circ \mu) = l_4. \]

Hence, \[ \psi \circ \mu \equiv \neg a \land \neg b \land \neg c \land \neg d \land \neg e. \]
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Revision Postulates [Alchourrón et al. 1985]

R1 $\psi \circ \mu$ implies $\mu$.

R2 If $\psi \land \mu$ is satisfiable then $\psi \circ \mu \equiv \psi \land \mu$.

R3 If $\mu$ is satisfiable then $\psi \circ \mu$ is satisfiable.

R4 If $\psi_1 \equiv \psi_2$ and $\mu_1 \equiv \mu_2$ then $\psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2$.

R5 If $(\psi \circ \mu) \land \phi$ implies $\psi \circ (\mu \land \phi)$.

R6 If $(\psi \circ \mu) \land \phi$ is satisfiable then $\psi \circ (\mu \land \phi)$ implies $(\psi \circ \mu) \land \phi$. 
Orders between interpretations

Let $\mathcal{I}$ be the set of all interpretations over a language $\mathcal{L}$. A preorder $\leq$ over $\mathcal{I}$ is a reflexive and transitive relation on $\mathcal{I}$. Define $<$ as $I < I'$ iff $I \leq I'$ and $I' \not\leq I$.

Suppose we assign every formula $\psi$, a preorder $\leq_\psi$ over $\mathcal{I}$. This assignment is faithful iff:

1. If $I, I' \in \text{Mod}(\psi)$ then $I <_\psi I'$ does not hold.
2. If $I \in \text{Mod}(\psi)$ and $I' \not\in \text{Mod}(\psi)$ then $I <_\psi I'$ holds.
3. If $\psi \equiv \phi$ then $\leq_\psi = \leq_\phi$.

For any $M \subseteq \mathcal{I}$, $\text{Min}(M, \leq_\psi)$ be the set of all interpretations $I$ s.t. $I$ is minimal in $M$ w.r.t. $\leq_\psi$. 
Soundness and Completeness

Theorem (Soundness and Completeness)

Revision operator $\circ$ satisfies postulates (R1)-(R6) iff there exists a faithful assignment that maps each KB $\psi$ to a total preorder $\leq_\psi$ s.t. $\text{Mod}(\psi \circ \mu) = \text{Min}(\text{Mod}(\mu), \leq_\psi)$. 
Suppose old KB is given by $\psi$ and new knowledge $\mu$, the knowledge update operator $\diamond$ is defined as:

**Definition (KB Update)**

$\psi \diamond \mu$ is the propositional theory s.t.

$$
\text{Mod}(\psi \diamond \mu) = \bigcup_{I \in \text{Mod}(\psi)} \text{closest}(\text{Mod}(\mu), I)
$$

Closeness could be the following notion: for any interpretations $I, J_1, J_2$, $J_1 \leq_I J_2$ iff $\text{diff}(J_1, I) \subseteq \text{diff}(J_2, I)$.

$\text{closest}(\text{Mod}(\mu), I) = \text{Min}(\text{Mod}(\mu), \leq_I)$, i.e the set of all minimal elements in $\text{Mod}(\mu)$ w.r.t. $\leq_I$ relation.
Example

Suppose now there are only two objects $A, B$, and the table. Proposition $a$ means “object $A$ is on the table”, similarly for $b$. Now our KB $\psi$ is s.t.

$$\psi \equiv (a \land \neg b) \lor (\neg a \land b),$$

and the new knowledge $\mu$ is s.t. $\mu \equiv b$.

$\text{Mod}(\psi) = \{I_1, I_2\}$, where $I_1 = \{a\}$, $I_2 = \{b\}$, and

$\text{Mod}(\mu) = \{I_3, I_4\}$, where $I_3 = \{b\}$, $I_4 = \{a, b\}$.

$\text{diff}(I_1, I_3) = \{a, b\}, \text{diff}(I_1, I_4) = \{b\}$, hence, $I_4 \leq_{I_1} I_3$

$\text{diff}(I_2, I_3) = \emptyset, \text{diff}(I_2, I_4) = \{a\}$, hence, $I_3 \leq_{I_2} I_4$
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diff$(I_1, I_3) = \{a, b\}$, $\text{diff}(I_1, I_4) = \{b\}$, hence, $I_4 \subseteq I_1 \ I_3$

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Example

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Example (Contd.)

Hence, we have

\[ \text{closest}(\text{Mod}(\mu), I_1) = I_4, \text{ and } \text{closest}(\text{Mod}(\mu), I_2) = I_3. \]

Hence, \[ \text{Mod}(\psi \diamond \mu) = \bigcup_{I \in \text{Mod}(\psi)} \text{closest}(\text{Mod}(\mu), I) = \{I_3, I_4\}, \]

and hence, updated KB \[ \psi \diamond \mu \equiv b \]

Whereas \[ \text{Mod}(\psi \circ \mu) = I_3, \text{ and } \]

hence, revised KB \[ \psi \circ \mu \equiv \neg a \land b. \]
Update Postulates

**U1** \( \psi \diamond \mu \) implies \( \mu \)

**U2** If \( \psi \) implies \( \mu \) then \( \psi \diamond \mu \) is equivalent to \( \psi \)

**U3** If both \( \psi \) and \( \mu \) are satisfiable then \( \psi \diamond \mu \) is also satisfiable.

**U4** If \( \psi_1 \equiv \psi_2 \) and \( \mu_1 \equiv \mu_2 \) then \( \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2 \).

**U5** \((\psi \diamond \mu) \land \phi \) implies \( \psi \diamond (\mu \land \phi) \).

**U6** If \( \psi \diamond \mu_1 \) implies \( \mu_2 \) and \( \psi \diamond \mu_2 \) implies \( \mu_1 \) then \( \psi \diamond \mu_1 \equiv \psi \diamond \mu_2 \).

**U7** If \( \psi \) is complete then \((\psi \diamond \mu_1) \land (\psi \diamond \mu_2) \) implies \( \psi \diamond (\mu_1 \lor \mu_2) \).

**U8** \((\psi_1 \lor \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \lor (\psi_2 \diamond \mu) \).
Lemma

If $\psi$ is inconsistent, then $\psi \bowtie \mu$ is inconsistent for any $\mu$. 
Orders between interpretations

Let $\mathcal{I}$ be the set of all interpretations over a language $\mathcal{L}$. Suppose we assign, to each interpretation $I$, a partial preorder $\leq_I$ over $\mathcal{I}$. This assignment is said to be faithful iff:

- For any $J \in \mathcal{I}$, if $J \neq I$ then $I <_I J$.

**Theorem (Soundness and Completeness)**

The update operator $\diamond$ satisfies postulates U1-U8 iff there exists a faithful assignment that maps each interpretation $I$ to a partial pre-order $\leq_I$ s.t.

$$
\text{Mod}(\psi \diamond \mu) = \bigcup_{I \in \text{Mod}(\psi)} \text{Min}(\text{Mod}(\mu), \leq_I).
$$
Outline

1. Introduction

2. Revision and Update
   - KB Revision
   - KB Update

3. Contraction and Erasure
   - Contraction
   - Erasure

4. Unifying Revision and Update Operations: Time Aspect

Survey of the paper [Katsuno & Mendelzon. 1991]
Example (Now Joe is certain.)

\[
\psi = \text{"Joe’s GF often cancels their dates lately"} \\
\wedge \text{"She is 30 minutes late now"} \\
\wedge \heartsuit \, : \text{"She is serious about Joe"}
\]

\[
\mu = \text{"Late because she was at a movie with another guy."}
\]

- \[
\psi \circ \mu = \psi \wedge \mu \wedge \neg \heartsuit \text{ makes } \psi \circ \mu \text{ inconsistent.}
\]
- **Contraction** operator: give up compromised beliefs (\(\heartsuit\) in our case).
Contraction

Eliminating sentences from the KB which are no longer trusted.

Postulates [Alchourrón et al. 1985]

C1 $\psi \Rightarrow \psi \bullet \mu$
C2 $\psi \not\rightarrow \mu \Rightarrow \psi \bullet \mu \equiv \psi$
C3 $\mu \not\equiv \top \Rightarrow \psi \bullet \mu \not\rightarrow \mu$
C4 $\psi_1 \equiv \psi_2 \land \mu_1 \equiv \mu_2 \Rightarrow \psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2$
C5 $(\psi \bullet \mu) \land \mu \Rightarrow \psi$
Contraction vs. Revision [Alchourrón et al. 1985]

R1 \( \psi \circ \mu \implies \mu \).
R2 \( \psi \wedge \mu \neq \bot \implies \psi \circ \mu \equiv \psi \wedge \mu \).
R3 \( \mu \neq \bot \implies \psi \circ \mu \neq \bot \).
R4 \( \psi_1 \equiv \psi_2 \land \mu_1 \equiv \mu_2 \)
\implies \psi_1 \circ \mu_1 \equiv \psi_2 \circ \mu_2 .

C1 \( \psi \implies \psi \bullet \mu \)
C2 \( \psi \nLeftarrow \mu \implies \psi \bullet \mu \equiv \psi \)
C3 \( \mu \neq \top \implies \psi \bullet \mu \nLeftarrow \mu \)
C4 \( (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \)
\implies \psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2
C5 \( (\psi \bullet \mu) \land \mu \implies \psi \)

Revision \( \Rightarrow \) Contraction

If \( \circ \) is a revision operator satisfying properties (R1)–(R4),
then \( \bullet \) defined as \( \psi \bullet \mu \equiv \psi \lor (\psi \circ \neg \mu) \) satisfies (C1)–(C5)

Contraction \( \Rightarrow \) Revision

If \( \bullet \) is a contraction operator satisfying (C1)–(C5),
then \( \circ \) defined as \( \psi \circ \mu \equiv (\psi \bullet \neg \mu) \land \mu \) satisfies (P1)–(P4)
Erasure: Contracting All Possible Worlds

Contraction only works for facts known for sure:

- Recall the postulate (C2) $\psi \not\rightarrow \mu \implies \psi \cdot \mu \equiv \psi$

Example (Original version)

$\psi = "Joe’s GF often cancels their dates lately” \\
\wedge "She is 30 minutes late now” \\
\wedge (\heartsuit : “She is serious about Joe” \lor \spadesuit : “... far less than about her cat”)

Contraction of $\heartsuit$ does nothing here: $\psi \cdot \heartsuit = \psi$, since $\psi \not\rightarrow \heartsuit$. That is, $\heartsuit$ is not part of all possible worlds.

Suppose Joe is fed up and decides to break up. He is determined and therefore sure that $\heartsuit$ should not be implied by any possible world.

The version of contraction that works on all possible worlds is called erasure. It is a form of update.
Erasure: Contracting All Possible Worlds

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- Recall the postulate (C2) \( \psi \not\rightarrow \mu \implies \psi \cdot \mu \equiv \psi \)

Example (Original version)

\( \psi = \text{"Joe's GF often cancels their dates lately"} \)
\( \wedge \text{"She is 30 minutes late now"} \)
\( \wedge (\heartsuit : \text{"She is serious about Joe"} \lor \clubsuit : \text{"...far less than about her cat"}) \)

Contraction of \( \heartsuit \) does nothing here: \( \psi \cdot \heartsuit = \psi \), since \( \psi \not\rightarrow \heartsuit \). That is, \( \heartsuit \) is not part of all possible worlds.

Suppose Joe is fed up and decides to break up. He is determined and therefore sure that \( \heartsuit \) should not be implied by any possible world.

The version of contraction that works on all possible worlds is called erasure. It is a form of update.
Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

E1 \[ \psi \implies \psi \diamond \mu \]
E2 \[ \psi \rightarrow \neg \mu \implies \psi \diamond \mu \equiv \psi \]
E3 \[ \psi \not\equiv \bot \land \mu \not\equiv \top \implies \psi \diamond \mu \not\rightarrow \mu \]
E4 \[ (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2 \]
E5 \[ (\psi \diamond \mu) \land \mu \implies \psi \]
E8 \[ (\psi_1 \lor \psi_2) \diamond \mu \iff (\psi_1 \diamond \mu) \lor (\psi_2 \diamond \mu) \]

Example (Erasure works on every possible world = disjunct)
Let \( \psi = \theta \land (\heartsuit \lor \clubsuit) \).
\[ \psi \diamond \heartsuit \stackrel{(E8)}{=} ((\theta \land \heartsuit) \diamond \heartsuit) \lor ((\theta \land \clubsuit) \diamond \heartsuit) \stackrel{(E3)}{=} \theta \lor (\theta \land \clubsuit) \diamond \heartsuit \]

Mathew Joseph, Vadim Savenkov
Survey of the paper [Katsuno & Mendelzon. 1991]
Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

E1 \[ \psi \implies \psi \diamond \mu \]
E2 \[ \psi \rightarrow \neg \mu \implies \psi \downarrow \mu \equiv \psi \]
E3 \[ \psi \neq \bot \wedge \mu \neq \top \implies \psi \downarrow \mu \not\rightarrow \mu \]
E4 \[ (\psi_1 \equiv \psi_2) \wedge (\mu_1 \equiv \mu_2) \implies \psi_1 \downarrow \mu_1 \equiv \psi_2 \downarrow \mu_2 \]
E5 \[ (\psi \downarrow \mu) \wedge \mu \implies \psi \]
E8 \[ (\psi_1 \lor \psi_2) \downarrow \mu \iff (\psi_1 \downarrow \mu) \lor (\psi_2 \downarrow \mu) \]

Example (Erasure works on every possible world = disjunct)
Let \[ \psi = \theta \wedge (\heartsuit \lor \clubsuit). \]
\[ \psi \downarrow \heartsuit \overset{(E8)}{=} ((\theta \wedge \heartsuit) \downarrow \heartsuit) \lor ((\theta \wedge \clubsuit) \downarrow \heartsuit) \overset{(E3)}{=} \theta \lor (\theta \wedge \clubsuit) \downarrow \heartsuit \]

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Erasure: Contraction-like Counterpart to Update

Postulates of the Erasure operator

E1 $\psi \implies \psi \diamond \mu$

E2 $\psi \rightarrow \neg \mu \implies \psi \diamond \mu \equiv \psi$

E3 $\psi \not\equiv \bot \land \mu \not\equiv \top \implies \psi \diamond \mu \nleftrightarrow \mu$

E4 $(\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2$

E5 $(\psi \diamond \mu) \land \mu \implies \psi$

E8 $(\psi_1 \lor \psi_2) \diamond \mu \iff (\psi_1 \diamond \mu) \lor (\psi_2 \diamond \mu)$

Example (Erasure works on every possible world = disjunct)

Let $\psi = \theta \land (\heartsuit \lor \clubsuit)$.

$\psi \diamond \heartsuit \overset{(E8)}{=} ((\theta \land \heartsuit) \diamond \heartsuit) \lor ((\theta \land \clubsuit) \diamond \heartsuit) \overset{(E3)}{=} \theta \lor (\theta \land \clubsuit) \diamond \heartsuit$

Mathew Joseph, Vadim Savenkov

Survey of the paper [Katsuno & Mendelzon. 1991]
Erasur vs. Update

\begin{align*}
\text{U1} & \quad \psi \Diamond \mu \implies \mu \\
\text{U2} & \quad \psi \rightarrow \mu \implies \psi \Diamond \mu \equiv \psi \\
\text{U3} & \quad \psi \land \mu \neq \bot \implies \psi \Diamond \mu \neq \bot \\
\text{U4} & \quad (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \\
& \quad \psi_1 \Diamond \mu_1 \equiv \psi_2 \Diamond \mu_2. \\
\ldots \\
\text{U8} & \quad (\psi_1 \lor \psi_2) \Diamond \mu \equiv (\psi_1 \Diamond \mu) \lor (\psi_2 \Diamond \mu). \\
\end{align*}

\begin{align*}
\text{E1} & \quad \psi \implies \psi \bullet \mu \\
\text{E2} & \quad \psi \rightarrow \neg \mu \implies \psi \bullet \mu \equiv \psi \\
\text{E3} & \quad \psi \neq \bot \land \mu \neq \top \implies \psi \bullet \mu \not\equiv \mu \\
\text{E4} & \quad (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \\
& \quad \psi_1 \bullet \mu_1 \equiv \psi_2 \bullet \mu_2 \\
\text{E5} & \quad (\psi \bullet \mu) \land \mu \implies \psi \\
\text{E8} & \quad (\psi_1 \lor \psi_2) \bullet \mu \equiv (\psi_1 \bullet \mu) \lor (\psi_2 \bullet \mu). \\
\end{align*}

\textbf{Theorem}

\begin{itemize}
\item \textbf{If an update operator} $\Diamond$ \textbf{satisfies (U1)--(U4) and (U8), then the erasure operator} $\bullet$ \textbf{defined by} $\psi \bullet \mu \equiv \psi \lor (\psi \Diamond \neg \mu)$ \textbf{satisfies (E1)--(E5) and (E8).}
\end{itemize}
Erasure vs. Update

U1  \( \psi \bowtie \mu \implies \mu \)
U2  \( \psi \rightarrow \mu \implies \psi \bowtie \mu \equiv \psi \)
U3  \( \psi \land \mu \neq \bot \implies \psi \bowtie \mu \neq \bot \)
U4  \( (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \psi_1 \bowtie \mu_1 \equiv \psi_2 \bowtie \mu_2 \).

\[ \ldots \]
U8  \( (\psi_1 \lor \psi_2) \bowtie \mu \equiv (\psi_1 \bowtie \mu) \lor (\psi_2 \bowtie \mu) \).

E1  \( \psi \implies \psi \diamond \mu \)
E2  \( \psi \rightarrow \neg \mu \implies \psi \diamond \mu \equiv \psi \)
E3  \( \psi \neq \bot \land \mu \neq \top \implies \psi \diamond \mu \not\equiv \mu \)
E4  \( (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2 \)
E5  \( (\psi \diamond \mu) \land \mu \implies \psi \)
E8  \( (\psi_1 \lor \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \lor (\psi_2 \diamond \mu) \).

**Theorem**

2. If an erasure operator \( \diamond \) satisfies (E1)–(E4) and (E8), then the update operator \( \bowtie \) defined by \( \psi \bowtie \mu \equiv (\psi \diamond \neg \mu) \land \mu \) satisfies (U1)–(U4) and (U8).
Erasure vs. Update

\[ \text{U1 } \psi \odot \mu \rightarrow \mu \]
\[ \text{U2 } \psi \rightarrow \mu \rightarrow \psi \odot \mu \equiv \psi \]
\[ \text{U3 } \psi \land \mu \neq \bot \rightarrow \psi \odot \mu \neq \bot \]
\[ \text{U4 } (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \rightarrow \psi_1 \odot \mu_1 \equiv \psi_2 \odot \mu_2. \]
\[ \text{U8 } (\psi_1 \lor \psi_2) \odot \mu \equiv (\psi_1 \odot \mu) \lor (\psi_2 \odot \mu). \]

\[ \text{E1 } \psi \rightarrow \psi \blacklozenge \mu \]
\[ \text{E2 } \psi \rightarrow \neg \mu \rightarrow \psi \blacklozenge \mu \equiv \psi \]
\[ \text{E3 } \psi \neq \bot \land \mu \neq \top \rightarrow \psi \blacklozenge \mu \land \mu \]
\[ \text{E4 } (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \rightarrow \psi_1 \blacklozenge \mu_1 \equiv \psi_2 \blacklozenge \mu_2 \]
\[ \text{E5 } (\psi \blacklozenge \mu) \land \mu \rightarrow \psi \]
\[ \text{E8 } (\psi_1 \lor \psi_2) \blacklozenge \mu \equiv (\psi_1 \blacklozenge \mu) \lor (\psi_2 \blacklozenge \mu). \]

**Theorem**

3 Suppose that an update operator \( \odot \) satisfies (U1)–(U4) and (U8). Then, we can define an erasure operator by \( \psi \blacklozenge \mu \equiv \psi \lor (\psi \odot \neg \mu) \). The update operator obtained from the erasure operator by \( \psi \odot \mu \equiv (\psi \blacklozenge \neg \mu) \land \mu \) is equal to the original update operator.
# Erasure vs. Update

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1 ( \psi \diamond \mu \implies \mu )</td>
<td>Erasure of update modifies the update.</td>
</tr>
<tr>
<td>U2 ( \psi \rightarrow \mu \implies \psi \diamond \mu \equiv \psi )</td>
<td>Erasure preserves the original update.</td>
</tr>
<tr>
<td>U3 ( \psi \land \mu \not\equiv \bot \implies \psi \diamond \mu \not\equiv \bot )</td>
<td>Erasure preserves the original update.</td>
</tr>
<tr>
<td>U4 ( (\psi_1 \equiv \psi_2) \land (\mu_1 \equiv \mu_2) \implies \psi_1 \diamond \mu_1 \equiv \psi_2 \diamond \mu_2 )</td>
<td>Update by equivalent pairs.</td>
</tr>
<tr>
<td>U8 ( (\psi_1 \lor \psi_2) \diamond \mu \equiv (\psi_1 \diamond \mu) \lor (\psi_2 \diamond \mu) )</td>
<td>Combination of updates.</td>
</tr>
</tbody>
</table>

### Theorem

Suppose that an erasure operator \( \psi \) satisfies (E1)–(E5) and (E8). Then, we can define an update operator by

\[
\psi \diamond \mu \equiv (\psi \downarrow \lnot \mu) \land \mu.
\]

The erasure operator obtained from the update operator by \( \psi \downarrow \mu \equiv \psi \lor (\psi \diamond \lnot \mu) \) is equal to the original erasure operator.
Outline

1. Introduction

2. Revision and Update
   - KB Revision
   - KB Update

3. Contraction and Erasure
   - Contraction
   - Erasure

4. Unifying Revision and Update Operations: Time Aspect
How to tell if $\mu$ is a revision or an update?

- Time parameter: $t$.
- Parameterized KB has the form $\langle \psi, t \rangle$.
- New operator: $\text{Tell}(\mu, t') \langle \psi, t \rangle = \begin{cases} \langle \psi \circ \mu, t \rangle & \text{if } t' = t \\ \langle \psi \diamond \mu, t' \rangle & \text{if } t' > t \end{cases}$

In this framework, the type of the change is done automatically based on the relationship between the time instant of the KB and that of the change:

- Change now ($t' = t$) $\Rightarrow$ That’s about the knowledge.
- Change in the future ($t' > t$) $\Rightarrow$ That’s about the world.
Example

Recall the example with two objects A, B on the table.

- $\langle \psi = (a \land \neg b) \lor (\neg a \land b), 10:00 \rangle$.

- **New knowledge:** it’s surely the object B on the table.
  
  $Tell(b, 10:00)\langle \psi, 10:00 \rangle$
  
  $\implies \langle \psi \circ b, 10:00 \rangle = \langle (b \land \neg a), 10:00 \rangle$

- Sent robot to put the object B on the table.
  
  $Tell(b, 10:05)\langle \psi, 10:00 \rangle$
  
  $\implies \langle \psi \diamond b, 10:05 \rangle = \langle b, 10:05 \rangle$
Example

Recall the example with two objects A, B on the table.

- $\langle \psi = (a \land \neg b) \lor (\neg a \land b), 10:00 \rangle$.

- New knowledge: it’s surely the object B on the table.
  
  $Tell(b, 10:00)\langle \psi, 10:00 \rangle$
  
  $\Rightarrow \langle \psi \circ b, 10:00 \rangle = \langle (b \land \neg a), 10:00 \rangle$

- Sent robot to put the object B on the table.
  
  $Tell(b, 10:05)\langle \psi, 10:00 \rangle$
  
  $\Rightarrow \langle \psi \diamond b, 10:05 \rangle = \langle b, 10:05 \rangle$
THANKS

Thanks for your attention

Questions?
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