1. **Search Space Reduction for the Tree Edit Distance**
   - Similarity Join and Search Space Reduction
   - Lower Bound: Traversal Strings
   - Upper Bound: Constrained Edit Distance
Outline

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Definition: Similarity Join

**Definition (Similarity Join)**

Given two sets of trees, \( S_1 \) and \( S_2 \), and a distance threshold \( \tau \), let \( \delta_t(T_i, T_j) \) be a function that assesses the edit distance between two trees \( T_i \in S_1 \) and \( T_j \in S_2 \). The similarity join operation between two sets of trees reports in the output all pairs of trees \( (T_i, T_j) \in S_1 \times S_2 \) such that \( \delta_t(T_i, T_j) \leq \tau \).
simJoin($S_1, S_2$)

for each $T_i \in S_1$ do
  for each $T_j \in S_2$ do
    if upperBound($T_i, T_j$) $\leq \tau$ then
      output($T_i, T_j$)
    elseif lowerBound($T_i, T_j$) $> \tau$ then
      /* do nothing */
    else if $\delta_t(T_i, T_j) \leq \tau$ then
      output($T_i, T_j$)
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Preorder and Postorder Traversal Strings

- Each node label is a single character of an alphabet $\Sigma$.
- Traversal Strings:
  - $pre(T)$ is the string of $T$'s node labels in preorder
  - $post(T)$ is the string of $T$'s node labels in postorder

Lemma (Tree Inequality)

Let $pre(T_1)$ and $pre(T_2)$ be the preorder strings, and $post(T_1)$ and $post(T_2)$ be the postorder strings of two trees $T_1$ and $T_2$, respectively. Then

$$pre(T_1) \neq pre(T_2) \lor post(T_1) \neq post(T_2) \Rightarrow T_1 \neq T_2$$

Proof.

The inversion of the argument is obviously true:

$$T_1 = T_2 \Rightarrow pre(T_1) = pre(T_2) \land post(T_1) = post(T_2)$$
If the traversal strings of two trees are equal, the trees can still be different:

\[ T_1 \quad \neq \quad T_2 \]

\[
T_1 = a \quad b \quad a \\
T_2 = a \quad b \\
\]

\[ pre(T_1) = aba = pre(T_2) = aba \]
Lower Bound

Theorem (Lower Bound)

If the trees are at tree edit distance \( k \), then the string edit distance between their preorder or postorder traversals is at most \( k \).

Proof.

Tree operations map to string operations (illustration on next slide):

- **Insertion** \( \text{ins}(v, p, k, m) \): Let \( t_1 \ldots t_f \) be the subtrees rooted in the children of \( p \). Then the preorder traversal of the subtree rooted in \( p \) is
  \[
  p \text{pre}(t_1) \ldots \text{pre}(t_{k-1})\text{pre}(t_k) \ldots \text{pre}(t_m)\text{pre}(t_{m+1}) \ldots \text{pre}(t_f).
  \]

  Inserting \( v \) moves the subtrees \( k \) to \( m \):
  \[
  p\text{pre}(t_1) \ldots \text{pre}(t_{k-1})v\text{pre}(t_k) \ldots \text{pre}(t_m)\text{pre}(t_{m+1}) \ldots \text{pre}(t_f).
  \]

  The string distance is 1. Analog rationale for postorder.

- **Deletion**: Inverse of insertion.

- **Rename**: With node rename a single string character is renamed. \( \square \)
Illustration for the Lower Bound Proof (Preorder)

\[
\text{ins}(v, p, k, m) \quad \overset{\leftrightarrow}{\Rightarrow} \quad \text{del}(v)
\]

\[
\begin{align*}
 p & \ pre(t_1) \ldots \ pre(t_{k-1}) \\
 & \ pre(t_k) \ldots \ pre(t_m) \\
 & \ pre(t_{m+1}) \ldots \ pre(t_f)
\end{align*}
\]
Lower Bound

- From the lower bound theorem it follows that

\[ \max(\delta_s(pre(T_1), pre(T_2)), \delta_s(post(T_1), post(T_2))) \leq \delta_t(T_1, T_2) \]

where \(\delta_s\) and \(\delta_t\) are the string and the tree edit distance, respectively.

- The string edit distance can be computed faster:
  - string edit distance runtime: \(O(n^2)\)
  - tree edit distance runtime: \(O(n^3)\)

- Similarity join: match all trees with \(\delta_t(T_1, T_2) \leq \tau\)
  - if \(\max(\delta_s(pre(T_1), pre(T_2)), \delta_s(post(T_1), post(T_2))) > \tau\)
  - then \(\delta_t(T_1, T_2) > \tau\)
  - thus we do not have to compute the expensive tree edit distance
Example: Traversal String Lower Bound

\[ pre(T_1) = f\text{d}a\text{c}be \quad pre(T_2) = f\text{c}d\text{a}be \]
\[ post(T_1) = a\text{b}c\text{d}e\text{f} \quad post(T_2) = a\text{b}d\text{c}e\text{f} \]

\[ \delta_s(pre(T_1), pre(T_2)) = 2 \]
\[ \delta_s(post(T_1), post(T_2)) = 2 \]
\[ \delta_t(T_1, T_2) = 2 \]
Example: Traversal String Lower Bound

- The string distances of preorder and postorder may be different.
- The string distances and the tree distance may be different.

\[
\begin{align*}
T_1 & \quad T_2 \\
b & \quad a \\
c & \quad a \\
a & \quad b \\
a & \quad c
\end{align*}
\]

\[
\begin{align*}
pre(T_1) & = abac & pre(T_2) & = abac \\
post(T_1) & = bcaa & post(T_2) & = acba
\end{align*}
\]

\[
\begin{align*}
\delta_s(pre(T_1), pre(T_2)) & = 0 \\
\delta_s(post(T_1), post(T_2)) & = 2 \\
\delta_t(T_1, T_2) & = 3
\end{align*}
\]
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Edit Mapping

- Recall the definition of the edit mapping:

**Definition (Edit Mapping)**

An edit mapping $M$ between $T_1$ and $T_2$ is a set of node pairs that satisfy the following conditions:

1. $(a, b) \in M \Rightarrow a \in N(T_1), b \in N(T_2)$
2. for any two pairs $(a, b)$ and $(x, y)$ of $M$:
   - (i) $a = x \Leftrightarrow b = y$ (one-to-one condition)
   - (ii) $a$ is to the left of $x$ $\Leftrightarrow$ $b$ is to the left of $y$ (order condition)
   - (iii) $a$ is an ancestor of $x$ $\Leftrightarrow$ $b$ is an ancestor of $y$ (ancestor condition)
3. Optional: $a = \text{root}(T_1)$ and $b = \text{root}(T_1) \Rightarrow (a, b) \in M$ (forbid deleting the root node)

\(^1\)i.e., a precedes $x$ in both preorder and postorder
We compute a special case of the edit distance to get a faster algorithm.

\( \text{lca}(a, b) \) is the lowest common ancestor of \( a \) and \( b \).

Additional requirement on the mapping \( M \):

(4) for any pairs \((a_1, b_1), (a_2, b_2), (x, y)\) of \( M \):

\[
\text{lca}(a_1, a_2) \text{ is a proper ancestor of } x \\
\iff \\
\text{lca}(b_1, b_2) \text{ is a proper ancestor of } y.
\]

Intuition: Distinct subtrees of \( T_1 \) are mapped to distinct subtrees of \( T_2 \).
Example: Constrained Edit Distance

- **Constrained** edit distance (dashed lines): \( \delta_c(T_1, T_2) = 5 \)
  - constrained mapping \( M_c = \{(a, a), (d, d), (c, i), (f, f)(g, g)\} \)
  - edit sequence: \( ren(c, i), del(b), del(e), ins(h), ins(e) \)

- **Unconstrained** edit distance (dotted lines): \( \delta_t(T_1, T_2) = 3 \)
  - mapping \( M_t = \{(a, a), (d, d), (e, e), (c, i), (f, f)(g, g)\} \)
  - edit sequence: \( ren(c, i), del(b), ins(h) \)
Example: Constrained Edit Distance

- $(e, e)$ violates the 4th condition of the constrained mapping:
  - $lca(e, f)$ in $T_1$ is $a$
  - $a$ is a proper ancestor of $d$ in $T_1$
  - assume $(e, e), (f, f), (d, d) \in M_c$
  - $lca(e, f)$ in $T_2$ is $h$
  - $h$ is not a proper ancestor of $d$ in $T_2$
Theorem (Complexity of the Constrained Edit Distance)

Let $T_1$ and $T_2$ be two trees with $|T_1|$ and $|T_2|$ nodes, respectively. There is an algorithm that computes the constrained edit distance between $T_1$ and $T_2$ with runtime

$$O(|T_1||T_2|).$$

Proof.

See [Zha95, GJK+02].
Theorem (Upper Bound)

Let $T_1$ and $T_2$ be two trees, let $\delta_t(T_1, T_2)$ be the unconstrained and $\delta_c(T_1, T_2)$ be the constrained tree edit distance, respectively. Then

$$\delta_t(T_1, T_2) \leq \delta_c(T_1, T_2)$$

Proof.

See [GJK+02].
The **constrained edit distance** can be computed faster:
- constrained edit distance runtime: $O(n^2)$
- unconstrained edit distance runtime: $O(n^3)$

**Similarity join**: match all trees with $\delta_t(T_1, T_2) \leq \tau$
- if $\delta_c(T_1, T_2) \leq \tau$ then also $\delta_t(T_1, T_2) \leq \tau$.
- thus we do not have to compute the expensive tree edit distance
Sudipto Guha, H. V. Jagadish, Nick Koudas, Divesh Srivastava, and Ting Yu.
Approximate XML joins.

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