Programming Paradigms

Unit 14 — Functors and Monads

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Outline

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1. **Functors Revisited**

2. Monads

Beefing Up Functors/1

- What happens if we want to use a function that takes two parameters with a functor?
- For example, lets multiply two values `Just 2` and `Just 5`

```
(Just 2) * (Just 5)
```

- This does not work, as the multplication operator `*` expects two numerical values, not two values wrapped in `Maybe`
  - Again: pure function `*`, impure parameters `Just 2` and `Just 5`
- We could push `*` into one of the functors

```
> :t fmap (*) (Just 2)
fmap (*) (Just 2) ::  Num a => Maybe (a -> a)
```

Beefing Up Functors/2

- That means, we now have a function wrapped in a `Just`
- We could also rewrite the above as

```
Just (*2)
```

- This is a partially evaluated function (remember currying!)
- But we still have a problem: *How do we apply a function that is wrapped inside a functor to values inside a functor box?*
- `fmap` only takes ordinary functions and maps them over a functor (box)
  - We saw how to map functions over a `Maybe a`, a list `[a]`, a tree `Tree a`, etc.
- However, `fmap` does **not** work in the following case:

```
fmap (Just(*2)) (Just 5)
```

- So, what do we do? Rewrite all our multi-parameter functions for functors?

Type Class Applicative

- Not really, there is a type class `Applicative` with two important functions

```
pure ::  a -> f a
(<*>) ::  f (a -> b) -> f a -> f b
```

- The function `pure` takes a value of any type and returns an applicative functor `f` with that value inside it
  - i.e., `pure` takes a value and wraps it in an applicative functor box
- The function `<*>`, also called "ap" or "apply",
  - takes a functor `f` that contains a function
  - and another functor that contains a's, and
  - extracts the function from the first functor and maps it over the second one
- This is exactly what we are looking for, remember:

```
> :t Just (*2)
Just (*2) ::  Num a => Maybe (a -> a)
```

```
> :t Just 5
Just 5 ::  Num a => Maybe a
```

- Compare `(<*>)` to `fmap ::  (a -> b) -> f a -> f b`

Using Applicative Functors/1

- Maybe is an instance of `Applicative`, so we can use its functions right out of the box
  - Well, we have to import the module `Control.Applicative` first ...

```
> import Control.Applicative
```

```
> (Just (*2)) <*> (Just 5)
Just 10
```

Success!

- This also works for values of `Nothing`

```
> (Just (*2)) <*> Nothing
Nothing
```

```
> Nothing <*> (Just 5)
Nothing
```

Using Applicative Functors/2

- As mentioned above, `pure` "wraps" a pure value into an impure context (an applicative functor box)
- We cannot combine pure and impure values in the same computation
- With applicative functors, we wrap the pure value into a (default) impure context:

```
> (Just (*2)) <*> 5
```

does not work

```
> (Just (*2)) <*> (pure 5)
Just 10
```

does work

Using Applicative Functors/3

- This does not stop at two parameters
- With applicative functors we can chain any number of functors

```
pure f <*> x <*> y <*> z <*> ...
```

- So, for example we define a function summing up three numbers

```
sum3 x y z = x + y + z
```

and then use it in a functor context

```
> pure sum3 <*> Just 4 <*> Just 9 <*> Just 2
Just 15
```

```
> pure sum3 <*> Just 4 <*> Nothing <*> Just 2
Nothing
```

Applicative Instance Implementation for Maybe

- This is how Maybe is defined as an instance of the type class Applicative

```
instance Applicative Maybe where
    pure = Just
    Nothing <*> _ = Nothing
    (Just f) <*> something = fmap f something
```

- The function to wrap a value inside a context is Just (recall that value constructors are functions)
- If the first parameter to (<*>) is Nothing, we cannot extract a function out of it, so the result is Nothing
- If the first parameter is Just with a function f inside, this function is mapped over the second parameter

More Examples of Using Applicative Functors

- Here are some more examples

```
> Just (+3) <*> Just 9
Just 12
```

```
> pure (+3) <*> Just 10
Just 13
```

```
> Just (++" world") <*> pure "Hello"
Hello world
```

```
> Just (++" world") <*> "Hello"
...  error!
```

```
> Just (++" world") <*> Nothing
Nothing
```

```
> Nothing <*> Just "Hi"
Nothing
```

- Notice that `pure` and `Just` have the same effect here

Outline

Monads

- We will now introduce the concept of monads with the help of an example
- Let's assume x persons want to divide up y things:

```
divideUp ::  Int -> Int -> Int
divideUp x y = div y x
```

- This is not going to work, as the following should fail (but it doesn't)

```
> divideUp 5 12
2
```

Second Try

- We could give back `Maybe Int`
  - If the function fails, we return `Nothing`
  - Otherwise, we return `Just` "the result"

```
divideAmong ::  Int -> Int -> Maybe Int
divideAmong x y =
    if mod y x /= 0 then
        Nothing
    else
        Just (div y x)

> divideAmong 5 12
Nothing

> divideAmong 6 12
Just 2
```

- So far, so good

Further Divisions/1

- What happens if we want to divide up one lot among further persons, i.e.:

  ```
  divideAmong 3 (divideAmong 2 12)
  ```

- This is not going to work, as `divideAmong` expects pure `Int`s as parameters, while it returns a `Maybe` functor (i.e., a `Maybe` box containing the value)
- Let's try using an applicative functor:

  ```
  > pure (divideAmong) <*> Just 2 <*> Just 12
  Just (Just 6)
  ```

  Nope, this adds yet another layer ...

Further Divisions/2

- Is implementing this manually the only option left?

```
divideAmongTwice ::  Int -> Int -> Int -> Maybe Int
divideAmongTwice x y z =
    if mod y x /= 0 then
        Nothing
    else
        if mod (div y x) z /= 0 then
            Nothing
        else
            Just (div (div y x) z)
```

- Keeping track of every step that can fail is very awkward and error-prone!
- Monads can help out here

Monads/1

- Monads are a type class with two important functions:

```
return ::  a -> m a
(>>=) ::  m a -> (a -> m b) -> m b
```

- The first function `return` wraps a pure value `a` into an impure context, termed a monad `m a`
  - Works like `pure` for applicative functors
- The second function `(>>=)`, called bind,
  - takes a monadic value `m a`, i.e., a value of type `a` inside a monadic context
  - and a function `a -> m b` that takes a pure value `a` and returns a monadic value `m b`
  - and applies the function to the first parameter (or feeds the parameter into the function), returning a monadic value `m b`
- That is, monads allow sequenced actions, i.e., to put together two actions, returning the result of the second one

Monads/2

- Now we can chain together calls of the function

```
> divideAmong 2 120 >>= divideAmong 3 >>= divideAmong 5
Just 4
```

- And Haskell will keep track of any failures on the way for us

```
> divideAmong 5 12 >>= divideAmong 3 >>= divideAmong 5
Nothing
```

```
> divideAmong 6 12 >>= divideAmong 3 >>= divideAmong 5
Nothing
```

Do Notation

- Monads are so important in Haskell that they have their own special notation: the do notation
- This notation allows you to chain together monadic function calls in a seemingly imperative way

```
routine ::  Maybe Int
routine = do
    x <- divideAmong 2 120
    y <- divideAmong 3 x
    divideAmong 4 y

routine
Just 5
```

- The statements are executed line by line
- With `<-` we bind a monadic `Maybe` value (impure) to a variable (pure)
- The result of the final execution is the result of `routine`

IO is a Monad

- Yes, you have seen this notation before in the context of IO
- And, yes, this means that IO is a monad!
- It doesn't end there:
  - There are monads for representing state
  - For dealing with indeterminism
  - Even lists can be interpreted as monads
- There are lots of other things to say about monads
  - All instances of monads need to follow certain laws (instances of (applicative) functors as well)
- But we are going to stop here

Mathematical Foundation

- The concepts used in Haskell did not just fall from the sky
- They are rooted in mathematical theory, category theory to be more specific
- In category theory, mathematicians try to capture the underlying properties of mathematical concepts
- Expressed in simplified terms, it is like finding and defining "type classes" for mathematical structures

Summary – Strengths of Haskell

- The type system (strong/static) prevents you from making a lot of mistakes
  - Nevertheless, it is quite flexible when it comes to extending it with user-defined types
- Haskell offers a lot in terms of expressiveness, yielding very concise code
- Haskell is a pure functional language, providing referential transparency
  - function give the same output for the same input
  - functions have no side effects
  - a variable can only be assigned a value once
- Hasekell uses curried functions in combination with partial evaluation of functions,
  - i.e., internally, functions have only one input parameter;
  - functions with multiple input parameters are decomposed into a sequence of partial functions, each having one parameter
- It is easier to show the correctness of your programs, due to the pure functional style
- It does lazy evaluation, which gives you an additional tool for writing programs efficiently
- It supports list comprehension and infinite lists

Summary – Weaknesses of Haskell

- The pure functional paradigm also has a price: dealing with messy real-world situations such as IO and state is not easy
- Haskell has a steep learning curve, i.e., it takes a while to learn how to wield the power of Haskell
- This may also explain the fact that the Haskell community is relatively small