1. Structures

2. Equality, Matching and Arithmetic

3. Lists

4. Examples
Outline

1. Structures

2. Equality, Matching and Arithmetic

3. Lists

4. Examples
If we want to say that Wallace and Wendolene own books, we could formulate the following facts:

owns(wallace, book).
owns(wendolene, book).

However, this means that Wallace owns the same object that Wendolene owns.

Specifying the title to distinguish may not help:

owns(wallace, perfume).
owns(wendolene, russell_the_sheep).

It’s not clear that we are talking about books here.

This can be solved by introducing a structure for books.
A structure in Prolog is a single object which consists of a collection of other objects, called components.

A structure can be decomposed into:
- a functor and
- one or more components.

The functor names the general kind of structure, and corresponds to a data type in other languages.

Using a structure for books, we have:

owns(wallace, book(perfume, suesskind)).
owns(wendolene, book(russell_the_sheep, scotton)).

Looking at book(perfume, suesskind):
- book is the functor of the structure
- perfume and suesskind are its components.
Nested Structures

- Structures can be **nested** (arbitrarily deep)
- Since there were three Brontë writers, we might want to present the author in more detail with another structure `author`, e.g.,
  
  `owns(gromit, book(wuthering_heights, author(emily,bronte))).`

- Prolog allows you to create arbitrarily complex structures to represent information/knowledge
- We could improve the book structure by adding an additional argument to represent *which copy* the book was
  
  - e.g., the third argument uniquely identifies the book
  
  `owns(gromit, book(wuthering_heights, author(emily,bronte), 3129)).`
Querying Structures

- Structures my participate in query processing by usin variables
- Structures are matched similar to facts
- For example, if we want to know if Gromit owns any books written by one of the Brontë sisters, we would query

```
?- owns(gromit,book(X,author(Y,bronte))).
X = wuthering_heights
Y = emily
```
Structures and Facts

- The syntax for structures and facts is identical
  - A predicate (used in facts and rules) is actually the functor of a structure
  - The arguments of a fact or rule are components of a structure

- So, Prolog programs are essentially structures, which has several advantages.

- All parts of Prolog, even Prolog programs themselves, are made up of constants, variables and structures.
Outline

1. Structures

2. Equality, Matching and Arithmetic

3. Lists

4. Examples
Equality and Matching

- Prolog has a number of built-in predicates
- One of them is equality written as “=”
- The expression $X = Y$ attempts to match $X$ and $Y$
  - i.e., tries to make $X$ and $Y$ equal
- The goal succeeds if $X$ and $Y$ match; otherwise it fails
- Following Prolog syntax, it should be written as $=\langle X, Y \rangle$
  - While this works, Prolog also allows you to use an infix notation: $X = Y$
Equality and Matching of Atoms and Numbers

Integers and atoms are always equal to themselves

?- wallace = wallace.
yes
?- cheese = cake.
no
?- 1066 = 1066.
yes
?- 1206 = 1583.
no
A variable always matches itself, i.e., $X = X$ always succeeds

?- X = X.
yes

If we match two different variables, i.e., $X = Y$, we have to distinguish three cases

Case 1: none of the variables is instantiated
- The goal always succeeds

?- X = Y.
X = Y
yes
Equality and Matching of Variables/2

- Case 2: one of the two variables (say Y) is instantiated
  - Goal succeeds, and X is instantiated with the value of Y

?- X = gromit.
X = gromit.

?- X = likes(wallace, toast).
X = likes(wallace, toast).

?- X = Y, likes(X, toast).
X = wallace.
Y = wallace.

Knowledge base

<table>
<thead>
<tr>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>likes(wallace, toast).</td>
</tr>
<tr>
<td>likes(wallace, cheese).</td>
</tr>
<tr>
<td>likes(gromit, cheese).</td>
</tr>
<tr>
<td>likes(gromit, cake).</td>
</tr>
<tr>
<td>likes(wendolene, sheep).</td>
</tr>
</tbody>
</table>
Equality and Matching of Variables/3

- **Case 3:** both variables are already instantiated
  - The values the two variables are instantiated with are compared
  - Might require the comparison of structures

Knowledge base

```
likes(wallace, toast).
likes(wallace, cheese).
likes(gromit, cheese).
likes(gromit, cake).
likes(wendolene, sheep).
```

?- likes(X,cheese), likes(Y,cake), X = Y.
X = gromit
Y = gromit

?- likes(X,toast), likes(Y,cake), X = Y.
no
Equality and Matching of Structures

- **Two structures are equal** if
  - they have the same functor and number of components and
  - all the corresponding components are equal

?- likes(gromit,cheese) = likes(gromit,X).
X = cheese

?- f(a,g(a,b)) = f(X,g(Y,Z)).
X = Y, Y = a,
Z = b.

?- a(b,C,d(e,F,g(h,i,J))) = a(B,c,d(E,f,g(H,i,j)))
B = b
C = c
E = e
F = f
H = h
J = j

?- letter(c) = word(c).
no.
Comparison and Matching

- Prolog also offers other built-in comparison operators
  
  ?- 2 > 3.
  no
  ?- 3 >= 2.
  yes
  ?- 3 =< 2.
  no
  ?- X \= Y.
  no

- The \= operator means that X cannot be made equal to Y
  - not(X=Y) could also be used
Arithmetic

- Prolog also offers the standard **arithmetic operators**: +, -, *, /, mod,
- Just typing in an arithmetic operation will not actually carry it out

```prolog
?- 3 + 4.
ERROR: toplevel: Undefined procedure: (+)/2 ...
?- X = 3 + 4.
X = 3 + 4.
?- 7 = 3 + 4.
no
```

- Using the **is** operator will evaluate the right-hand side and match it to the left-hand side

```prolog
?- 7 is 3 + 4.
yes
?- X is 3 + 4.
X = 7.
```
Arithmetic Example/1

- Given the following fact base, compute the population density of countries

  pop(usa,313).
  pop(italy,61).
  pop(uk,63).
  area(usa,9.826).
  area(italy,0.301).
  area(uk,0.243).

- The following rule computes the density

  density(X,Y) :- pop(X,P),
  area(X,A),
  Y is P/A.

- This rule is read as follows:
  - The population density of country X is Y, if:
    - The population of X is P, and
    - The area of X is A, and
    - Y is calculated by dividing P by A.
Arithmetic Example/2

- Compute population density of USA

  \[-\text{density(usa,Y).}\
  Y = 31.854264197028289\]

  yes

- Compute all densities

  \[-\text{density(X,Y).}\
  X = \text{usa}\
  Y = 31.854264197028289 ? ;\

  X = \text{italy}\
  Y = 202.65780730897012 ? ;\

  X = \text{uk}\
  Y = 259.25925925925924 .\]
Outline

1. Structures
2. Equality, Matching and Arithmetic
3. Lists
4. Examples
We have already seen structures as a construct to build more complicated data types.

Another important type supported by Prolog is a list.

The elements of a list are enclosed in square brackets \([\quad]\).

?- [1,2,3] = [1,2,3].
yes

?- [1,2,3] = [X,Y,Z].
X = 1,
Y = 2,
Z = 3.

Lists are matched similar to structures.
The elements of a list can be any terms – constants, variables, structures, lists...

... and they can be mixed

Examples of valid lists are

- []
- [2,3,5,a,b]
- [the,cat,sat,[on,the,mat]]
- [a,V,b,[X,Y]]
- [the,book([programming,in,prolog]),by,authors(C,M)]
Internal Representation of Lists

- Internally, lists are represented as compound terms using:
  - the functor "."/2 (dot, list constructor), where the first argument is the first element and the second argument is the rest of the list, and
  - the atom [] representing the empty list, which is the second argument on the innermost level.

- For example, the list

  \[a,b,c]\n
corresponds to the compound term

  \(.a, .(b, .(c, [])))\n
- So, \[1,2,3\] is just a more convenient notation for an important structure.

- We can verify this in Prolog:

  ?- X = .(a, .(b, .(c, []))).
  X = [a, b, c]
  Yes

- In SWI Prolog v7, the functor "." has been replaced by the functor "[|]"
Splitting Lists in Head and Tail/1

- We can split lists into a head and tail using the ”|” operator
  - Head is the first element of the list
  - Tail is the (possibly empty) rest of the list, and it is a list

?- [Head|Tail] = [1,2,3].
Head = 1,
Tail = [2,3].

?- [Head|Tail] = [].
no.

?- [Head|Tail] = [1].
Head = 1,
Tail = [].

Splitting Lists in Head and Tail/2

Some more examples

?- [H|T] = [[the,cat],sat].
H = [the,cat],
T = [sat].

?- [H|T] = [the,[cat,sat],down].
H = the,
T = [[cat,sat],down].

?- [H|T] = [X+Y,x+y].
H = X+Y,
T = [x+y].
Let's assume we want to find out if an element is part of a list

- Prolog has the built-in predicate `member(X,Y)`, but define our own predicate
- We have to do this **recursively** in Prolog
  - There are no loops like in other programming languages
- **Recursion in Prolog** means that a predicate appears on the left- and the right-hand side of a rule
- For example, *an element is in a list* if it is
  - *the head* of the list or
  - *in the tail* of the list

\[
\text{is\_in}(X,[H|\_]) :\text{ X = H.}
\]
\[
\text{is\_in}(X,[\_|T]) :\text{ is\_in}(X,T).
\]

?- is\_in(d,[a,b,c,d,e,f]).
true
Step-by-step execution of the goal on the previous slide

\[
\text{is\_in}(X, [H|\_]) :- X = H.
\]

\[
\text{is\_in}(X, [\_|T]) :- \text{is\_in}(X, T).
\]

\[-\text{is\_in}(d, [a, b, c, d, e, f]).\]

true

<table>
<thead>
<tr>
<th>(Recursive) call</th>
<th>Rule 1</th>
<th>Rule 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{is_in}(d, [a, b, c, d, e, f])</td>
<td>(X = d, H = a \rightarrow \text{false})</td>
<td>(X = d, T = [b, c, d, e, f])</td>
</tr>
<tr>
<td>\text{is_in}(d, [b, c, d, e, f])</td>
<td>(X = d, H = b \rightarrow \text{false})</td>
<td>(X = d, T = [c, d, e, f])</td>
</tr>
<tr>
<td>\text{is_in}(d, [c, d, e, f])</td>
<td>(X = d, H = c \rightarrow \text{false})</td>
<td>(X = d, T = [d, e, f])</td>
</tr>
<tr>
<td>\text{is_in}(d, [d, e, f])</td>
<td>(X = d, H = d \rightarrow \text{true})</td>
<td></td>
</tr>
</tbody>
</table>
Does the \texttt{is\_in} predicate cover all cases?

Having a closer look at the recursion, we observe that there are actually two base cases for the \texttt{is\_in} predicate:

- Base case 1: element $X$ is the head of the list (first predicate)
- Base case 2: element $X$ is not in the list, then the list is empty

However, the second base case need not to be implemented, as none of the two predicates matches an empty list as second parameter.

However, we could add the following clause for the second base case:

\[
\texttt{is\_in}(X,L) :- L = [], \texttt{fail}.
\]

- Predicate \texttt{fail} returns false.

The termination of \texttt{is\_in} is guaranteed as in the recursive call the list passed to the goal is shorter, hence:

- eventually $X$ is encountered as first element of the list (base case 1),
- or the list is empty (base case 2)
Enumerating Elements and Generating Lists

- The predicate `is_in` can also be used to enumerate all elements of a list

  ```prolog
  ?- is_in(X,[1,2,a]).
  X = 1;
  X = 2;
  X = a;
  false
  ``

- We can even use it to generate lists

  ```prolog
  ?- is_in(a,L).
  L = [a|_G5033893];
  L = [_G5033893, a|_G5033898]
  ``

- `_G5033893, ...` are variables
Finding the last element of a list

\[
\text{last}(X,[X]). \\
\text{last}(X,[\_|T]) :- \text{last}(X,T).
\]

?- \text{last}(X,[\text{talk,of,the,town}]). \\
X = \text{town}
List Predicates – next_to/2

Checking for two consecutive elements of a list

next_to(X,Y,[X,Y|_]).
next_to(X,Y,[_|Z]) :- next_to(X,Y,Z).

?- next_to(X,Y,[talk,of,the,town]).
X = talk,
Y = of ;
X = of,
Y = the ;
X = the,
Y = town
List Predicates – append/3

- **append** is a very useful built-in predicate that can be used in a flexible way
- **Appending** two lists
  
  ```prolog
  append([], L, L).
  append([X|L1], L2, [X|L3]) :- append(L1, L2, L3).
  
  ?- append([i,like], [prolog], L).
  L = [i,like,prolog]
  
  **Generating** sublists
  
  ?- append(X, Y, [i,like,prolog]).
  X = [], Y = [i,like,prolog] ;
  X = [i], Y = [like,prolog] ;
  X = [i,like], Y = [prolog] ;
  X = [i,like,prolog], Y = [] ;
  no
  ``

- **Computing the difference** between lists
  
  ?- append([i], Y, [i,like,prolog]).
  Y = [like,prolog]
Prolog is very flexible with regard to the initialization of parameters
  e.g., in `append` any of two parameters can be initialized
We can easily implement `last`, `next_to`, and `is_in` using `append`

\[
\text{last}(E, \text{List}) :- \text{append}(_, [E], \text{List}).
\]

\[
\text{next_to}(X, Y, \text{List}) :- \text{append}(_, [X, Y | _], \text{List}).
\]

\[
\text{is_in}(X, \text{List}) :- \text{append}(_, [X | _], \text{List}).
\]
Strings in Prolog can be quite confusing if you come from another language.

- There are two "types" of strings.
- Strings enclosed in single quotes (') are atoms.

```prolog
?- 'hello' = S.
S = hello.

write('hello')
hello
true.
```

As this class of strings are atoms, they naturally cannot be manipulated.
Strings in Prolog/2

- Strings (or terms) written in **double quotes (""**) are immediately converted to a list of character codes (ASCII)

  ?- "hello" = L.
  write("hello").
  true.

- As of SWI-Prolog v7, only **back quoted** text is converted,
  - e.g., ‘text‘ is represented as [116, 101, 120, 116],
  whereas text enclosed in double quotes is read as a sequence of characters
Sometimes, single-quoted strings need to be converted to character lists, e.g., to print the first character of a string or to search for a substring.

This can be done by the `name` predicate.

```
?- name('hello', L).
```

SWI-Prolog provides a large number of built-in predicates for strings, e.g., concatenate strings, string length, conversion between terms and strings, etc.
The following predicate verifies, whether a string $S1$ is a prefix of another string $S2$.

```
prefix(S1, S2) :-
    atom(S1),
    atom(S2),
    name(S1, L1),
    name(S2, L2),
    append(L1, _, L2).
```

We can use it as follows:

?- prefix('hello', 'hello world').
true.

?- prefix("hello", "hello world").
false.
Outline

1 Structures

2 Equality, Matching and Arithmetic

3 Lists

4 Examples
The Towers of Hanoi/

- Goal: move a stack of \( n \) disks from one peg to another with the help of an auxiliary peg, where
  - Only one disk can be moved at a time
  - A move can only take the upper disk of a stack
  - A larger disk can never be placed on top of a smaller disk

- Legend: Somewhere in the surrounding of Hanoi, there is a monastery, where the monks have to perform this task assigned to them by God when the world was created with \( n = 64 \) golden disks. At the moment they complete their task, the world will collapse.
- The minimum number of moves to solve a Tower of Hanoi puzzle is \( 2^n - 1 \)
  - This is roughly 585 billion years for the monks, if they move the disks at a rate of one move per second
The Towers of Hanoi/2

- Recursive solution
  - Termination: there are no disks on peg A
  - Move \( n - 1 \) disks from peg A to C (notice the recursive move!)
  - Move disk \( n \) from peg A to B
  - Move \( n - 1 \) disks from peg C to B

- Predicate \text{Move}(N,A,B,C)\) moves \( n \) disks from peg A to peg B with the help of C

\[
\text{hanoi}(N) :- \text{move}(N,\text{peg}A,\text{peg}B,\text{peg}C).
\]

\[
\text{move}(1,A,B,_) :- \text{write}([\text{move},\text{disc},\text{from},A,\text{to},B]), \text{nl}.
\]

\[
\text{move}(N,A,B,C) :-
\begin{align*}
&N > 1, \\
&M \text{ is } N-1, \\
&\text{move}(M,A,C,B), \\
&\text{move}(1,A,B,_) , \\
&\text{move}(M,C,B,A).
\end{align*}
\]
Sudoku is a logic-based, combinatorial number-placement puzzle.

The objective is to fill a 9x9 grid with digits so that each column, each row, and each of the nine 3x3 sub-grids (boxes) contains all of the digits from 1 to 9.

A partially completed grid is given, which has a unique solution.
Sudoku/2

- We make the problem easier and consider a 4x4 sudoku, where rows, columns, and boxes have to be filled with a permutation of the numbers 1, ..., 4.
- We can model the Sudoku problem in Prolog using list permutations:
  - Each row must be a permutation of [1, 2, 3, 4]
  - Each column must be a permutation of [1, 2, 3, 4]
  - Each 2x2 box must be a permutation of [1, 2, 3, 4]
- The Sudoku is represented by a list of lists:
  - [[[X11, X12, X13, X14],
    [X21, X22, X23, X24],
    [X31, X32, X33, X34],
    [X41, X42, X43, X44]]}
sudoku([R1, R2, R3, R4]) :-
    R1 = [X11, X12, X13, X14],
    R2 = [X21, X22, X23, X24],
    R3 = [X31, X32, X33, X34],
    R4 = [X41, X42, X43, X44],
    % rows
    permutation([X11, X12, X13, X14], [1, 2, 3, 4]),
    permutation([X21, X22, X23, X24], [1, 2, 3, 4]),
    permutation([X31, X32, X33, X34], [1, 2, 3, 4]),
    permutation([X41, X42, X43, X44], [1, 2, 3, 4]),
    % cols
    permutation([X11, X21, X31, X41], [1, 2, 3, 4]),
    permutation([X12, X22, X32, X42], [1, 2, 3, 4]),
    permutation([X13, X23, X33, X43], [1, 2, 3, 4]),
    permutation([X14, X24, X34, X44], [1, 2, 3, 4]),
    % boxes
    permutation([X11, X12, X21, X22], [1, 2, 3, 4]),
    permutation([X13, X14, X23, X24], [1, 2, 3, 4]),
    permutation([X31, X32, X41, X42], [1, 2, 3, 4]),
    permutation([X33, X34, X43, X44], [1, 2, 3, 4]).
Binary Search Trees/1

- **Binary search trees** can be represented in Prolog by a recursive structure with three arguments \( \text{bst}(K, L, R) \), where
  - \( K \) is the key of the root
  - \( L \) is the left sub-tree
  - \( R \) is the right sub-tree

- The empty (null) tree is usually represented as the constant \text{nil}.

- Example tree with 6 nodes:
  
  \[
  \text{bst}(6, \\
  \quad \text{bst}(4, \\
  \quad \quad \text{bst}(2, \text{nil}, \text{nil}), \\
  \quad \quad \text{bst}(5, \text{nil}, \text{nil})), \\
  \quad \text{bst}(9, \\
  \quad \quad \text{bst}(7, \text{nil}, \text{nil}), \\
  \quad \quad \text{nil})
  \]

```prolog
\[
\text{bst}(6, \\
\quad \text{bst}(4, \\
\quad \quad \text{bst}(2, \text{nil}, \text{nil}), \\
\quad \quad \text{bst}(5, \text{nil}, \text{nil})), \\
\quad \text{bst}(9, \\
\quad \quad \text{bst}(7, \text{nil}, \text{nil}), \\
\quad \quad \text{nil})
\]
```
Binary Search Trees/2

- A basic operation is `bstmem(Tree, X)`, which succeeds if `X` is contained in `Tree`.

  
  \[
  \begin{align*}
  \text{bstmem}(\text{bst}(X,_,_), X). \\
  \text{bstmem}(\text{bst}(K,L,\_), X) & : \text{\quad} X < K, \\
  & \quad \text{bstmem}(L, X). \\
  \text{bstmem}(\text{bst}(K,\_,R), X) & : \text{\quad} X > K, \\
  & \quad \text{bstmem}(R, X)
  \end{align*}
  \]

- Examples:
  
  ?- bstmem(nil, 3).
  No
  ?- bstmem(bst(5,bst(8,nil,nil),nil),nil),8).
  Yes
Another basic operation is `inorder(Tree, L)` that succeeds if `L` contains the keys in `Tree` in inorder.

```
inorder(bst(K,L,R), List) :-
    inorder(L, LL),
    inorder(R, LR),
    append(LL, [K|LR], List).
inorder(nil, []).
```

Examples:

```
?- inorder(bst(5,bst(4,nil,nil),bst(8,nil,nil)),L).
L = [4,5,8]
```

Modify the above predicate to a predicate

- `preorder(Tree,L)` that succeeds if `L` contains the keys in `Tree` in preorder
- `postorder(Tree,L)` that succeeds if `L` contains the keys in `Tree` in postorder