

Data Structures and Algorithms

Exam

Prof. Dr. M. Böhlen

February 3, 2009

11:00 - 13:00

As auxiliary material you may use 1 A4 sheet with your notes. The exam consists of three exercises that have identical weight. It is important that you argue for your answers and that you present your solutions in a readable form. Write clearly in terms of math, language, and legibility. The clarity of your explanations impacts your grade. The exam lasts 2 hours. Write your name and ID on each solution sheet. All the best.

1 Koch Snowflake

The Koch snowflake is a fractal curve and is generated through a recursive process that starts with an equilateral triangle. Figure 1 shows the first four iterations of Koch's snowflake.

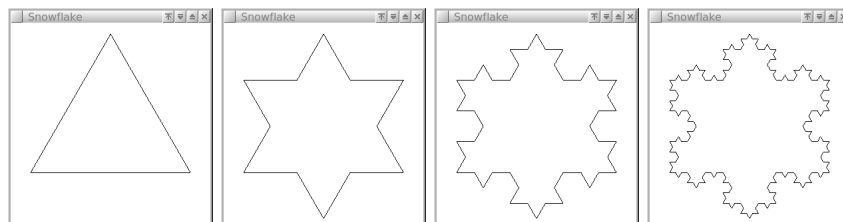


Figure 1: First 4 iterations of the Koch Snowflake

- 1a) Write a function `void drawKoch(..., int i)` that can be used to draw the i -th iteration of Koch's snowflake. Show how to call `drawKoch`. Assume a function `void lineTo(float dx, float dy)` that draws a line dx units to the right (or to the left, if negative) and dy units to the top (or to the bottom, if negative).

- 1b) State the exact running time $T(i)$ as a recurrence. Assume that the cost of a call of `lineTo` is b .
- 1c) Solve the recurrence and determine the simplest $f(i)$ such that $T(i) = \Theta(f(i))$.
- 1d) Determine the length of the line of the i th iteration of the snowflake.

2 Binary Search Trees

Consider the binary search tree T in Figure 2.

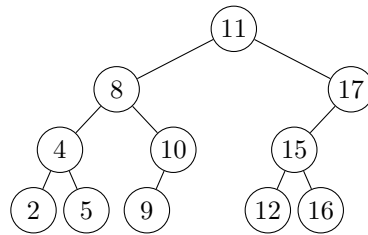


Figure 2: The Binary Search Tree T .

- 2a) Draw the tree after adding value 14 to tree T .
- 2b) Draw the tree after deleting value 8 from tree T .
- 2c) Draw the tree after left-rotating node 11 in tree T .
- 2d) Write an algorithm that determines if a binary tree is balanced. Assume the following definition of a balanced tree:

Definition 2.1. A binary tree is balanced iff the depths difference of any two leaves is at most 1.

- 2e) Write an algorithm that determines if a binary tree is balanced. Assume the following definition of a balanced tree:

Definition 2.2. A binary tree is balanced iff for each node it is true that the number of nodes in the left subtree and the number of nodes of the right subtree differ by at most 1.

- 2f) Compare Definition 2.1 and Definition 2.2. Which definition enforces more compact trees? Illustrate your answer by an example.

3 Longest Common Subsequence

- 3a) Write a recursive version of the LCS-Length algorithm that determines the length of the longest common subsequence of two strings. Show the details of calling your algorithm.
- 3b) What is the complexity of your algorithm in the best and worst case, respectively? State the recurrences and solve them.
- 3c) Fill in the table in Figure 3 with the result of dynamic programming version of LCS-Length algorithm for the strings 'SI VIS PACEM' and 'PARA BELLUM'.
- 3d) List all possible longest common subsequences.

	j	0	1	2	3	4	5	6	7	8	9	10	11	12
i		S_1	S	I		V	I	S		P	A	C	E	M
0	S_2													
1	P													
2	A													
3	R													
4	A													
5														
6	B													
7	E													
8	L													
9	L													
10	U													
11	M													

Figure 3: LCS-Length result.