Data Structures and Algorithms

Exercise 4

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Naming Conventions

- For each assignment create a folder *nn_yourlogin*
  - *nn* – is a number of the assignment (01, 02, . . . , 10)
  - *yourlogin* – is your login to the university network
- Name your files as task#.c or task#.pdf, where '#' is the number of the corresponding task in the assignment, for example:

```
/ 01_minnerebner
   task12.pdf
   task3.c
   task4.c
```

- Put your name and student ID in all files
- Send the solution folder via e-mail archived as *nn_yourlogin*.zip or *nn_yourlogin*.tar.gz file, for example:

```
01_minnerebner.tar.gz
```

Solve the following recurrences

a) $T(n) = 15T(n/4) + n^2$

b) $T(n) = bT(n-1) - (b-1)$

c) $T(n) = 6T(n/2) + n^2$

d) $T(n) = T(n-1) + \lg n$

e) $T(n) = 9^c T(n/3) + n^{2c}$

Use the QUICKSORT algorithm described in the lecture notes to sort the character array $A = \{'Q', 'U', 'I', 'C', 'K', 'S', 'O', 'R', 'T'\}$ in increasing order.

a) Write a C program that uses the Hoare partitioning. Modify in the code PARTITION from slides 31/Week4 by setting the pivot element (x) to the most left side.

b) Show the call stack and the state of the array after each call of PARTITION(A,l,r) for each partitioning method.

	Q	U	I	C	K	S	O	R	T
1									
2									
3									
4									
5									
6									
7									
8									

a) Analyze the runtime of your algorithm.
b) Illustrate worst and best case and give for each case the suitable example.
c) How would you modify the algorithm to sort into non-increasing order?

- Solve the following recurrence using:
  a) Master method,
  b) substitution method

$$T(n) = \begin{cases} 4 & \text{, if } n = 1 \\ 2T(n/2) + 5n + 1 & \text{, if } n > 1 \end{cases}$$

Solution Using Master Method

$$n^{\log_b a} = n \implies \text{Case 2}$$

$$f(n) = 5n + 1 \in \Theta(n) \implies T(n) \in \Theta(n \lg n)$$

Solution Using Substitution Method

$T(n) = 2T(\frac{n}{2}) + 5n + 1 = 2(2T(\frac{n}{4}) + 5\frac{n}{2} + 1) + 5n + 1 =$
$4T(\frac{n}{4}) + 5n + 4 + 5n + 2 + 5n + 1 =$
$4(2T(\frac{n}{8}) + 5\frac{n}{4} + 1) + 5n + 2 + 5n + 1 = 8T(\frac{n}{8}) + 5n + 4 + 5n + 2 + 5n + 1$

$2^i T(\frac{n}{2^i}) + i \cdot 5n + 2^i - 1$

$\frac{n}{2^i} = 1$ when $i = \lg n$

$2^{\lg n} \cdot 4 + \lg n \cdot 5n + 2^{\lg n} - 1 = 5n \cdot \lg n + 6n - 1$

Our Guess

$T(n) \leq cn \lg n$

Proof with Induction

- $T(\frac{n}{2}) \leq c\frac{n}{2} \lg \frac{n}{2}$

? $2 \cdot c\frac{n}{2} \lg \frac{n}{2} + 5n + 1 \leq cn \lg n$

Hints

Task 2: Quicksort. TODO add example