Modeling and Querying Current Movement: Location Updates

Background

Context: Objects moving around currently, MOST, FTL.

Movements described by dynamic attributes / motion vectors.

Objects need to transmit location updates to keep the **deviation** to the **database location** bounded.

How often and when should updates be sent? Tradeoff between

- cost of update
- imprecise knowledge about location

Various possible **location update policies**:

- periodically
- **dead-reckoning**: update whenever deviation exceeds threshold

"Current location is (x, y) with a deviation of at most 100 meters."

Two forms of imprecision that both lead to costs in terms of incorrect decision making

- deviation
- uncertainty

The Information Cost of a Trip

Find a cost model that allows one to judge the quality of location update policies. Three kinds of cost:

- update cost
- deviation cost
- uncertainty cost

Deviation Cost

Depends on

- the size of the deviation
- the duration (e.g. assume one query per time unit).

Let $d(t)$ denote the deviation over time. Penalty for each unit of deviation during one unit of time: constant 1. The **deviation cost function** is

$COST_d(t_1, t_2) = \int_{t_1}^{t_2} d(t)\, dt$ Called the **uniform** deviation cost function. Other example of a deviation cost function: **step** deviation cost function.

Update Cost

The cost for transmitting a single update: C_1

Ratio between update cost and deviation cost

$$\frac{\text{update cost}}{\text{deviation cost}} = \frac{C_1}{1} = C_1$$

Uncertainty Cost

Penalty for each unit of uncertainty during one unit of time: C_2

Ratio between uncertainty cost and deviation cost

$$\frac{\text{uncertainty cost}}{\text{deviation cost}} = \frac{C_2}{1} = C_2$$

Let $u(t)$ be the uncertainty over time. The **uncertainty cost function** is

$$COST_u(t_1, t_2) = \int_{t_1}^{t_2} C_2\, u(t)\, dt$$

Information cost of a trip between two updates at t_1 and t_2 within the half-open interval $[t_1, t_2[$:

$$COST_I([t_1,t_2[) = C_1 + COST_d([t_1,t_2[) + COST_u([t_1,t_2[)$$

Assume trip start at time 0, then updates are sent at times t_1, ..., t_n, trip ends at time t_{n+1}. **Total information cost of a trip**:

$$COST_I([0,t_{n+1}[) = COST_d([0,t_1[) + COST_u([0,t_1[) + \sum_{i=1}^{n} COST_I([t_i,t_{i+1}[)$$

So far a general model regardless of location update policy.

Cost-Based Optimization for Dead-Reckoning Policies

Back to the question: How often and when should updates be sent?

Dead-reckoning: essential feature a threshold *th* on the deviation. Object sends update when threshold is reached.

Update consists of

- current location
- predicted speed
- new threshold K

How should K be set to minimize the information cost?

General strategy:

- Moving object m predicts the future behaviour of the deviation
- Based on this the average cost per time unit between now and the next update is determined as a function $f(K)$ of the threshold K
- K is set to minimize $f(K)$

Simple model for deviation: grows **linearly** with time.

$$d(t) = a(t - t_1)$$

We will optimize K for this model.

Insert assumptions into the general cost model:

$$\begin{aligned} COST_I([t_1,t_2[) &= C_1 + COST_d([t_1,t_2[) + COST_u([t_1,t_2[) \\ &= C_1 + \int_{t_1}^{t_2} a(t-t_1)\,dt + \int_{t_1}^{t_2} C_2 K\,dt \\ &= C_1 + \frac{1}{2}a(t_2-t_1)^2 + C_2K(t_2-t_1) \end{aligned}$$

The **average cost per time unit** is $f(t_2) =$

$$\frac{C_1 + 0.5a(t_2-t_1)^2 + C_2K(t_2-t_1)}{t_2-t_1} = \frac{C_1}{t_2-t_1} + \frac{1}{2}a(t_2-t_1) + C_2K$$

$$f(t_2) = \frac{C_1}{t_2 - t_1} + \frac{1}{2}a(t_2 - t_1) + C_2 K$$

We know that the message is sent at time t_2 when threshold K is reached, hence

$$K = a(t_2 - t_1)$$
$$(t_2 - t_1) = K / a$$

Rewrite average cost per time unit as $f(K)$:

$$\begin{aligned} f(K) &= \frac{C_1 a}{K} + \frac{1}{2}a\frac{K}{a} + C_2 K \\ &= C_1 a\, K^{-1} + (C_2 + \frac{1}{2})K \end{aligned}$$

Minimize $f(K)$:

$$f'(K) = \frac{-C_1 a}{K^2} + C_2 + \frac{1}{2} = 0$$

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$$C_2 + 1/2 = \frac{C_1 a}{K^2}$$

$$K^2 = \frac{C_1 a}{C_2 + 1/2} = \frac{2C_1 a}{2C_2 + 1}$$

$$K = \sqrt{\frac{2C_1 a}{2C_2 + 1}}$$

Remember what this means:

- moving object *m* predicts at time t_1 a linear growth of its deviation (with factor *a*)
- then sends this value *K* as the new threshold within the location update message

A Variation: Continually Decreasing Threshold

Motivation: Disconnection detection. Object does not send updates due to disconnection.

Idea: If the threshold decreases, object has to send an update after some time.

Mathematically suitable strategy: **fractionally decreasing threshold**

Time unit	Threshold
1	K
2	K/2
3	K/3
4	K/4
...	...

Again optimize information cost by choosing (initial) threshold K.

Still assume $d(t) = a(t - t_1)$.

What is the information cost in this case?

$$COST_I([t_1,t_2[) \quad = C_1 + COST_d([t_1,t_2[) + COST_u([t_1,t_2[)$$

$$= C_1 + \frac{a(t_2-t_1)^2}{2} + C_2K(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{t_2-t_1})$$

The deviation at time t_2 is $a(t_2 - t_1)$.

The threshold at time t_2 is $K / (t_2 - t_1)$.

Therefore, $K / (t_2 - t_1) = a(t_2 - t_1)$ and $K = a\,(t_2 - t_1)^2$.

So,

$$\sqrt{K/a} = t_2 - t_1$$

So, the function is $f(K)$ $= \dfrac{C_1 + K/2 + C_2K(1+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{\sqrt{K/a}})}{\sqrt{K/a}}$

So the function is

$$f(K) = \frac{C_1 + K/2 + C_2 K\left(1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{\sqrt{K/a}}\right)}{\sqrt{K/a}}$$

Minimize *f*(*K*):

Need a closed form. The *n*-th harmonic number is

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n} \approx \ln n + 1$$

Approximate *f*(*K*) by *g*(*K*)

$$g(K) = \frac{C_1 + K/2 + C_2 K(\ln\sqrt{K/a} + 1)}{\sqrt{K/a}}$$

Derivative of $g(K)$ is 0 when K is the solution to the equation:

$$\ln k = \frac{d_1}{K} - d_2$$

with constants $d_1 = \frac{2C_1}{C_2}$ and $d_2 = \frac{1}{C_1} + 4 - \ln a$

Can be solved numerically by the **Newton-Raphson** method:

given an equation $f(x) = 0$, compute x_0, x_1, x_2, ... by

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

Dead-Reckoning Location Update Policies

Discuss three policies:

- **speed dead-reckoning** policy (*sdr*)
- **adaptive dead-reckoning** policy (*adr*)
- **disconnection detection dead-reckoning** policy (*dtdr*)

Speed Dead-Reckoning

- At the beginning of trip, determine threshold *K* in an ad-hoc manner
- *K* remains unchanged for the whole trip
- Update: current location and current speed

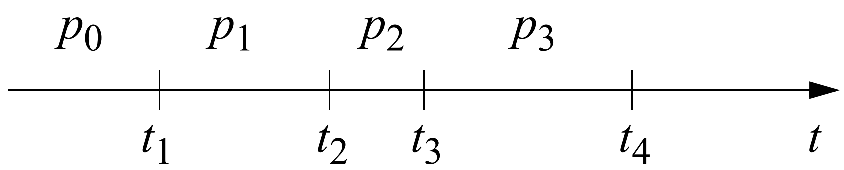
  (variations:

  - average speed since last update
  - average speed since beginning of the trip
  - speed predicted on terrain knowledge)

Adaptive Dead-Reckoning

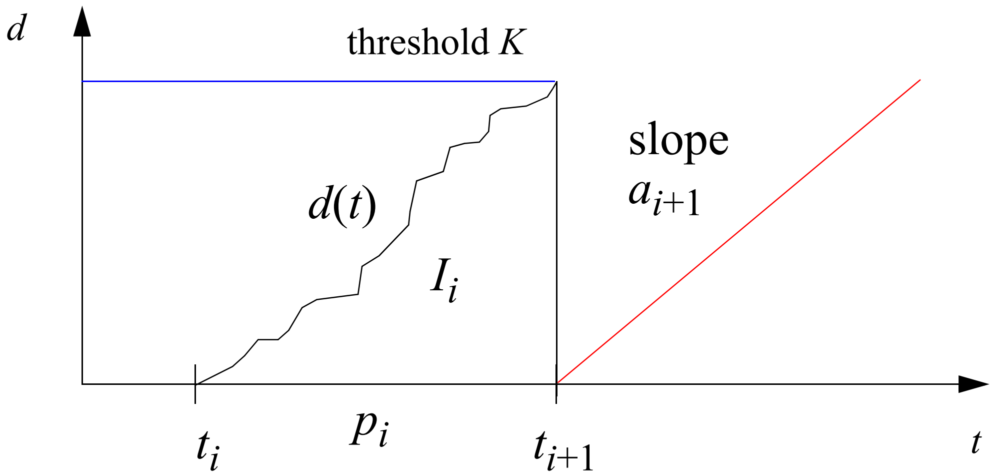
- At the beginning of trip, determine threshold *K* in an ad-hoc manner
- Whenever the threshold is reached, determine a new threshold based on **observing the deviation** during the period since the last update, and transmit it in the update
- Update consists of current location, current speed, and new threshold

Determining the new threshold



$p_i = t_{i+1} - t_i$

Assuming a linear deviation function:



$$I_i = \frac{1}{2} a\, p_i^2$$

$$a = \frac{2 I_i}{p_i^2}$$

Example:

Assume that at the beginning of the trip, the moving object *m* sends the following update: *loc.speed* = *0.2* miles/min, *loc.uncertainty* = *0.5* miles. After 4 min, the deviation exceeds the threshold of 0.5 miles. The deviation cost between 0 and 4 min is $I_1 = 1$, the update cost is $C_1 = 8$, and the deviation cost factor is $C_2 = 1$.

We compute the new value for *loc.uncertainty* according to the adaptive dead-reckoning location update policy:

$$a_1 = 2I_1 / t_1^2 = 2 * 1/4^2 = 0.125$$

$$loc.uncertainty = \sqrt{(2a_1C_1)/(2C_2+1)} =$$

$$\sqrt{(2 \cdot 0{,}125 \cdot 8)/(2 \cdot 1 + 1)} = 0.82$$

Disconnection Detection Dead-Reckoning

- At the beginning of trip, determine threshold th_0 in an ad-hoc manner
- Moving object *m* sets threshold to a fractionally decreasing value starting with $K = th_0$. Hence it is K for time unit 1, $K/2$ for time unit 2, and so forth.
- Moving object *m* starts tracking the deviation.
- Let $a_1 = \frac{2I_0}{p_0^2}$. Use a_1 to compute the next threshold th_2, using the formula for $g(K)$ above.
- Update consists of current location, current speed, and new threshold.
- Repeat until end of trip.

Example:

Assume that at the beginning of the trip, the moving object *m* sends the following update: *loc.speed* = *0.2* miles/min, *loc.uncertainty* = *0.5* miles. After 2 min, the deviation exceeds the threshold of 0.5/2 = 0.25 miles. The deviation cost between 0 and 2 min is $I_1 = 0.5$, the update cost is $C_1 = 8$, and the deviation cost factor is $C_2 = 1$.

We compute the new value for *loc.uncertainty* according to the disconnection detection location update policy:

$a_1 = 2I_1 / t_1^2 = 2 * 0.5/2^2 = 0.25$

The new value for *loc.uncertainty* is the solution to the equation

$$\ln k = \frac{d_1}{K} - d_2 \text{ with constants } d_1 = \frac{2C_1}{C_2} \text{ and } d_2 = \frac{1}{C_1} + 4 - \ln a$$

This leads to $ln(K) = 16/K - 1 - 4 + ln(0.25) = 16/K - 6.39$.

By using the Newton-Raphson method, we obtain $K = 2.226 = loc.uncertainty$.