Review of Approximation Algorithms

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It is recommended that you solve the tasks without auxiliary material to mirror the setting at the exam.

Task 1:

Let $G = (V, E)$ be a complete weighted graph and let $w : E \Rightarrow \mathbb{R}$ be a weight function that satisfies the triangular inequality.

1. Describe an approximation of the TSP problem. How do you quantify the quality of your approximation? Prove the approximation ratio of the algorithm.
2. Assume G is embedded in the Euclidean space and the edge weights are given by the Euclidean distance. Does an optimal TSP tour have crossing edges?

Task 2:

Assume n objects. Object o_i has size s_i. For all objects $0 < s_i < 1$. The task is to place all objects in a minimal number of bins of capacity 1.

The first-fit heuristics takes each object in turn and places it in the first bin that can accommodate it. Give an approximation algorithm for bin packing.

Prove that the approximation ratio of the first-fit heuristic is 2. Hints:

- relate the optimal number of bins and the number of bins returned by the first-fit algorithm.
- show that the first fit strategy leaves at most one bin less than half full.

Task 3:

A heuristic to solve the vertex cover problem is to repeatedly select the vertex of highest degree and remove all incident edges.

The bipartite graph in Figure 1 illustrates a worst case scenario for this algorithm. The graph is defined as follows:

- $|L| = m$
- $R = R_1 \cup ... \cup R_m$
- $|R_i| = \lfloor L/i \rfloor$
- each vertex in R_i has edges to i vertexes in L
- all edges from vertices in R_i lead to different vertexes in L

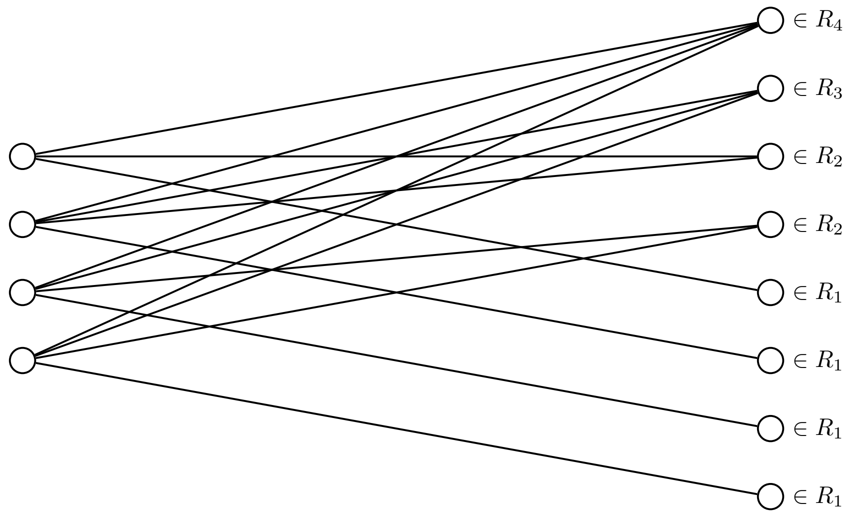

Figure 1: Example Bipartite Graph

Provide an algorithm that implements the proposed heuristics and determine its complexity.