

Advanced Algorithms

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Lab 5 – Solution of assignments

Assignment 04

Exercise 5.6 page 149 DPV

- Let $G = (V, E)$ be an undirected graph
- Prove that if all its edge weights are distinct, then it has a unique minimum spanning tree



Assignment 04

Exercise 5.6 page 149 DPV

Solution

- Suppose the graph has two different MSTs T_1 and T_2
- Let e be the lightest edge which is present in exactly one of the trees (there must be some such edge since the trees must differ in at least one edge)
- Without loss of generality, say $e \in T_1$
- Then adding e to T_2 gives a cycle
- This cycle must contain an edge e' which is (strictly) heavier than e , since all lighter edges are also present in T_1 , where e does not induce a cycle
- Then adding e to T_2 and removing e' gives a (strictly) better spanning tree than T_2 which is a contradiction



Assignment 04 (cont.)



Exercise 5.15 page 150 DPV

- We use Huffman's algorithm to obtain an encoding of alphabet $\{a, b, c\}$ with frequencies f_a, f_b, f_c
- In each of the following cases, either give an example of frequencies (f_a, f_b, f_c) that would yield the specified code and say if the code is optimal, or explain why the code cannot possibly be obtained (no matter what the frequencies are)
 - a. Code: $\{0, 10, 11\}$
 - b. Code: $\{0, 1, 00\}$
 - c. Code: $\{10, 01, 00\}$

Assignment 04 (cont.)



Exercise 5.15 page 150 DPV

Solution

- a. $(f_a, f_b, f_c) = (2/3, 1/6, 1/6)$ gives the code $\{0, 10, 11\}$
- b. This encoding is not possible, since the code for a (0), is a prefix of the code for c (00)
- c. This code is not optimal since $\{1, 01, 00\}$ gives a shorter encoding. Also, it does not correspond to a full binary tree and hence cannot be produced by the Huffman algorithm

Assignment 04 (cont.)



Exercise 7.2 page 223 DPV

- Duckwheat is produced in Kansas and Mexico and consumed in New York and California
- Kansas produces 15 shnupells of duckwheat and Mexico 8.
- Meanwhile, New York consumes 10 shnupells and California 13
- The transportation costs per shnupell are \$4 from Mexico to New York, \$1 from Mexico to California, \$2 from Kansas to New York, and \$3 and from Kansas to California.
- Write a linear program that decides the amounts of duckwheat (in shnupells and fractions of a shnupell) to be transported from each producer to each consumer, so as to minimize the overall transportation cost

Assignment 04 (cont.)



Exercise 7.2 page 223 DPV

Solution

- We will use concatenation of the first letters of the two cities for the shnupells of duckwheat transported between those cities (i.e. MN for the quantity of shnupells between Mexico and New York etc.)
- The linear program will be the following:

$$\text{minimize } 4MN + MC + 2KN + 3KC$$

$$MN + KN = 10$$

$$MC + KC = 13$$

$$MN + MC = 8$$

$$KN + KC = 15$$

$$MN, MC, KN, KC \geq 0$$

Assignment 04 (cont.)



Exercise 7.3 page 223 DPV

- A cargo plane can carry a maximum weight of 100 tons and a maximum volume of 60 cubic meters. There are three materials to be transported, and the cargo company may choose to carry any amount of each, up to the maximum available limits given below
 - Material 1 has density 2 tons/cubic meter, maximum available amount 40 cubic meters, and revenue \$1,000 per cubic meter
 - Material 2 has density 1 ton/cubic meter, maximum available amount 30 cubic meters, and revenue \$1,200 per cubic meter
 - Material 3 has density 3 tons/cubic meter, maximum available amount 20 cubic meters, and revenue \$12,000 per cubic meter
- Write a linear program that optimizes revenue within the constraints

Assignment 04 (cont.)



Exercise 7.3 page 223 DPV

Solution

- Let q_i denote the quantity (in cubic meters) of material i
- The linear program will be the following:

$$\begin{aligned}
 &\text{maximize } 1000q_1 + 1200q_2 + 12000q_3 \\
 &2q_1 + q_2 + 3q_3 \leq 100 \\
 &q_1 + q_2 + q_3 \leq 60 \\
 &q_1 \leq 40 \\
 &q_2 \leq 30 \\
 &q_3 \leq 20 \\
 &q_1, q_2, q_3 \geq 0
 \end{aligned}$$