Continuous Imputation of Missing Values in Streams of Pattern-Determining Time Series

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Freie Universität Bozen
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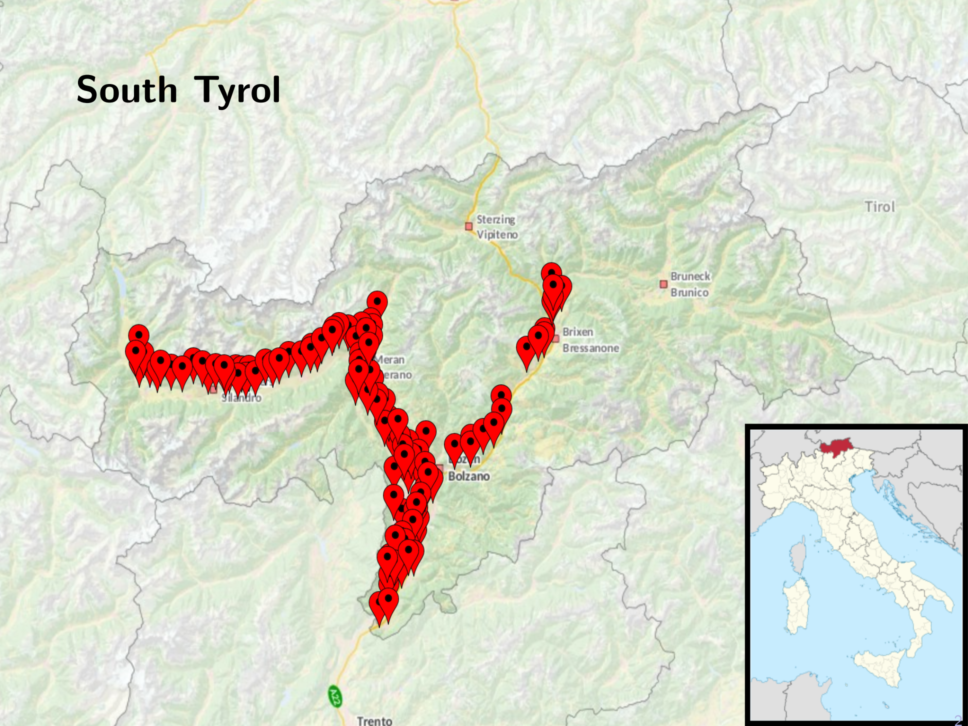
South Tyrol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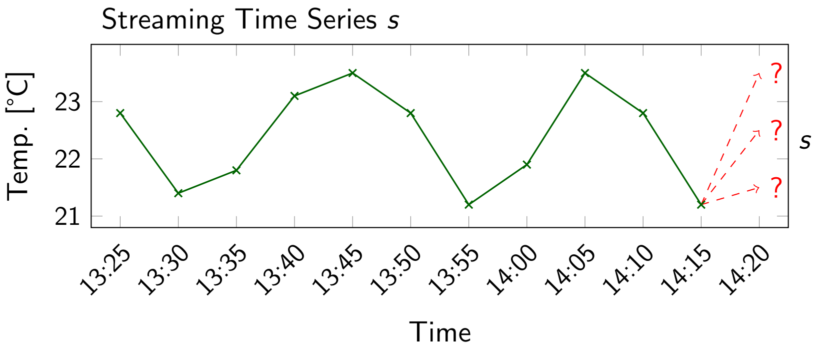
Overview

Problem. Streaming time series often have **missing values**, e.g. due to sensor failures or transmission delays!

Goal. Accurately **impute** (i.e. recover) the latest measurement by exploiting the **correlation** among streams.

Challenge. Streaming time series are often **non-linearly correlated**, e.g. due to **phase shifts**.

Example



- The latest value at time 14:20 is **missing** and needs to be **imputed** (i.e. recovered).

Approach

Top-k Case Matching (TKCM)

Intuition. Impute a missing value in time series s with past values from s when a set of correlated **reference time series** exhibited similar **patterns**.

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1. Draw query pattern over most recent values

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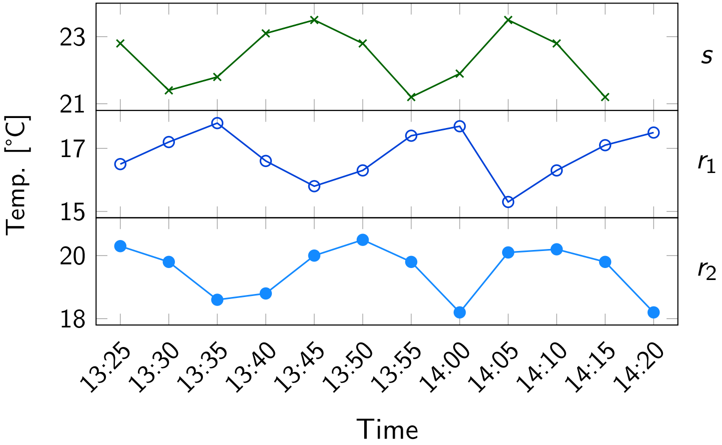
Imputation Steps:

1. Draw query pattern over most recent values
2. Find k most similar non-overlapping patterns
3. Impute missing value using the k most-similar patterns

Applying TKCM



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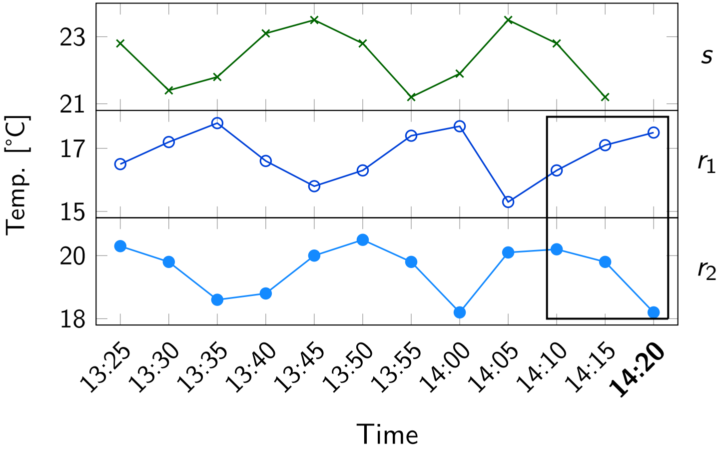
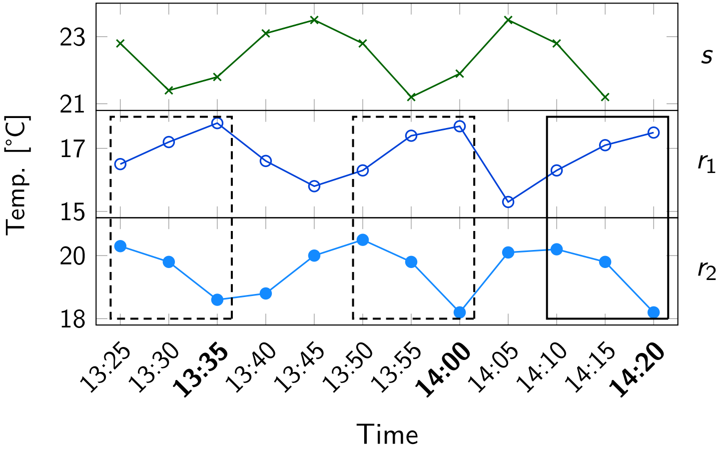


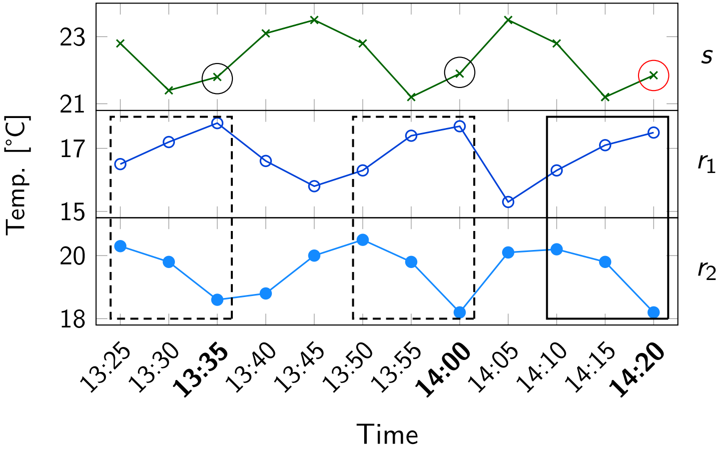
1. Define query pattern $P(14{:}20)$ over $d = 2$ reference time series $\{r_1, r_2\}$ in a time frame of $l = 10$ minutes

Applying TKCM



2. The $k = 2$ most similar non-overlapping patterns are $P(14{:}00)$ and $P(13{:}35)$

Applying TKCM



3. Missing value is imputed as
$\hat{s}(14{:}20) = \frac{1}{2}(s(14{:}00) + s(13{:}35)) = 21.85°C$

Query Pattern

Pattern length $l = 3$

16.3	17.1	17.5	r_1
20.2	19.9	18.2	r_2
14:10	14:15	14:20	

reference time series $d = 2$

- With $l > 1$, TKCM takes the temporal context into account and captures how time series change over time
- Pattern length l is important to deal with **non-linear correlations**

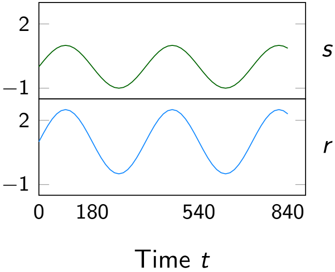
Related Work

1. **Centroid Decomposition (CD)**
   - M. Khayati, M. H. Böhlen, and J. Gamper. Memory-efficient centroid decomposition for long time series. ICDE 2014
   - Singular Value Decomposition (SVD) that expects **linear correlations**
2. **SPIRIT**
   - S. Papadimitriou, J. Sun, and C. Faloutsos. Streaming pattern discovery in multiple time-series. VLDB 2005
   - Principal Component Analysis (PCA) that expects **linear correlations**
3. **MUSCLES**
   - B. Yi, N. Sidiropoulos, T. Johnson, H. V. Jagadish, C. Faloutsos, and A. Biliris. Online data mining for co-evolving time sequences. ICDE 2000
   - Multi-variate linear regression that expects **linear correlations**

Linear vs. Non-Linear Correlations

Linear Correlations

$s(t) = \text{sind}(t)$
$r(t) = 1.5 \times \text{sind}(t) + 1$



- Time series s and r have different **amplitude** and **offset**

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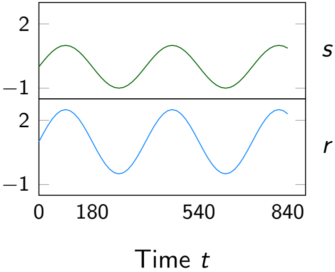

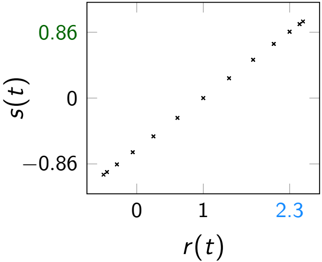

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- They are **linearly correlated** and their Pearson Correlation Coefficient is 1!

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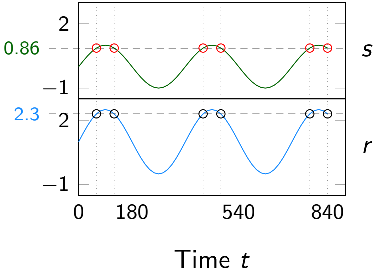

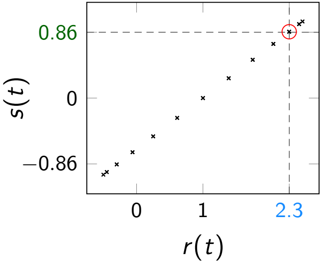

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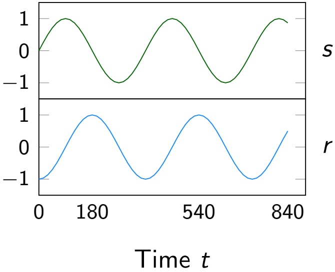
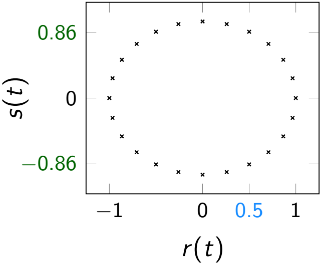
Non-Linear Correlations

$$s(t) = \text{sind}(t)$$
$$r(t) = \text{sind}(t - 90)$$





- Time series s and r are **phase-shifted** by 90 degrees
- They are **non-linearly correlated** and their Pearson Correlation Coefficient is 0!

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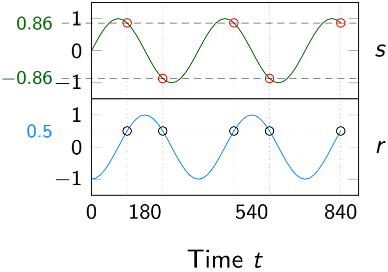

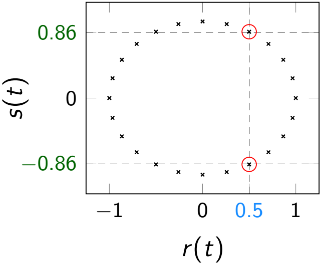

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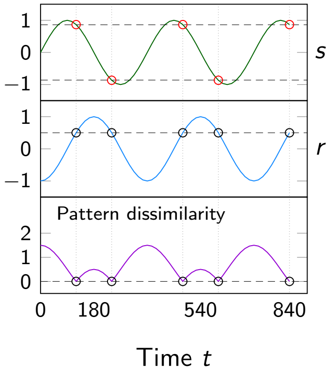
Pattern Length l and Non-Linear Correlations

Pattern length $l = 1$



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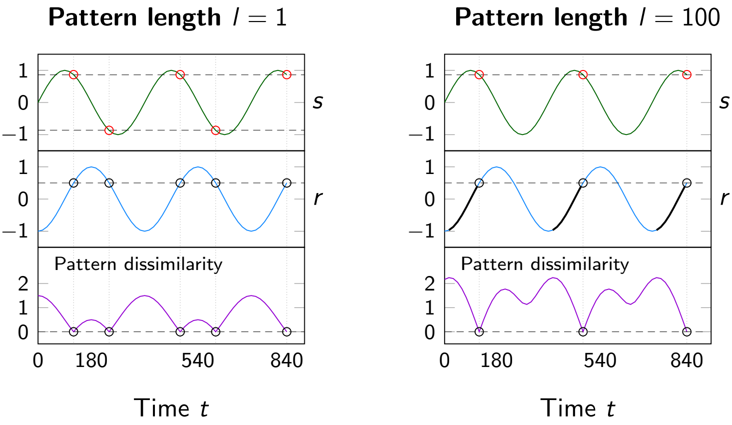
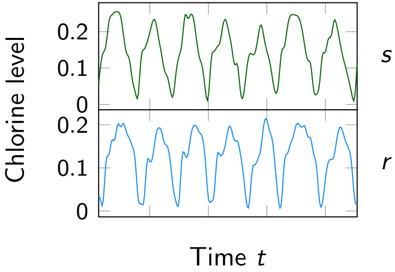
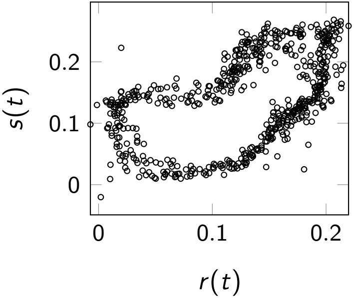


- With $l > 1$ there are less patterns with pattern dissimilarity 0

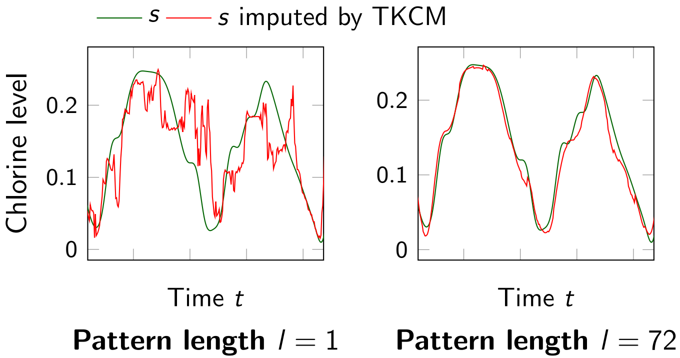
Chlorine Dataset

- Chlorine dataset is **phase-shifted** and hence **non-linearly correlated**





Importance of Pattern Length l



- A larger pattern length decreases the oscillation in the imputed time series

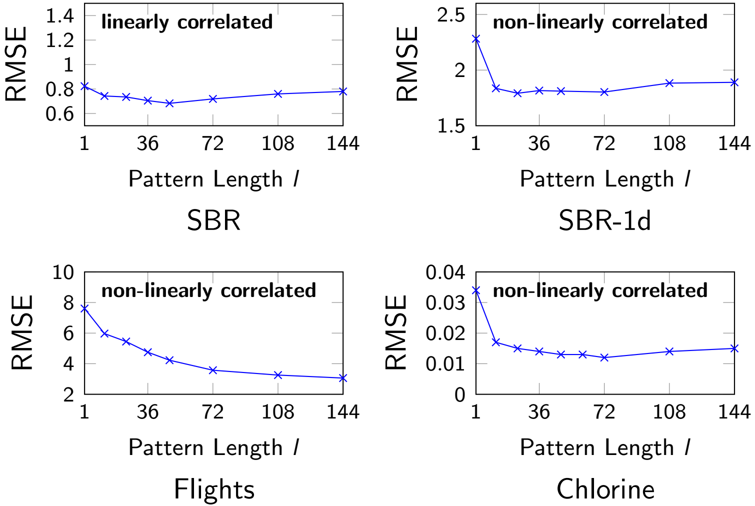
Experiments

Datasets

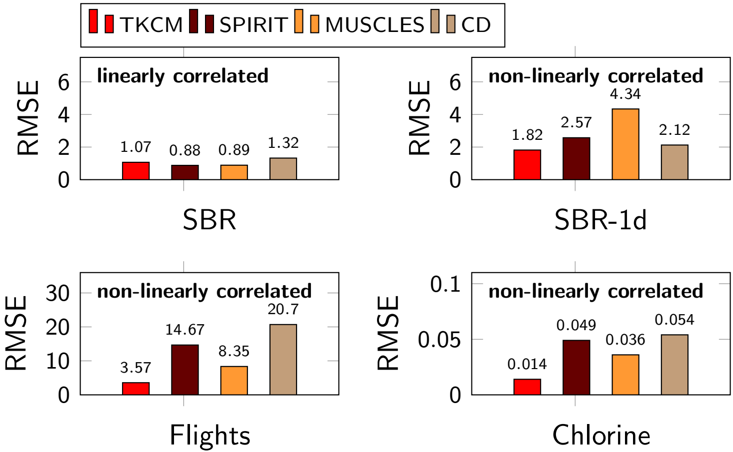
We use 4 datasets:

1. SBR
   - 130 meteorological time series from South Tyrol
   - **linearly correlated**
2. SBR-1d
   - SBR dataset shifted up to 1 day
   - **non-linearly correlated**
3. Flights
   - 8 time series
   - **non-linearly correlated**
4. Chlorine
   - 166 time series
   - **non-linearly correlated**

Pattern Length l



Comparison



- TKCM is more accurate on all **non-linearly correlated** datasets (SBR-1d, Flights, and Chlorine).

Conclusion & Future Work

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- TKCM imputes the current missing value in a stream using reference time series
- TKCM exploits **linear** and **non-linear correlations** among time series

Future work

- Automatically choose reference time series
- Improve efficiency of TKCM by pruning candidate patterns

Thanks!